ae

II 1

Zeszyty zabrane
z zakładu
(z Gołębiej 13)
przeważnie z zakresu
termodynamiki

Nernst Theor. Allg. Chemie [illegible]

[illegible] 202 E, 6

[illegible] Boltzmann Wied. Ann. 57 p. 783

[illegible]

Maxwell Physic. Papers 1

Helmholtz { Wied. Ann. 7 p. 337 (1879)
= Ges. Abhandl. I p 855

Dorn Wied. A. 9 p. 513
10 p 46 (1880)

} Strömungsströme etc.

Hardy Journ. ph. Chem. 4 p 335 (1900)
Z. ph. Ch. 33 p 385

Für wirkliche x:

$$x! = x^x e^{-x} \sqrt{2\pi x}$$

Jeśli gaz ma ciśnienie p i temp. θ, jakie prawdopod. zboczenia z normalnej gęstości $\bar\rho = \frac{p}{R\theta}$ n.p. w jednym mm³ ?

N.p. prawdopod. gęstości $\alpha\bar\rho$?

$\alpha\bar\rho$ będzie jeśli w owym mm³ (ω) będą $\alpha\bar N$ cząsteczek

Prawdop. ażeby pewna drobina znajdowała się w owem $\omega = W = \frac{\omega}{v}$

ażeby (równocześnie) $\nu = \alpha\frac{\omega}{v}n$ się znajdowały $= \left(\frac{\omega}{v}\right)^{\nu} = \left(\frac{\omega}{v}\right)^{\alpha n\frac{\omega}{v}}$

Jeśli nic nie wiadomo o indywidualności tych drobin, więc jakakolwiek kombinacja możliwa

to należy owe wyrażenie pomnożyć przez ilość możliwych ~~permutacji~~ kombinacyj

rzędu $\alpha n\frac{\omega}{v}$ (bez powt.) z n elementów $= \binom{n}{\alpha n\frac{\omega}{v}}$

więc w całem:

$$\boxed{\left(\frac{\omega}{v}\right)^{\alpha n\frac{\omega}{v}}\cdot\frac{n(n-1)(n-2)\cdots(n-\alpha n\frac{\omega}{v}+1)}{1\cdot 2\cdot 3\cdots\alpha n\frac{\omega}{v}}}\ (n-\alpha n\tfrac{\omega}{v})(n-\alpha n\tfrac{\omega}{v}-1)\cdots$$

$$\binom{n}{\alpha n\frac{\omega}{v}} = \frac{n!}{(\alpha n\frac{\omega}{v})!\,(n-\alpha\frac{n\omega}{v})!} = \frac{\Gamma(n)}{\Gamma(\alpha n\frac{\omega}{v})\cdot\Gamma(n-\frac{\alpha n\omega}{v})}$$

$$\frac{\alpha n\frac{\omega}{v}\cdots 3.2.1}{(n-\alpha n\frac{\omega}{v}+1)\cdots(n-\alpha n\frac{\omega}{v}+1+\alpha n\frac{\omega}{v}-1)} = \frac{\Gamma(n-\alpha n\frac{\omega}{v})}{(\alpha n\frac{\omega}{v})^{n-\alpha n\frac{\omega}{v}}}$$

$$W = \frac{\left(\frac{\omega}{v}\right)^{\alpha n\frac{\omega}{v}}(\alpha n\frac{\omega}{v})^{n}}{(\alpha n\frac{\omega}{v})^{\alpha n\frac{\omega}{v}}\,\Gamma(n-\alpha n\frac{\omega}{v})} = \frac{(\alpha n\frac{\omega}{v})^{n}}{(\alpha n)^{\alpha n\frac{\omega}{v}}\,\Gamma(n-\alpha n\frac{\omega}{v})}$$

$$W = \frac{\omega}{v}^{\alpha n \frac{\omega}{v}} \frac{n^{\alpha n \frac{\omega}{v}}}{\left(\alpha n \frac{\omega}{v}\right)^{\alpha n \frac{\omega}{v}} e^{-\alpha n \frac{\omega}{v}} \sqrt{2\pi \alpha n \frac{\omega}{v}}} = \left(\frac{e}{\alpha}\right)^{\alpha n \frac{\omega}{v}} \frac{1}{\sqrt{2\pi \alpha n \frac{\omega}{v}}}$$

więc oznaczając liczbę cząstek --

$$W = \left(\frac{e}{\alpha}\right)^{\nu} \frac{1}{\sqrt{2\pi\nu}}$$

$$\frac{\partial W}{\partial \nu} = \left(\frac{e}{\alpha}\right)^{\nu} \left[\frac{1}{\sqrt{2\pi\nu}} \lg \frac{e}{\alpha} - \frac{\frac{1}{2}}{\sqrt{(2\pi\nu)^3}}\right] = 0$$

$$\cancel{\lg \frac{e}{\alpha} = \frac{1}{4\pi\nu} = 1 - \lg \alpha}$$

$$\cancel{\lg \alpha = 1 - \frac{1}{4\pi\nu}}$$

$$\cancel{\int_{\alpha=0}^{\infty} W d\alpha = \frac{e^{\nu}}{\sqrt{2\pi\nu}} \int \frac{d\alpha}{\alpha^{\nu}}} = -\frac{e^{\nu}}{\sqrt{2\pi\nu}} \frac{1}{\alpha^{\nu+1}}\Big|_0^{\infty}$$

$$\lg \frac{e}{a} = \frac{1}{4\pi\nu} + 1 = 1 - \lg a$$

~~jeśli wziąć to~~ $\lg \alpha = -\frac{1}{4\pi\nu}$

$$\alpha = e^{-\frac{1}{4\pi\nu}} \neq 1$$

najprawdop. $\alpha = 1$

Równie prawdop. jakaś bądź z $n!$ Permutacyj $= \bar{W}_1$

z tych będzie $\dfrac{n!}{\binom{n}{\alpha n \frac{\omega}{v}}}$ takich przy których dane elementy znajdują się w drążkach

a tego prawdop. $\bar{W}_1 \dfrac{n!}{\binom{n}{\alpha n \frac{\omega}{v}}}$ równa się zatem $\left(\frac{\omega}{v}\right)^{\nu}$

z czego otrzymuje się prawdop. $\bar{W}_1$ dla jednej z pierwotnych możliwych Permutacyj:

$$\bar{W}_1 = \left(\frac{\omega}{v}\right)^{\nu} \frac{\binom{n}{\alpha n \frac{\omega}{v}}}{n!} = \left(\frac{\omega}{v}\right)^{\nu} \frac{1}{\left(\alpha n \frac{\omega}{v}\right)! \left(n - \alpha \frac{n\omega}{v}\right)!}$$

$n\frac{\omega}{v} = m$

$$1 = \int_0^\infty \bar{W}\, d\alpha = \int_0^\infty \left(\frac{e}{\alpha}\right)^{\alpha m} \frac{1}{\sqrt{2\pi \alpha m}}\, d\alpha \qquad \alpha m = \nu, \quad \alpha = \frac{\nu}{m}$$

$$= \frac{1}{m}\int_0^\infty \left(\frac{e m}{\nu}\right)^{\nu} \frac{1}{\sqrt{2\pi\nu}}\, d\nu$$

$\left(\frac{\omega}{v}\right)^{\nu}$ prawdop. po włożeniu pierwszych ν cząstek

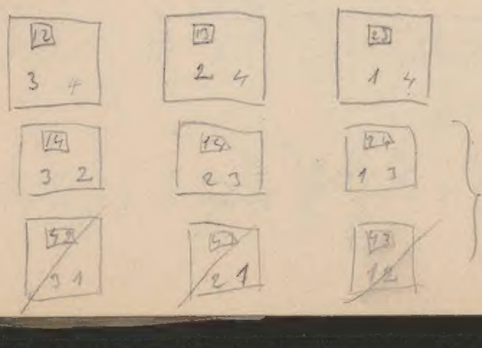
wkładamy $\nu+1$ q ; prawdop. że ona ~~nie~~ będzie poza ω: $\frac{v-\omega}{v}$

teraz nam nie chodzi o identyczność jej lecz jakaś bądź

więc prawdop. żeby jakaś kolwiek z ν była wewnątrz zewnątrz : $\left(\frac{\omega}{v}\right)^{\nu} \frac{v-\omega}{v} \cdot (\nu+1)$

wkładamy ν+drugą ; prawdop. że będzie zewnątrz a inne wewnątrz : $\left(\frac{\omega}{v}\right)^{\nu}\left(\frac{v-\omega}{v}\right)^2 (\nu+1)$

może się zamienić z ν będącemi wewnątrz



$$\left(\frac{\omega}{v}\right)^{\nu}\left(\frac{v-\omega}{v}\right)^2 (\nu+1)$$

~~N w $1\,cm^3 = 21 \cdot 10^{18}$~~ (?)

~~prawdop. że w jednym mm^3: $21 \cdot 10^{15}$~~

~~$W_0 = e^{21 \cdot 10^{15}}$~~

~~Prawdop. że n w będą $\frac{n\omega}{v} = W_0$~~

~~że będą~~ że n będą we v: $W_0 = 1$

w $v - dv$: $dW =$

Zobacz Pascal D. Mathem. p. 518:

$\mu = n \qquad p = \frac{\omega}{v} \qquad q = \frac{v-\omega}{v}$

$n = \nu$

$$W = \frac{n!}{\nu!\,(n-\nu)!} \left(\frac{\omega}{v}\right)^{\nu} \left(\frac{v-\omega}{v}\right)^{n-\nu} \qquad x! = \left(\frac{x}{e}\right)^{x} \sqrt{2\pi x}$$

$$= \frac{\left(\frac{n}{e}\right)^{n} \sqrt{2\pi n}}{\left(\frac{\nu}{e}\right)^{\nu} \sqrt{2\pi\nu} \left(\frac{n-\nu}{e}\right)^{n-\nu} \sqrt{2\pi(n-\nu)}} \left(\frac{\omega}{v}\right)^{\nu} \left(1 - \frac{\omega}{v}\right)^{n-\nu}$$

$$= \frac{n^{n}}{\nu^{\nu} (n-\nu)^{n-\nu}} \sqrt{\frac{n}{2\pi\nu(n-\nu)}} \left(\frac{\omega}{v}\right)^{\nu} \left(1 - \frac{\omega}{v}\right)^{n-\nu}$$

$$= \frac{n^{n-\nu} \cdot n^{\nu}}{(n-\nu)^{n-\nu}\, \nu^{\nu}} \cdots = \frac{1}{\left(1-\frac{\nu}{n}\right)^{n-\nu} \left(\frac{\nu}{n}\right)^{\nu}} \sqrt{\frac{1}{2\pi\nu\left(1-\frac{\nu}{n}\right)}}$$

$$\lim \left(1 - \frac{\nu}{n}\right)^{n} = \left(1 - \frac{\nu}{n}\right)^{\frac{n}{\nu}\cdot\nu} = e^{\nu} \qquad = \left(\frac{n}{e\nu}\right)^{\nu} \frac{1}{\sqrt{\nu \cdot 2\pi}}$$

$$W = \frac{1}{\sqrt{\nu}}\left(\frac{n}{e\nu}\right)^{\nu}\left(\frac{\omega}{v}\right)^{\nu}\cdot[\quad] \quad \left(1-\frac{\omega}{v}\right)^{n-\nu} = \frac{\left(1-\frac{\omega}{v}\right)^{n}}{\left(1-\frac{\omega}{v}\right)^{\nu}} = \frac{\left(1-\frac{\omega}{v}\right)^{\frac{v}{\omega}\cdot\frac{n\omega}{v}}}{\left(1-\frac{\omega}{v}\right)} = e^{-(n-\nu)\frac{\omega}{v}}$$

$$= \left(\frac{1}{e\alpha}\right)^{\nu}\frac{1}{\sqrt{\nu\cdot 2\pi}}\cdot e^{-\frac{\nu}{\alpha}} \qquad \neq e^{-\frac{\nu}{\alpha}}$$

$$n\frac{\omega}{v} = \frac{\nu}{\alpha}$$

$$\cancel{= \left(\frac{\omega}{v}\right)^{\nu} - \binom{n-\nu}{1}\left(\frac{\omega}{v}\right)^{\nu+1}}$$

$$\cancel{W_0 = e^{21.10^{15}}\frac{1}{\sqrt{2\pi.21.10^{15}}}e^{-\frac{21.10^{15}}{2.7}}}$$

$$W = \frac{1}{(e\alpha)^{\nu}e^{\frac{\nu}{\alpha}}\sqrt{2\pi\nu}} = \frac{1}{\sqrt{2\pi\nu}\cdot\alpha^{\nu}\cdot e^{\nu+\frac{\nu}{\alpha}}}$$

N.p. $\alpha = 1$

$$W_0 = \frac{1}{\sqrt{2\pi\nu}\; e^{2\nu}}$$

$$\frac{W}{W_0} = \frac{e^{2\nu}}{\alpha^{\nu}\cdot e^{\nu+\frac{\nu}{\alpha}}} = \frac{e^{\nu-\frac{\nu}{\alpha}}}{\alpha^{\nu}} = \left(\frac{e}{\alpha}\right)^{\nu}\left(\frac{1}{e}\right)^{\frac{\nu}{\alpha}} \qquad \Bigg\| \qquad \frac{W_{\alpha=e}}{W_{\alpha=1}} = \left(\frac{1}{e}\right)^{\frac{\nu}{e}}$$

$$\frac{W_{\alpha=\frac{e}{2}}}{W_0} = 2^{\nu}\left(\frac{1}{e}\right)^{\frac{2\nu}{e}}$$

N.p. Kostki o długości boków 1μ: $\nu_0 = 21.10^6$

10^{-5} cm: $\nu_0 = 21.10^3$

10^{-6} cm: $\nu_0 = 21$ | przy ciśnieniu $\frac{760}{21} = 36$ mm

" $\frac{1}{100}\mu$ | jedna drobina na taką kostkę

$$\log\frac{W_{\alpha=\frac{e}{2}}}{W_0} = \nu\log 2 - \frac{2\nu}{e}\log e = \nu\left[\log 2 - \frac{2}{e}\right]$$

0·30103 ~~2306~~
−0·43429
0·86673 −1

0·7357
0·3010
−0·4347

$\alpha = 1+\delta$

$$W = \frac{1}{[e(1+\delta)]^{\nu}\, e^{\frac{\nu}{1+\delta}} \sqrt{2\pi\nu}}$$

$$(1+\delta)^{\nu} = (1+\delta)^{\frac{1}{\delta}\cdot\nu\delta} = e^{\nu\delta}$$

$$= \frac{1}{e^{\nu+\nu\delta+\frac{\nu}{1+\delta}}\sqrt{2\pi\nu}} = \frac{1}{e^{2\nu}\sqrt{2\pi\nu}}$$

$$= \frac{\mu^{\mu}}{n^{n}(\mu-n)^{\mu-n}\sqrt{2\pi n\frac{\mu-n}{\mu}}} = \frac{\mu^{\mu}}{n^{n-\mu}\, n^{\mu}\,(\mu-n)^{\mu-n}}$$

$n<\mu$

$$= \mu^{\mu}\left(\frac{p}{n}\right)^{n}\left(\frac{q}{\mu-n}\right)^{\mu-n}\sqrt{2\pi\frac{n}{\mu}(\mu-n)}$$

$$= \mu^{\mu}\left(\frac{p}{n}\right)^{n}\left(\frac{1}{\mu+\frac{h}{q}}\right)^{\mu-n}$$

$\mu p - n = h$

$\mu - n = \mu - \mu p + h$

$= \mu(1-p) + h$

$= \mu q + h$

Dwie kulki spadające w cieczy nieściśl.: Czy one się przyciągają?

Jaka zmiana układu Strömungsstrom?

$$u = A \frac{\partial^2 L}{\partial x^2} + \dots$$

$$v = A \frac{\partial^2 L}{\partial x \partial y} - \dots$$

$$\frac{\partial \varphi}{\partial r} = -4\pi\sigma$$

$$\varepsilon = -\frac{\partial^2 \varphi}{\partial r^2}$$

$$u = -\frac{p}{4\mu L}(r^2 - R^2)$$

$$2\pi R \frac{pR}{2\mu L} \int \varepsilon \, d\sigma = \frac{\pi R^2 p}{\mu L} \frac{\partial \varphi}{\partial r}\Big|$$

$$\frac{\partial u}{\partial r} = -\frac{pR}{2\mu L}$$

$$= \frac{\pi R^2 p}{\mu L} \frac{\partial \varphi}{\partial r}\Big|_R = \frac{E}{L} \frac{R^2 \pi}{\sigma}$$

$$E = \frac{p}{4\pi\mu} \sigma \frac{\partial \varphi}{\partial r}$$

$u = -\frac{1}{\omega} \quad \cdots \qquad v = \frac{1}{\omega} \quad \cdots$

$$u = c\left[1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3} - \frac{3}{4}\frac{a}{r^3}\left(1 - \frac{a^2}{r^2}\right)x^2\right]$$

$$v = -c\,\frac{3}{4}\frac{a}{r^3}\left(1 - \frac{a^2}{r^2}\right)x\,\omega$$

$$u = c\left[1 - \frac{3}{4}\frac{a}{r}\left(1 + \cos^2\theta\right) - \frac{1}{4}\frac{a^3}{r^3}\left(1 - 3\cos^2\theta\right)\right]$$

$$v = -\frac{3}{4}\frac{ac}{r}\left(1 - \frac{a^2}{r^2}\right)\sin\theta\cos\theta$$

$$u = u_{r=a} + \rho \cdot \frac{\partial u}{\partial r}\Big|_{r=a} + \cdots$$

$$\frac{\partial u}{\partial r} = c\left[\frac{3}{4}\frac{a}{r^2}\left(1 + \cos^2\theta\right) + \frac{3a^3}{4r^4}\left(1 - 3\cos^2\theta\right)\right]$$

$$= \frac{3}{4}\frac{c}{a}\left[1 + \cos^2\theta + 1 - 3\cos^2\theta\right]_{k=r} = \frac{3}{2}\frac{c}{a}\sin^2\theta$$

$$\frac{\partial v}{\partial r} = -\frac{3}{4}ac\sin\theta\cos\theta\left[\underbrace{\left(1 - \frac{a^2}{r^2}\right)\frac{-1}{r^2} + \frac{2a^2}{r^4}}_{-\frac{1}{r^2} + \frac{3a^2}{r^4}}\right] = \cancel{\frac{3}{4}\frac{c}{a}\left[1 - \frac{3a^2}{r^2}\right]}$$

$$= -\frac{3}{2}\frac{c}{a}\sin\theta\cos\theta$$

$$u = \frac{3}{2}\frac{c}{a}\sin^2\theta \cdot \rho$$

$$v = -\frac{3}{2}\frac{c}{a}\sin\theta\cos\theta \cdot \rho$$

$$vel_{\parallel} = \sqrt{u^2 + v^2} = \frac{3}{2}\frac{c}{a}\sin\theta \cdot \rho$$

$$vel_{\perp} = -(u\sin\theta + v\cos\theta) = -\frac{3}{2}\frac{c}{a}\sin\theta\,\rho\left(\sin^2\theta + \cos^2\theta\right)$$

$$= \frac{3}{2}\frac{c}{a}\sin\theta \cdot \rho$$

więc cały prąd ~~…~~ w bliskości powierzchni równoległy do niej

zatem prąd elektr. ~~równoległy~~ całkowity do powierzchni (w szerokim pasie θ, : pod założeniem że grubość warstwy mała w porównaniu do a)

(pro cm obwodu)

$$i=\int_0 u\,\varepsilon\,d\rho = \frac{-1}{4\pi}\int u\,\frac{\partial^2\varphi}{\partial\rho^2}\,d\rho$$

$$= \frac{-1}{4\pi}\,\frac{3}{2}\,\frac{c}{a}\,\sin\theta \int \underbrace{\frac{\partial^2\varphi}{\partial\rho^2}\,\rho\,d\rho}$$

$$= \frac{\partial\varphi}{\partial\rho}\cdot\rho\Big|_0^\infty - \int \frac{\partial\varphi}{\partial\rho}\,d\rho = \varphi_i-\varphi_e$$

$$= \frac{3}{2}\,\frac{c}{a}\,\frac{\sin\theta}{4\pi}\,(\varphi_e-\varphi_i)$$

~~Zatem zadanie~~ Zatem różnica potencjału

$$J=\lambda\,\dots = \frac{\lambda}{a}\frac{\partial\lambda}{\partial\theta}$$

$E=\frac{\partial U}{\partial x}$

$$U=\oint \frac{1}{\lambda}\,\frac{3}{2}\,\frac{c}{a}\,\frac{\varphi_e-\varphi_i}{4\pi}\;a\int\sin\theta\,d\theta$$

Zatem pot. na zewn. powierzchni: $\frac{3}{2}\,\frac{c}{4\pi}\,\frac{\varphi_e-\varphi_i}{\lambda}\cos\theta$

Jaki system prądów powstanie z rozkładu potencjału $U=A\cos\theta$?

$$U_1=\frac{M_1}{r_1}\qquad U_2=-\frac{M_1}{r_2}$$

$$U=U_1+U_2=M_1\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=\frac{M_1}{r^2}\cos\theta$$

więc jeżeli $\frac{M_1}{a^2}=\frac{3}{2}\,\frac{c}{4\pi\lambda}(\varphi_e-\varphi_i)$

to będzie w polu $U=\frac{3}{2}\,\frac{c}{4\pi\lambda}(\varphi_e-\varphi_i)\,\frac{a^2}{r^2}\cos\theta$

$\int a^2\sin\theta\,d\theta\;J_a=$

związku $\sum i\,(U_{+\theta}-U_{-\theta})=$

$$di = \frac{3}{2}\frac{c}{a}\frac{\varphi_e-\varphi_i}{4\pi}\cos\theta\, d\theta$$

$$W = \int_0^{\frac{\pi}{2}} 2\pi a \sin\theta \;\frac{3}{2}\frac{c}{a}\frac{\varphi_e-\varphi_i}{4\pi}\cos\theta \cdot 2\frac{3}{2}\frac{c}{4\pi}\frac{\varphi_e-\varphi_i}{\lambda}\cos\theta\, d\theta$$

$$= 4\pi \left(\frac{3}{2}\frac{c}{4\pi}\right)^2 \frac{1}{\lambda}(\varphi_e-\varphi_i)^2 \int \underbrace{\cos^2\theta \sin\theta\, d\theta}_{=\frac{\cos^3\theta}{3}}$$

$$= \frac{3c^2}{16\pi^2}\frac{(\varphi_e-\varphi_i)^2}{\lambda}$$

zatem opór ... $\frac{iJ}{c} = \frac{3c}{\lambda}\left(\frac{\varphi_e-\varphi_i}{4\pi}\right)^2$

który ...

... : $6\pi u a c$

$$1-\frac{6x^2}{r^2}+\underbrace{\frac{9x^4}{r^4}+\frac{9x^2\omega^2}{r^4}}_{\frac{9x^2}{r^2}} \qquad 1+\frac{3x^2}{r^2}$$

$v = \frac{Ca^2}{r^3}\sqrt{1+ ...}$

$$C\frac{a^2}{r^2}\cos\theta = U = C\frac{a^2x}{r^3}$$

$$U = \frac{Ca^2x}{(x^2+\omega^2)^{3/2}}$$

$$v_x = -\frac{\partial U}{\partial x} = -\frac{Ca^2}{r^3}\left[1-\frac{3x^2}{r^2}\right]$$

$$v_\omega = -\frac{\partial U}{\partial \omega} = +\frac{3Ca^2x\omega}{r^5}$$

$$-\frac{dx}{d\omega} = \frac{\frac{\partial\chi}{\partial\omega}}{\frac{\partial\chi}{\partial x}} = -\frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial\omega}}$$

$$-\frac{3x\omega}{r^2}\overset{dx}{\frac{\partial\chi}{\partial\omega}} + \left[1-\frac{3x^2}{r^2}\right]\overset{d\omega}{\frac{\partial\chi}{\partial x}} = 0$$

$$\cancel{\frac{\partial\chi}{\partial x} - \frac{3x}{r}\frac{\partial\chi}{\partial\omega}\frac{\omega}{r} + \frac{\partial\chi}{\partial x}\frac{x}{r} = 0}$$

$\frac{\partial r}{\partial\omega}$ $\frac{\partial r}{\partial x}$

$$\left(1-\frac{3x^2}{r^2}\right)d\omega + \frac{3x\omega}{r^2}dx = 0$$

$$\underset{\frac{\partial\chi}{\partial\omega}}{(\omega^2-2x^2)}\,d\omega + \underbrace{3\omega x}_{\frac{\partial\chi}{\partial x}}\,dx = 0$$

$\frac{dx}{d\omega}$ + ...

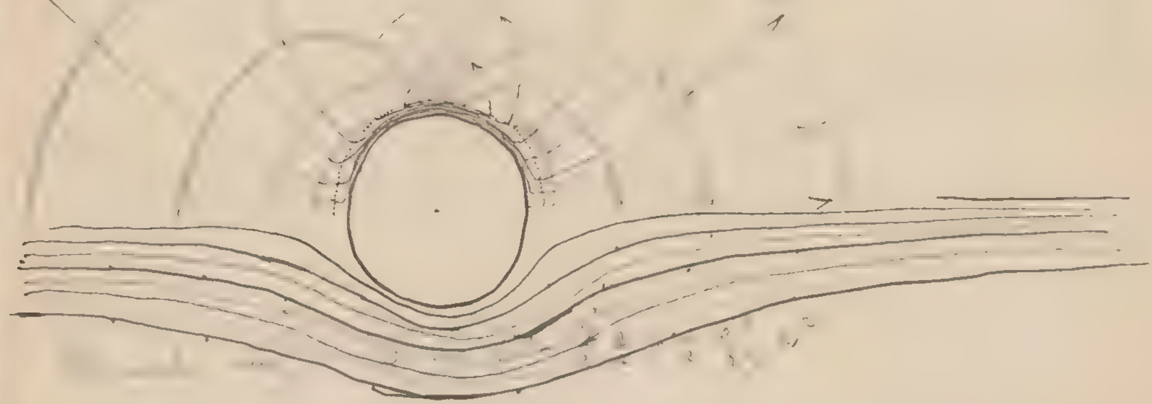
$\boxed{\frac{\omega^2}{r^2}}$ ... $2(\omega^2+x^2) ... r^2$

$$d\left(\frac{\omega^2}{r^3}\right) = \left(\frac{2\omega}{r^3} - \frac{3\omega^3}{r^5}\right) d\omega - 3\frac{\omega^2 x}{r^5}\,dx$$

$$= \frac{\omega}{r^5}\left[(2x^2 + 2\omega^2 - 3\omega^2)\,d\omega - 3\,\omega x\,dx\right]$$

$$= \frac{\omega}{r^5}\left[\,(2x^2 - \omega^2)\,d\omega - 3\,\omega x\,dx\right]$$

Więc linie prądu : $\frac{\omega^2}{r^3} = \text{const}$



$$\frac{v_\omega}{u} = \frac{-\frac{3}{4}\frac{ca}{r^3}\left(1-\frac{a^2}{r^2}\right)x\,\omega}{-\frac{3}{4}\frac{ca}{r^3}\left(1-\frac{a^2}{r^2}\right)x^2 + c\left(1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right)}$$

$$= \frac{\omega}{x - \frac{4r^3}{3ax}\frac{1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}}{1-\frac{a^2}{r^2}}} = \frac{\omega}{x - \frac{4r^3-3ar^2-a^3}{3ax\left(1-\frac{a^2}{r^2}\right)}}$$

$$\frac{m}{\omega^2} = 1 - \frac{3}{2}\frac{a}{r} + \frac{1}{2}\frac{a^3}{r^3}$$

		$m=1$	$m=\frac{1}{4}$	$m=\frac{1}{9}$
$x=0 \quad r=\omega$	$\frac{m}{\omega^2} = 1 - \frac{3}{2}\frac{a}{r} + \frac{1}{2}\frac{a^3}{r^3}$			
$r = 1a$	$\frac{1}{\omega^2} = 0$	$\omega=$		
$r = \frac{3}{2}a$	$\frac{m}{\omega^2} = 1 - 1 + \frac{1}{2}\frac{8}{27} = \frac{4}{27}$	$\sqrt{\frac{27}{4}} = 2.6$	1.3	0.87
$r = 2a$	$= 1 - \frac{3}{2}\frac{1}{2} + \frac{1}{2}\frac{1}{8} = \frac{16-12+1}{16} = \frac{5}{16}$	1.8	0.9	0.6
$r = 3a$	$= 1 - \frac{3}{2}\frac{1}{3} + \frac{1}{2}\frac{1}{27} = \frac{14}{27}$	1.4	0.7	0.47
$r = 4a$	$= 1 - \frac{3}{2}\frac{1}{4} + \frac{1}{2}\frac{1}{64} = \frac{81}{128}$	1.26	0.63	0.42

$m = 1$

Przy przepływie przez rurki

$$U = \frac{\pi P r^4}{8 \kappa L}$$

$$i = \frac{P\,6\,(\varphi_i - \varphi_a)}{4\pi k^2}$$

$$J = \frac{E\,r^2}{6}$$

Praca $= E J = \frac{P^2 R^2\, 6\,(\varphi_i-\varphi_a)^2}{4 k^2 L \cdot 4\pi k^2} = U \cdot \frac{(\varphi_i-\varphi_a)^2\, 6\, P}{2 R^2 \pi^2 k^2}$

$\cancel{\frac{1{\cdot}3 \cdot 4 \cdot 6 \cdot 4 \cdot 7}{1 \cdot (0{\cdot}01)^2}}\ \frac{1}{5 \cdot 10^{10}} = \Delta p \cdot U$

Δp jest ciśnieniem dodatkowem, spowodowanem elektr.:

$$\Delta p = \frac{(\varphi_i - \varphi_a)^2\, 6\, P}{2 R^2 \pi^2 k^2}$$

N. p. $\varphi_i - \varphi_a = 4\,V = 4$

~~1 elst. $\frac{l\,m}{t^2} \cdot t = \frac{l\,m}{t}$~~

~~1 calorj. $\frac{l^3\,m}{t^4} \cdot \frac{l^2}{t^2}$~~

$\cancel{\frac{10^3 \cdot 10^9}{3 \cdot 10^{10}} \cdot \frac{10^6}{10^3} = 10^7}$

Conductance:

Siła prądu $= c \cdot 4\pi \frac{\delta\varphi}{\delta x} \cdot q = \frac{l}{t} \cdot \frac{l^2}{l^2}\, \frac{l^{\frac{1}{2}} m^{\frac{1}{2}}}{t} = \frac{l^{+\frac{3}{2}} m^{\frac{1}{2}}}{t^2} =$ elst. jedn. czas

~~$\frac{l^{3/2}\, m^{1/2}}{t^2} = l^{5/2}\, m^{1/2}$~~

elemen. nie jest to samo w ...

$$Hg = \frac{10^4 \cdot 10^9}{9 \cdot 10^{20}} = \frac{1}{9 \cdot 10^7}\ (\text{est.})$$

$$\frac{4{\cdot}6 \cdot 10^{-8} \cdot 10^{16}}{1{\cdot}17} =$$

$$\frac{4{\cdot}6}{1{\cdot}17} \cdot 10^8\ (\text{Hg} \ldots)$$

$$\frac{1{\cdot}3 \cdot 1{\cdot}7 \cdot 1}{3 \cdot \ldots\ 3{\cdot}24} \cdot \frac{1}{9 \ldots} =$$

$i_{\gamma} = \frac{10^{10} \cdot \ldots}{1.4} = 4 \cdot 10^{8} = \cancel{\frac{4}{9} \cdot 10^{\ldots} \cdot 10^{-11} = \frac{4}{9} \cdot 10^{-3}}$ (CSS)

$k = 0.018$

$\frac{1}{\ldots} 10^{9} = 2.5$

$i = \frac{10^{4}}{\ldots 5} = 4 \cdot 10^{8}$

$$\frac{4\pi \; 0.018 \; 2.2}{\frac{4}{9} \cdot 10^{-7} \cdot 10^{6} \cdot 1.3} =$$

$\ldots 10^{-4} \ldots = \frac{10^{-15}}{9}$ CSS

$= 1.1 \cdot 10^{16}$

$\cancel{4\pi \cdot 2.2 \cdot 0.003 = \ldots 2.2 \cdot 0.01 = 0.12}$

$$\frac{4\pi \cdot 2.2 \cdot 0.18}{1.3 \quad 1.4} = \cancel{\frac{4\pi \cdot 1.4}{1.3 \cdot 1.4}} = 49$$

$4\pi \cdot \ldots = \pi \cdot \frac{2}{1.3} = 6$

$$4\pi \cdot \frac{\frac{1.78}{1.80} \cdot 2.113}{4.65 \cdot 10^{-8} \cdot 1.33 \cdot 10^{8}} = \frac{3.14 \cdot 2.11}{1.1 \ldots \cdot 1.3} = \frac{\ldots}{1.5} = 4$$

$$\frac{\Delta p}{p} = \frac{(\varphi_i - \varphi_a)^2 \, 6}{2 R^2 \pi^2 \eta}$$

$\varphi_i - \varphi_a = 4 V = \frac{4}{300}$ (CSS)

$6 = \frac{4}{9} \cdot 10^{-7} = 4.6 \cdot 10^{-8}$

$\eta = 0.010$

$R = 0.2294$ mm

$$= \left(\frac{4}{300 \cdot 0.0229 \cdot 3.14}\right)^2 \frac{4.6 \cdot 10^{-8}}{2 \cdot 0.01}$$

$$= \left(\frac{4}{3.14 \cdot 3 \cdot 0.23}\right)^2 2.3 \cdot 10^{-4} = \left(\frac{4}{2.2}\right)^2 \cdot 2.3 \cdot 10^{-4} = \frac{4}{1.21} \cdot 2.3 \cdot 10^{-4}$$

$= 0.8 \cdot 10^{-3}$

dielektr.

$$\frac{\text{Opór dielektr.}}{\text{Opór tarcia}} = \frac{\frac{3a}{\lambda}\left(\frac{\varphi_a-\varphi_i}{4\pi}\right)^2}{6\pi\mu a c} = \frac{\frac{1}{\lambda}\left(\frac{\varphi_a-\varphi_i}{4\pi}\right)^2}{2\pi a \eta}$$

Np. $\varphi_a - \varphi_i = 4V = \frac{4}{300}$ CGS

$$\frac{1}{\lambda} = \sigma = 4{\cdot}6 .\, 10^{-8}$$

$$\eta = 0{\cdot}010$$

$$\frac{4{\cdot}6 .\, 10^{-8} \cdot \left(\frac{4}{300 .\, 4 .\, 3{\cdot}14}\right)^2}{2 .\, 3{\cdot}14 .\, 0{\cdot}01 .\, a} = \frac{2{\cdot}3 .\, 10^{-6} \left(\frac{1}{1000}\right)^2}{3{\cdot}14 \;.\; a}$$

Wtedy opór el. = oporowi tarcia jeśli $a = 10^{-12}$ cm !

Np. opór el. = opór tarcia jeśli:

$$\left(\frac{4\ 1}{4 .\, 300 .\, 3{\cdot}14}\right)^2 \cdot \frac{4{\cdot}6 .\, 10^{-8}}{0{\cdot}01 .\; a^2} = 1 = \left(\frac{1}{0{\cdot}942}\right)^2 \cdot \frac{4{\cdot}6 .\, 10^{-12}}{a^2}$$

$$a^2 = \frac{4{\cdot}6}{0{\cdot}88} .\, 10^{-12} = 5 .\, 10^{-12}$$

$$a \doteq 2 .\, 10^{-6}\ \text{cm} = 2 .\, 10^{-2}\,\mu = 0{\cdot}02\,\mu$$

Rozwiązanie równania $U = \frac{M\cos\theta}{r^2}$ otrzymam

więc prąd w kierunku r : $J_r = 2\lambda\frac{M\cos\theta}{r^3}$ na powierzchni $J_a = 2\lambda\frac{M\cos\theta}{a^3}$

zatem całkowity prąd od $\theta=0$ do θ :

$$\int 2a^2\pi\sin\theta\,d\theta\,\frac{2\lambda M\cos\theta}{a^3} = \frac{4\pi\lambda M}{a}\int_0^\theta \underbrace{\sin\theta\cos\theta\,d\theta}$$

$$\frac{1}{4}\int \sin 2\theta\,d2\theta = \frac{1-\cos 2\theta}{4} = \frac{2\sin^2\theta}{4}$$

$$= \frac{2\pi\lambda M}{a}\sin^2\theta$$

to musi przechodzić przez równoleżnik θ

zatem natężenie prądu na 1 cm długości równoleżnika : $\frac{\frac{2\pi\lambda M\sin^2\theta}{a}}{2a\pi\sin\theta} = \frac{\lambda M\sin\theta}{a^2}$

jeśli to ma być równe $\frac{3}{2}\frac{c}{a}\sin\theta\frac{\varphi_e-\varphi_i}{4\pi}$

to musi być: $M = \frac{3}{2}\frac{ac}{\lambda}\frac{\varphi_e-\varphi_i}{4\pi}$ zatem: $U = \frac{3}{2}\frac{ac}{\lambda}\frac{\varphi_e-\varphi_i}{4\pi}\frac{\cos\theta}{r^2}$

$$\frac{\partial U}{\partial r_a} = \frac{3c}{\lambda a^2}\frac{\varphi_e-\varphi_i}{4\pi}\cos\theta$$

Praca całkowita:

$$W = 2\int_0^{\frac{\pi}{2}} 2a^2\pi\sin\theta\,d\theta\,.\,U_{a\theta}\,\lambda\frac{\partial U_a}{\partial r}$$

$$= 4a^2\pi\,\frac{3}{2}\frac{c}{\lambda a}\frac{\varphi_e-\varphi_i}{4\pi}\cdot\lambda\frac{3c}{\lambda a^2}\frac{\varphi_e-\varphi_i}{4\pi}\int_0^{\frac{\pi}{2}}\underbrace{\cos^2\theta\sin\theta\,d\theta}_{\frac{\cos^3\theta}{3} = \frac{1}{3}}$$

$$= \frac{9c^2(\varphi_e-\varphi_i)^2}{8\pi a\lambda}\cdot\frac{1}{3} = \frac{3c^2(\varphi_e-\varphi_i)^2}{8\pi a\lambda} \qquad \frac{W}{c} = \Delta p = \frac{3c(\varphi_e-\varphi_i)^2}{8\pi a\lambda}$$

Siła elektr. / do tarcia $= \frac{\frac{3(\varphi_e-\varphi_i)^2}{8\pi a\lambda}}{6\pi\eta a} = \frac{(\varphi_e-\varphi_i)^2}{16\pi^2 a^2\eta\lambda} = \left(\frac{\varphi_e-\varphi_i}{4\pi}\right)^2\frac{1}{\eta\lambda a^2}$

Jaki bezpośredni wpływ ładunków elektr.?

jeśli różnica pot. dana to ładunek prop. a^2

zatem siła $\sim \frac{a^2 \cdot a^2}{r^2}$ najmniejsze ~~r~~ możliwe $= 2a$

$\therefore \; f \sim a^2$ zatem przy zmniejszeniu cząstek wz. (?) zmniejsza się wpływ. Zatem koagulacja z rachunku wyjść nie może.

Uważając że „Doppelschicht" zatem z dala ładunek przeciwny zewnątrz

$\varphi_i = -\varphi_e$ zatem wpływ na zewnątrz $= 0$

W polu elektr. zewnętrznem:

Gdyby φ_i i φ_e były sobie związane, toby skutek był $= 0$.

tak jednak φ_e ~~po~~ będzie miało dążność w jednym kierunku φ_i w drugim

---

$$\left(\nabla^2 U \frac{\partial U}{\partial x}\right) = \frac{\partial}{\partial x}\left(U \nabla^2 U\right) - U \nabla^2 \frac{\partial U}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{1}{4\pi} U \nabla^2 U - p\right) = U \nabla^2 \frac{\partial U}{\partial x} - k \nabla^2 u$$

$$\frac{\partial}{\partial y}\left(\qquad\right) = U \nabla^2 \frac{\partial U}{\partial y} - k \nabla^2 v$$

$$\frac{\partial U}{\partial y} = -\frac{3}{2} c \frac{a^3 xy}{r^5}$$

$$\frac{\partial U}{\partial z} = -\frac{3}{2} \frac{c a^3 xz}{r^5}$$

$$\frac{\partial U}{\partial w} = -\frac{3}{2} \frac{c a^3 x w}{r^5}$$

~~$\frac{\partial U}{\partial (x\omega)} = \frac{\partial U}{\partial x}$~~

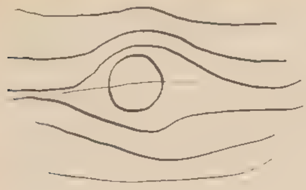
$$r = a \;\Big\|\; \frac{\partial U}{\partial x} = \frac{3}{2} c \left[1 - \frac{x^2}{a^2}\right]$$

$r^2 = t^2 + r_0^2$

$\frac{\partial U}{\partial t} = \frac{3t}{2}\frac{a^3 c x}{r^5}$

$\left.\frac{\partial^2 U}{\partial t^2}\right|_0 = \frac{3}{2}\frac{cx}{a}$

Jaki rozkład przy działaniu zewnętrznej siły X



$$\nabla^2 U = 0 \qquad \left.\frac{\partial U}{\partial r}\right|_{r=a} = 0$$

$$U = cx\left[1+\frac{a^3}{2r^3}\right] = cr\cos\theta\left[1+\frac{a^3}{2r^3}\right]$$

$$\frac{\partial U}{\partial x} = c\left[1+\frac{a^3}{2r^3}\right] - \left.\frac{3cx^2a^3}{2r^5}\right|_{r=\infty} = c$$

$$\frac{\partial U}{\partial r} = c\cos\theta\left[1+\frac{a^3}{2r^3}\right] - cr\cos\theta\,\frac{3a^3}{2r^4} = c\cos\theta\left[1+\frac{a^3}{2r^3} - \frac{3a^3}{2r^3}\right]_{r=a} = 0$$

$$= c\cos\theta\left[1-\frac{a^3}{r^3}\right]$$

Na powierzchni kuli:

$$\frac{\partial U}{\partial (a\theta)} = \frac{-1}{a}\,cr\sin\theta\left[1+\frac{a^3}{2r^3}\right]_{r=a} = -\frac{3c}{2}\sin\theta$$

$$\frac{\partial^2 U}{\partial r^2} = c\cos\theta\left[\frac{3a^3}{r^4}\right]_{r=a} = \frac{3c\cos\theta}{a}$$

Ogólne zadanie:

Ruch cieczy:

$$X - \frac{\partial p}{\partial x} = -\overset{\mu/\rho}{k}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \qquad X = \varepsilon\,\xi$$

ξ = natężenie zewn. pola elektr. (zazwyczaj modyfikacja wskutek własnych prądów)

ogólnie postać:

$$X = \varepsilon\frac{\partial U}{\partial x} =$$

$$\frac{1}{4\pi}\nabla^2 U\cdot\frac{\partial U}{\partial x} - \frac{\partial p}{\partial x} = -k\nabla^2 u \qquad \Big|\ \frac{\partial}{\partial x}$$

$$\frac{1}{4\pi}\nabla^2 U\cdot\frac{\partial U}{\partial y} - \frac{\partial p}{\partial y} = -k\nabla^2 v \qquad \Big|\ \frac{\partial}{\partial y}$$

$$\frac{1}{4\pi}\nabla^2 U\cdot\frac{\partial U}{\partial z} - \frac{\partial p}{\partial z} = -k\nabla^2 w \qquad \Big|\ \frac{\partial}{\partial z}$$

$$\nabla^2 p = \frac{1}{4\pi}\left\{(\nabla^2 U)^2 + \frac{\partial U}{\partial x}\nabla^2\frac{\partial U}{\partial x} + \dots\right\}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} = 0$$

$\nabla^2 u = 0$ wszędzie z wyjątkiem warstwy grubości δ na powierzchni

We warstwach powierzchniowych:

$$\frac{\partial^2 U}{\partial r^2} = \frac{\partial}{\partial r}\left\{ c\cos\theta \left[1 - \frac{a^3}{r^3} \right] \right\}$$

$$= c\cos\theta \, \frac{3a^3}{r^4} \Big|_{r=a} = \frac{3c\cos\theta}{a}$$

$$\frac{\partial^3 U}{\partial r^3} = -c\cos\theta \, \frac{12a^3}{r^5} \Big|_{r=a} = -\frac{12c\cos\theta}{a^2}$$

Zatem $$U = U_a + \left(\frac{\partial U}{\partial a}\right)_a \rho + \frac{\rho^2}{1.2}\left(\frac{\partial^2 U}{\partial r^2}\right)_a + \cdots$$

$$= \frac{\rho^2}{1.2}\,\frac{3c\cos\theta}{a} - \frac{\rho^3}{1.2.3}\,\frac{12c\cos\theta}{a^2} - \cdots$$

$$\underbrace{U = \frac{3c\cos\theta}{2a}\rho^2}- \frac{2c\cos\theta}{a^2}\rho^3 + \cdots$$

Prąd w kierunku stycznym (pro cos obwodu równoleżnika):

$$\lambda \frac{\partial U}{\partial (r\theta)}_{r=\mathrm{const}} = \frac{\lambda}{r}\frac{\partial U}{\partial \theta} = -\lambda c \sin\theta \left[1 + \frac{a^3}{2r^3} \right]$$

$$= \cancel{-\lambda c \sin\theta \left[1 + \right.}$$

$$= -\frac{3}{2}\lambda c \sin\theta \left[1 + \frac{\rho}{a} \right]$$

Więc punkt niedostępny c wywołujący prąd

$$i = \frac{3}{2}\,\frac{c}{a}\,\frac{\varphi_e - \varphi_i}{4\pi}\,\sin\theta \quad [\text{po ciu długości równoleżnika } \theta]$$

spowoduje przez to układ potencyjałów:

$$-\frac{3}{2}\,\alpha\lambda\,\sin\theta = \frac{3}{2}\,\frac{c}{a}\,\frac{\varphi_e - \varphi_i}{4\pi}\,\sin\theta$$

$$\therefore \qquad \alpha = -\frac{c}{a}\,\frac{\varphi_e - \varphi_i}{4\pi\lambda}$$

$$\therefore \quad U = -\frac{c}{a}\,\frac{\varphi_e - \varphi_i}{4\pi\lambda}\,r\cos\theta\left[1 + \frac{a^3}{2r^3}\right]$$

Albo też odwrotnie: jeżeli mamy

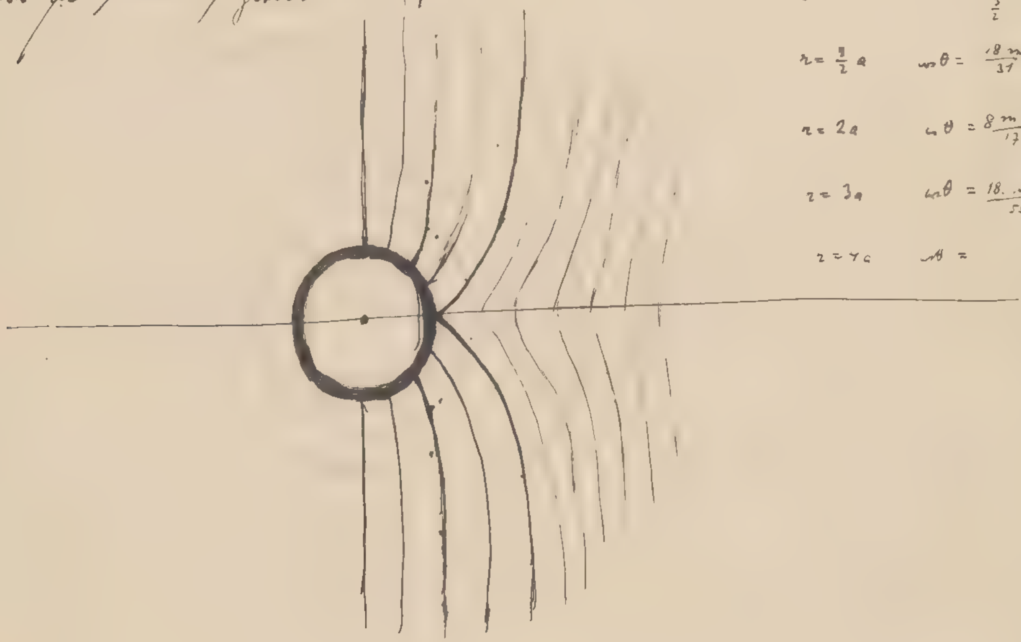
$r = a \qquad \cos\theta = \frac{m}{\frac{3}{2}} = \frac{2}{3}m$

$r = \frac{3}{2}a \qquad \cos\theta = \frac{18\,m}{31}$

$r = 2a \qquad \cos\theta = \frac{8\,m}{17}$

$r = 3a \qquad \cos\theta = \frac{18\,m}{55} \neq \frac{1}{3}m$

$r = 4a \qquad \cos\theta = \qquad \frac{1}{4}m$



Wzięte właściwie zadanie:

Znaleźć potencjał $\mathcal{U}$:

$\Delta \mathcal{U} = 0$ a na powierzchni kuli dane wartości dla $\frac{\partial \mathcal{U}}{\partial r}$

Ponieważ prąd z kuli przypływa do cieczy z daną siłą

$$\cancel{\frac{i' - i}{a\,d\theta} = \frac{1}{a}\frac{di}{d\theta} = \frac{3}{2}\frac{c}{a^2}\frac{\varphi_e - \varphi_i}{4\pi}\cos\theta}$$

Prąd przez cały równoleżnik: $\frac{2\pi i}{a \sin\theta} = \frac{3}{4} c (\varphi_e - \varphi_i) \sin^2\theta$

różnica $\frac{dJ}{\cancel{d\theta}} = \frac{3c}{2}(\varphi_e - \varphi_i) \sin\theta \cos\theta \, d\theta$

podzielone przez powierzchnię: $2\pi a^2 \cos\theta \, d\theta$.

~~natężenie~~ zatem musi ubywać w kierunku normalnym na cm²:

$$\frac{3c}{4\pi} \frac{\varphi_e - \varphi_i}{a^2} \cos\theta = \lambda \left(\frac{\partial \mathcal{U}}{\partial r}\right)_{r=a,\,\theta}$$

Więc zadanie znaleźć $\mathcal{U}$ danej na kuli ~~jako~~ przez warunek

$$\left(\frac{\partial \mathcal{U}}{\partial r}\right)_{r=a} = m \cos\theta \qquad \text{a w nieskończ.} \quad \begin{matrix} r = \infty \\ \mathcal{U} = 0 \end{matrix}$$

taki $\mathcal{U}$ jest np.: (jako funkcja prędkości) : $\mathcal{U} = \frac{M \cos\theta}{r^2}$

$$\lambda \left(\frac{\partial \mathcal{U}}{\partial r}\right)_{r=a} = -\frac{2\lambda M \cos\theta}{a^3} = \frac{3c}{4\pi} \frac{\varphi_e - \varphi_i}{a^2} \cos\theta$$

$$M = -\frac{3ca}{8\lambda\pi} \, \varphi_e - \varphi_i$$

zatem: $\mathcal{U} = -\frac{3ca}{2\lambda} \frac{\varphi_e - \varphi_i}{4\pi} \frac{\cos\theta}{r^2}$

$u = u_0 + u_1$

$$-\frac{\partial p}{\partial x} = -k\nabla^2 u_0$$

$$-\frac{\partial p}{\partial y} = -k\nabla^2 v_0$$

$$\frac{1}{4\pi}\nabla^2 U \frac{\partial U}{\partial x} = -k\nabla^2 u_1$$

$$\frac{1}{4\pi}\nabla^2 U \frac{\partial U}{\partial y} = -k\nabla^2 u_2$$

$$\frac{1}{4\pi}\nabla^2 U \frac{\partial U}{\partial z} = -k\nabla^2 u_3$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \quad - = 0$$

$$\frac{\partial u_0}{\partial x} + \quad - = 0$$

$$\frac{\partial u_1}{\partial x} + \quad = 0$$

$$\cancel{\int U\nabla^2 U = \int U \frac{\partial U}{\partial n} df - \int \left(\frac{\partial U}{\partial x}\frac{\partial U}{\partial x} - -\right)}$$

~~przyśpieszenie w kierunku x~~

~~Ogólne równania:~~

$$\cancel{\frac{1}{4\pi}\nabla^2 U \cdot \frac{\partial U}{\partial x} = -k\nabla^2 u + \frac{\partial p}{\partial x}}$$

$$\cancel{\frac{1}{4\pi}\nabla^2 U \frac{\partial U}{\partial y} = -k\nabla^2 v + \frac{\partial p}{\partial y}}$$

$$\cancel{\frac{1}{4\pi}\nabla^2 U \frac{\partial U}{\partial z} = -k\nabla^2 w + \frac{\partial p}{\partial z}}$$

Przyjmujemy teraz dla U: superpozycję rozkładu przy działaniu zewnętrznego pola X i rozkładu pot. warstwy podwójnej:

$$U = A x \left[1 + \frac{a^3}{2r^3}\right] + U_0$$

przyśpieszenie składowe: ~~w kierunku prostopadłym (r = const) = q~~ $\cancel{\frac{\partial U}{\partial x}}_{r=a}$

~~w kierunku r~~

$$\cancel{\frac{\partial q}{\partial \psi}} = 0 \qquad \frac{\partial s}{\partial \psi} = 0$$

$$\int \nabla^2 U^2\, dx\,dy\,dz = \int \cancel{\frac{\partial U}{\partial x}\frac{\partial x}{\partial x}} + \cancel{\frac{\partial U}{\partial y}\frac{\partial z}{\partial y}} + \frac{\partial U}{\partial z}\cdot df - !$$

jeżeli można uważać grubość warstwy za bardzo małą wobec zmian i krzywizny

to układając ~~…~~ oś x w kierunek stycznej $= s$, Γ będziemy mieć

$$\frac{1}{4\pi}\nabla^2 U \frac{\partial U}{\partial s} = -k\nabla^2 v_s + \frac{\partial p}{\partial s}$$

w kierunku r $(\perp s)$ $\frac{\partial U}{\partial s}$ ~~…~~ mało zmienne

$$= -\frac{3}{2} A \sin\theta \left[1 + \frac{\rho}{a}\right]$$

A całkując to $\int \rho\, d\rho\, \underbrace{\nabla^2 U}_{= \frac{\partial^2 U}{\partial \rho^2}}$ będziemy

$$= \rho \frac{\partial U}{\partial \rho} - \int \frac{\partial U}{\partial \rho} = \varphi_3 - \varphi_0$$

$\frac{\partial p}{\partial s}$ takie przyśpieszenie stałe w długości ρ

zatem $\int \rho\, d\rho \frac{\partial p}{\partial s} = \frac{\partial p}{\partial s}\frac{\rho^2}{2} \neq 0$

$$\frac{3}{2} A \sin\theta \; \frac{\varphi_a - \varphi_i}{4\pi} = k\left(\cancel{v_{si}} - v_{s0}\right) + \rho\frac{\partial v_s}{\partial \rho}$$

można to stąd $= -v_{s0} k$

jeżeli $\lim_{\rho=0} \rho\frac{\partial v}{\partial \rho} = 0$ [co każdym razem dopiero musi być a posteriori dowiedzione]

To odnosi się do warstwy powierzchniowej, bo w reszcie przestrzeni

$\varepsilon = 0$ $\cancel{\text{rot } \nabla^2 u_1 = 0}$

$k\nabla^2 u = \frac{\partial p}{\partial x}$ etc.

zadanie będzie więc znaleźć ruch cieczy z toruem który zapewni

Kuli spełnia warunek: $v_s\Big|_{r=a} = \frac{3}{2}\frac{A}{k}\sin\theta\ \frac{\varphi_i - \varphi_a}{4\pi}$

t.j. $u = -\frac{3}{2}\frac{A}{k}\frac{\varphi_i - \varphi_a}{4\pi}\left(1 - \frac{x^2}{a^2}\right)$

$\sin\theta\cos\theta\cos\varphi = \frac{yx}{a^2}$

$\sin\theta\cos\theta\sin\varphi =$

$v = \qquad \frac{xy}{a^2}$

$w = \qquad \frac{xz}{a^2}$

Próbujmy owszem: $u = \frac{M}{2\mu r^3}(r^2 + x^2) + \frac{N}{r^5}(r^2 - 3x^2) + c\ \frac{3c}{2}\left(1 - \frac{x^2}{a^2}\right)$

$v = \frac{M}{2\mu r^3}xy - \frac{3N}{r^5}xy \qquad -\frac{3}{2}c\frac{xy}{a^2}$

$w = \frac{M}{2\mu r^3}xz - \frac{3N}{r^5}xz$

Dla $r=a$: $\frac{M}{2\mu a^3}(a^2 + x^2) + \frac{N}{a^5}(a^2 - 3x^2) + c = -\frac{3}{2}\frac{A}{k}\cdots\left(1 - \frac{x^2}{a^2}\right)$

$\frac{M}{2\mu a} - \frac{3N}{a^3} = \frac{3}{2}\frac{A}{k}\cdots$

$\frac{M}{2\mu a} + \frac{N}{a^3} + c = -\frac{3}{2}\frac{A}{k}$

$\frac{4N}{a^3} + c = -3\frac{A}{k}\cdots \qquad \Big| \qquad \frac{4M}{2\mu a} + 3c = -2\,\frac{3}{2}A\cdots$

$$M = -\frac{3}{2}\left[\frac{A}{\mu}\frac{\varphi_a-\varphi_i}{4\pi} + c\right]\mu a \qquad N = -\left[\frac{3A}{\mu}\frac{\varphi_a-\varphi_i}{4\pi} + c\right]\frac{a^3}{4}$$

$M = 0$

$N = \frac{c a^3}{2}$

Warunek $c = 0 \quad r = \infty$

$$u = -\frac{3}{4}A\frac{\varphi_a-\varphi_i}{4\pi}\frac{(r^2+x^2)a}{r^3\mu} - \frac{3}{4}A\frac{\varphi_a-\varphi_i}{4\pi}\frac{(r^2-3x^2)a^3}{r^5}$$

$$u = -\frac{3}{4}\frac{A}{k}\frac{\varphi_a-\varphi_i}{4\pi}\frac{a}{r}\left[1 + \frac{x^2}{r^2} + \frac{a^2}{r^2} - \frac{3a^2x^2}{r^4}\right] \qquad \Bigg\| \quad \frac{\partial u}{\partial r} = \dots \frac{1}{r^2}\left[1 + \frac{2\ldots x^2}{r^2} - \frac{4.3a^2x^2}{r^4}\right]$$

$$v = -\frac{3}{4}\frac{A}{k}\frac{\varphi_a-\varphi_i}{4\pi}\frac{axy}{r^3}\left[1 - \frac{3a^2}{r^2}\right]$$

$v =$

Jak odwrotnie będzie ruch kuli w cieczy nieruchomej?

Dajmy na to że ostatecznie wynikłe ruch kuli z prędkością q

to odejmując tę prędkość od wszystkiego będziemy mieli zadanie:

kula nieruchoma w cieczy która w nieskończ. ma ruch $-q$

A na powierzchni kuli $v_{s/r=a} = \frac{3}{2}\frac{A}{k}\sin\theta\,\frac{\varphi_i-\varphi_a}{4\pi}$

i ostatecznie wypadkowe $= 0$

Chodzi więc o oznaczenie q..

$$ux+vy+wz = \frac{M}{2\mu r^3}\underbrace{\left[x(r^2+x^2)+xy^2+xz^2\right]}_{2r^2x} + \frac{N}{r^5}\left(\underbrace{r^2x-3x^3-3xy^2-3xz^2}_{-2r^2x}\right)+cx$$

$$= \frac{M}{\mu}\frac{x}{r} - \frac{2N}{\cancel{\mu}}\frac{x}{r^3} + cx$$

$$p = p_0 + \cancel{M}\frac{x}{r^3}$$

$$u = \frac{M}{2\mu}\left(\frac{1}{r}+\frac{x^2}{r^3}\right) + \frac{N}{,}\left(\frac{1}{r^3}-\frac{3x^2}{r^5}\right)+c$$

$$r\frac{d}{dr} = -\frac{M}{2\mu}\left(\frac{1}{r}+\frac{3x^2}{r^3}\right) - N\left(\frac{3}{r^3}-\frac{15x^2}{r^5}\right)+$$

$$+\left(\frac{M}{2\mu}\frac{1}{r^3}-\frac{3N}{r^5}\right)r\frac{d(x^2)}{dr}$$

$$= x\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}+z\frac{\partial}{\partial z} = 2x^2$$

$$= \frac{M}{2\mu}\left[-\frac{1}{r}-\frac{3x^2}{r^3}+\frac{2x^2}{r^3}\right] + 3N\left[-\frac{1}{r^3}+\frac{5x^2}{r^5}-\frac{2x^2}{r^5}\right]$$

$$r\frac{du}{dr} = -\frac{M}{2\mu}\left[\frac{1}{r}+\frac{x^2}{r^3}\right] - 3N\left[\frac{1}{r^3}-\frac{3x^2}{r^5}\right]$$

$$-u = -\frac{M}{2\mu}\left[\cdots\right] - N\left[\cdot\ -\ \right]-c$$

$$\mu\Big[\qquad = -\frac{M}{\cancel{\mu}}\left[\frac{1}{r}+\frac{x^2}{r^3}\right] - 4N_\mu\left[\frac{1}{r^3}-\frac{3x^2}{r^5}\right]-c\mu$$

$$\mu\frac{\partial}{\partial x}(ux+vy+wz) = \frac{M}{\cancel{\mu}}\left[\frac{1}{r}-\frac{x^2}{r^3}\right] - 2N_\mu\left[\frac{1}{r^3}-\frac{3x^2}{r^5}\right]+c\mu$$

$$-px = -p_0x - M\frac{x^2}{r^3}$$

$$r p_{rx} = -p_0x - 3M\frac{x^2}{r^3} - 6N_\mu\left[\frac{1}{r^3}-\frac{3x^2}{r^5}\right]$$

$$p_{rx}\Big|_{r=a} = -p_0\frac{x}{a} - \frac{6N\mu}{a^4} - 3\frac{x^2}{a^4}\left[M - \frac{6N\mu}{a^2}\right]$$

$$= -p_0\frac{x}{a} + \frac{3\mu}{2a}\left[3\frac{A}{\mu}\frac{\varphi_a-\varphi_i}{4\pi} + c\right] - \frac{3x^2}{a^2}\frac{\mu}{a}\left[-\frac{3}{2}\left(\frac{A}{\mu}\frac{\varphi_a-\varphi_i}{4\pi}\right) + \frac{3}{2}\,3\frac{A}{\mu}\frac{\varphi_a-\varphi_i}{4\pi}\right]$$

$$p_{rx} = -p_0\frac{x}{a} + \frac{3}{2}\left[3\frac{A}{a}\frac{\varphi_a-\varphi_i}{4\pi} + \frac{c\mu}{a}\right] - \frac{3x^2}{a^3}\,2A\,\frac{\varphi_a-\varphi_i}{4\pi}$$

$$\cancel{P = \iint p_{rx}\, dS = 2\pi}$$

$$= -p_0\cos\theta + \frac{3}{2a}\left[3A\frac{\varphi_a-\varphi_i}{4\pi} + c\mu\right] - 6\cos^2\theta\,\frac{A}{a}\frac{\varphi_a-\varphi_i}{4\pi}$$

$$P_x = 2\pi a^2\int_0^{\pi}\sin\theta\, d\theta \cdot p_{rx}$$

$$= 2\pi a^2\left\{3\,\frac{1}{a}\left[3A\frac{\varphi_a-\varphi_i}{4\pi} + c\mu\right] - 6\,\frac{A}{a}\frac{\varphi_a-\varphi_i}{4\pi}\,\frac{2}{3}\right\}$$

$$P_x = 2\pi a\left[5A\frac{\varphi_a-\varphi_i}{4\pi} + 3c\mu\right]$$

Jeśli to ma być $=0$ to wynika: $c = -\dfrac{5A\frac{\varphi_a-\varphi_i}{4\pi}}{3\mu}$

zatem 1). prędkość niezależna od a

2). prop. A

3). odwr. prop μ zatem szybciej przy podw. temp.

$c = b A$ $\qquad i = A\lambda$ $\qquad J = A\lambda q$

$= b\,\frac{E}{l}$ $\qquad Q = i q t = \frac{E}{l}\lambda q t$

$s = ct = b\,\frac{E}{l}t = b\,\frac{Q}{\lambda q} = \frac{b}{\lambda q}\cdot Q$

więc droga zakreślona przez część zależna od ilości elektr. [nie od E, ani od l ale od q]

$c = b\,\frac{J}{\lambda q} = \frac{b}{\lambda q}\cdot J$ tak samo

porówn. Winkelm. p. 503 Quincke P. Ann 113 (1861)

$\int_0^\delta \nabla^2 V\,dV = V_2 - V_1 = \int_0^\delta 4\pi\varepsilon\,d\delta$

$\int \varepsilon\,d\delta = \sigma\delta$

Elektr. Doppelschichten

Można sobie wyobrazić $\sigma = \frac{1}{4\pi}\frac{V_2-V_1}{\delta}$

na dwóch zbiorach oddzielonych przez δ

~~Jak to będzie gdy krzywizna jest ...~~

$W = \frac{1}{2}\sum \varphi V = \frac{1}{2}\sum\left(\frac{\partial V}{\partial n}\right)^2$

pro cm² powierzchni:

$W = \frac{1}{2}\,\frac{1}{4\pi}\left(\frac{V_2-V_1}{\delta}\right)^2$

Jeśli się to stosuje różne mają oddziaływać.

ε dla wody (powierzchni) $= 80$ C.G.S. dla wody / pewnie ...

$V_2 - V_1 = $ N. p. $4^V = \frac{4}{300}$

$80 = \frac{1}{8\pi}$ $\quad \delta = \frac{1}{8\pi}\,\frac{1}{80}\left(\frac{4}{300}\right)^2$

$= \frac{1}{31}\left(\frac{1}{600}\right)^2 = \frac{1}{31 \cdot 3.6}\, 10^{-6} = 10^{-7}$ cm

To nie może być zjawisko identyczne
bo np. dodanie $\frac{1}{10}$ % NaCl zmienia $\varphi_1 - \varphi_2$ od 0.1 V do 2.7 V podczas gdy
a woda nie zmienia. Doświadcz.!

W każdym razie należy przy wszystkich zjaw. elektrok. także uwzgl. zmiany …

Przy kondens. lub stężeniu emulsyi powierzchnia zmniejsza się ogromnie
zostaje wypar. pow. elekt.
zatem energia elektr.

ilość $\Omega 6$ z jednej i z drugiej strony się wyswobodzi

(rozwiązanie i tak jest zatem prąd równoległy do niej)

To stosuje się zarówno do każdej powierzchni:

W kierunkach x y w powierzchni z normalnie

$$\nabla^2 U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}$$

$$\underbrace{\phantom{\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}}}_{\text{bardzo małe}}$$

$$\frac{1}{4\pi}\nabla^2 U \cdot \frac{\partial U}{\partial x} = -\mu\nabla^2 u + \frac{\partial p}{\partial x}$$

$$\frac{1}{4\pi}\nabla^2 U \frac{\partial U}{\partial y} = -\mu\nabla^2 v + \frac{\partial p}{\partial y}$$

$\frac{\partial U}{\partial x}$ i $\frac{\partial U}{\partial y}$ mało zmienne w obszarze δ

zatem $\int_0^\delta \nabla^2 U \frac{\partial U}{\partial x} \cdot z\,dz = \underbrace{\left|\frac{\partial U}{\partial x}\right|}_{\text{średnia wartość}} \int_0^\delta \nabla^2 U\, z\, dz$

$$\frac{1}{4\pi}\nabla^2 U \cdot \frac{\partial U}{\partial z} = -\mu\nabla^2 w + \frac{\partial p}{\partial z}$$

Prąd prawie równoległy do pow. bo $u_0 = 0 \quad v_0 = 0 \quad w_0 = 0$

$$u = z\left(\frac{\partial u}{\partial z}\right)_0 + \frac{z^2}{2}\left(\frac{\partial^2 u}{\partial z^2}\right)_0$$

$$v = z\left(\frac{\partial v}{\partial z}\right)_0 + \frac{z^2}{2}\left(\frac{\partial^2 v}{\partial z^2}\right)_0$$

$$w = \cancel{z\left(\frac{\partial w}{\partial z}\right)_0} + \frac{z^2}{2}\left(\frac{\partial^2 w}{\partial z^2}\right)_0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$z\left(\frac{\partial^2 u}{\partial x \partial z}\right) + \frac{z^2}{2}\left(\frac{\partial^3 u}{\partial x \partial z^2}\right) + z\left(\frac{\partial^2 v}{\partial z \partial y}\right) + \frac{z^2}{2}\left(\frac{\partial^3 v}{\partial y \partial z^2}\right) + \frac{\partial w}{\partial z} + z\frac{\partial^2 w}{\partial z^2} = 0$$

z tego wynika zwłaszcza z dostat. małe że także $\left(\frac{\partial w}{\partial z}\right)_0 = 0$

Oprócz tego:

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial z}\right)_0 + \frac{\partial}{\partial y}\left(\frac{\partial v}{\partial z}\right)_0 + \left(\frac{\partial^2 w}{\partial z^2}\right)_0 = 0$$

$$\frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial z^2}\right)_0 + \frac{\partial}{\partial y}\left(\frac{\partial^2 v}{\partial z^2}\right)_0 + \left(\frac{\partial^3 w}{\partial z^3}\right)_0 = 0$$

$$\nabla^2 u = 2\left[\frac{\partial^2}{\partial x^2}\left(\frac{\partial u}{\partial z}\right)_0 + \frac{\partial^2}{\partial y^2}\left(\frac{\partial u}{\partial z}\right)_0\right] + \frac{z^2}{2}\left[\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u}{\partial z^2}\right)_0 + \frac{\partial^2}{\partial y^2}\left(\frac{\partial^2 u}{\partial z^2}\right)_0\right] + \left(\frac{\partial^2 u}{\partial z^2}\right)_0$$

więc dla dostatecznie małych z: $\nabla^2 u = \left(\frac{\partial^2 u}{\partial z^2}\right)_0$ W tem niedokładność!

$$\frac{1}{4\pi}\,\overline{\frac{\partial U}{\partial x}} \underbrace{\int_0^\delta \frac{\partial^2 U}{\partial z^2}\, z\, dz}_{\frac{\partial U}{\partial z} z - \int \frac{\partial U}{\partial z} dz} = -\mu \int_0^\delta \left(\frac{\partial^2 u}{\partial z^2}\right) z\, dz + \overline{\frac{\partial p}{\partial x}} \int z\, dz$$

δ obieramy tam gdzie $\frac{\partial U}{\partial z} = 0$

dla dost. małych δ:

$$\int z \frac{\partial^2 u}{\partial z^2} dz = z\frac{\partial u}{\partial z} - u$$

$$\frac{1}{4\pi}\,\overline{\frac{\partial U}{\partial x}}\,(U_i - U_a) = \cancel{\mu (u_a - u_i) = \mu u_a}$$

$$\frac{1}{4\pi}\,\overline{\frac{\partial U}{\partial y}}\,(U_i - U_a) = \cancel{\mu (v_a - v_i) = \mu v_a}$$

$$\int \nabla^2 u\, z\, dz = \frac{z^2}{2}\left(\frac{\partial^2 u}{\partial z^2}\right)_0 + \frac{z^3}{3}\left[\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial u}{\partial z}\right)_0\right] + \frac{z^4}{2.4}\left[\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial^2 u}{\partial z^2}\right)_0\right] + \left(\frac{\partial^2 u}{\partial z^2}\right)_0$$

$$\frac{1}{4\pi}\nabla^2 u \frac{\partial u}{\partial x} = -k\nabla^2 u_1$$ | $\frac{\partial}{\partial x}\nabla^2 u$ będzie tylko f.(ξ)

$$\frac{1}{4\pi}\nabla^2 u \frac{\partial u}{\partial y} = -k\nabla^2 v_1$$ | $\frac{\partial}{\partial y}$

$$\frac{1}{4\pi}\nabla^2 u \frac{\partial u}{\partial z} = -k\nabla^2 w_1$$

$$\frac{1}{4\pi}\left[\nabla^2 u\left(\nabla^2 u - \frac{\partial^2 u}{\partial z^2}\right) + \frac{\partial}{\partial z}\left(\nabla^2 u \frac{\partial u}{\partial z}\right)\right] = 0$$

$$\left(\nabla^2 u\right)^2 + \frac{\partial}{\partial z}\left(\nabla^2 u\right)\frac{\partial u}{\partial z} = 0$$

Ścisłe: $\underbrace{\left(\frac{\partial^2 u}{\partial z^2}\right)^2 + \frac{\partial u}{\partial z}\cdot\frac{\partial^3 u}{\partial z^3}}$ $= 0$

$$\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial z^2}\right) = 0$$

$$\underbrace{\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial z^2}}_{\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\right)^2} = \text{const}$$

$$\left(\frac{\partial u}{\partial z}\right)^2 = cz + a$$

$$u = \left(\frac{\partial u}{\partial \rho}\right)_0 \rho$$

$$\frac{1}{4\pi}\int\left(\frac{\partial^2 u}{\partial x^2}\right)\left(\frac{\partial u}{\partial \rho}\right)_0 \rho\, d\rho$$

$$i = \frac{1}{4\pi}\left(\frac{\partial u}{\partial \rho_0}\right)(\varphi_a - \varphi_i)$$

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = \alpha$$

$$\sim \varphi + \frac{y^2}{r}\frac{\partial \varphi}{\partial r} \quad \varphi + \cancel{\frac{x^2}{r}\ldots} = \cancel{2}\varphi + r\frac{d\varphi}{dr} = \alpha$$

$u = u_0 + u_1$

Rozłóżmy szeregami v

$$\left\{\begin{aligned} -\frac{\partial p}{\partial x} &= \mu \nabla^2 u_0 \\ -\frac{\partial p}{\partial y} &= \mu \nabla^2 v_0 \\ -\frac{\partial p}{\partial z} &= \mu \nabla^2 w_0 \end{aligned}\right. \qquad \left.\begin{aligned} \varepsilon\frac{\partial U}{\partial x} &= -\mu \nabla^2 u_1 \\ \varepsilon\frac{\partial U}{\partial y} &= -\mu \nabla^2 v_1 \\ \varepsilon\frac{\partial U}{\partial z} &= -\mu \nabla^2 w_1 \end{aligned}\right\}$$

$$\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} + \frac{\partial w_0}{\partial z} = 0$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0$$

~~[illegible]~~ $u_1 = v_1 = w_1 = 0$ na powierzchni kuli $\big|\, r = a$

$$\frac{\partial U}{\partial x}(\text{dla } r=\infty) = A \qquad \frac{\partial U}{\partial y} = \frac{\partial U}{\partial z} = 0$$

$$U = \cancel{[illegible]}\; A\, x\left[1 + \frac{a^3}{2r^3}\right]$$

$\cancel{[illegible]}^{\varepsilon}$ tylko $f(r+\Delta r)$

~~[illegible]~~

Wiemy tylko tyle że $\int z \rho\, d\rho = \varphi_a - \varphi_i$

$$\int_0 \frac{\partial^2 v_s}{\partial x^2}\rho\, d\rho = \frac{\partial v_s}{\partial \rho}\cdot\rho\Big|_0^{\delta} - \underbrace{\int \frac{\partial v_s}{\partial \rho}\, d\rho}_{v_s\big|_0^{\delta}} = \frac{\partial v_s}{\partial \rho}\Big|_{\delta}\,\delta - v_s\Big|_{\delta}$$

$$2\left[\left(\frac{\partial v}{\partial z}\right)_0 + z\left(\frac{\partial^2 v}{\partial z^2}\right)_0\right] - \left[z\frac{\partial v}{\partial z} + \frac{z^2}{2}\left(\frac{\partial^2 v}{\partial z^2}\right)\right]$$

$$" \ \frac{z^2}{2}\left(\frac{\partial^2 v}{\partial z^2}\right)$$

$$\frac{\varphi_1 - \varphi_0}{4\pi}\cdot\frac{\varphi}{J\delta} = \nabla^2 u_1$$

Poza warstwą elektr. będzie

$$\underbrace{\nabla^2 u_1 = 0 \quad \nabla^2 v_1 = 0 \quad \nabla^2 w_1 = 0}_{\nabla^2 \mathfrak{v} = 0} \qquad \underbrace{\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0}_{\operatorname{div}\mathfrak{v} = 0}$$

$$\mathfrak{v} = \operatorname{curl}\mathfrak{A}$$

$$\operatorname{curl}\operatorname{curl}\mathfrak{v} = \nabla\operatorname{div}\mathfrak{v} - \nabla^2\mathfrak{v} = 0$$

$$\operatorname{curl}\mathfrak{v} = \nabla A = \nabla\operatorname{div}\mathfrak{A} - \nabla^2\mathfrak{A}$$

dla nieskończoności żadnego przepływu, zatem $\mathfrak{v} = 0$ $\quad \nabla A = 0$

$$\operatorname{div}\operatorname{curl}\mathfrak{v} = \nabla^2 A = 0$$

[illegible] beweis:

$$\cancel{\frac{\partial u}{\partial t}} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = X - \frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{\mu}{\rho}\nabla^2 u$$

[illegible] v [illegible] [illegible]
Dle [illegible] dané u v w
i p

$u_1 \quad v_1 \quad w_1 \quad p_1$

$$u\frac{\partial u}{\partial x} + \ldots \quad - u_1\frac{\partial u_1}{\partial x} \ldots \quad = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = u_1\frac{\partial u_1}{\partial x} + v_1\frac{\partial u_1}{\partial y} + w_1\frac{\partial u_1}{\partial z}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = u_1\frac{\partial v_1}{\partial x} + v_1\frac{\partial v_1}{\partial y} + w_1\frac{\partial v_1}{\partial z}$$

$$u\frac{\partial w}{\partial x} + \qquad =$$

$$\frac{\partial(u-u_1)}{\partial x} + \ldots = 0$$

$$\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y} - \frac{\partial w}{\partial z}$$

$$\left(v\frac{\partial u}{\partial y} - u\frac{\partial v}{\partial y}\right) + \left(w\frac{\partial u}{\partial z} - u\frac{\partial w}{\partial z}\right) = \ldots$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{\partial}{\partial x}\left(\frac{u^2}{2}\right) + \frac{\partial(uv)}{\partial y} + \frac{\partial(uw)}{\partial z} - \underbrace{u\frac{\partial v}{\partial y} - u\frac{\partial w}{\partial z}}_{= +u\frac{\partial u}{\partial x} = \frac{1}{2}\frac{\partial u^2}{\partial x}}$$

$$= \frac{\partial}{\partial x}(u^2) + \frac{\partial}{\partial y}(uv) + \frac{\partial}{\partial z}(uw)$$

Równania z zaniedbaniem „inertia terms"

Stacjonarne; n. p. ruch kołowy $u = -\omega \frac{y}{r}$ $v = \omega \frac{x}{r}$

$= -y\varphi$ $= x\varphi$

$$\frac{\partial u}{\partial x} = \frac{yx}{r}\frac{d\varphi}{dr}$$

$$\frac{\partial u}{\partial y} = \varphi + \frac{y^2}{r}\frac{d\varphi}{dr}$$

$$\frac{\partial^2 u}{\partial x^2} = y\left(\frac{1}{r}\frac{d\varphi}{dr} - \frac{x^2}{r^3}\frac{d\varphi}{dr} + \frac{x^2}{r^2}\frac{d^2\varphi}{dr^2}\right)$$

$$\cancel{\frac{\partial^2 u}{\partial y^2} = x\left(\frac{1}{r}\frac{d\varphi}{dr} - \frac{y^2}{r^3}\frac{d\varphi}{dr} + \frac{x^2}{r^2}\frac{d^2\varphi}{dr^2}\right)}$$

$$\frac{\partial^2 u}{\partial y^2} = 3\frac{d\varphi}{dr}\frac{y}{r} - \frac{y^3}{r^3}\frac{d\varphi}{dr} + \frac{y^3}{r^2}\frac{d^2\varphi}{dr^2}$$

~~Stąd~~

$$\Delta^2 u = 3\frac{y}{r}\frac{d\varphi}{dr} + y\frac{d^2\varphi}{dr^2}$$

$p = f(r)$ $\frac{\partial p}{\partial x} = \frac{x}{r}\frac{dp}{dr}$

$$\frac{\partial p}{\partial x} = \mu \nabla^2 u$$

$$\frac{x}{r}\frac{dp}{dr} = -y\left[\frac{3}{r}\frac{d\varphi}{dr} + \frac{d^2\varphi}{dr^2}\right] \quad x$$

$$\frac{y}{r}\frac{dp}{dr} = x\left[\frac{3}{r}\frac{d\varphi}{dr} + \frac{d^2\varphi}{dr^2}\right] \quad y$$

$$r\frac{dp}{dr} = 0$$

$p = \text{const}$

$r^3 \frac{d\varphi}{dr} = \text{const}$

$\varphi = \frac{1}{r^2} + \dots$

Tutaj tylko jedna możliwa

Przy

Strunę, której dwa końce drgania harm. o różnym okresie:

$$\frac{\partial^2\xi}{\partial t^2} = a^2\frac{\partial^2\xi}{\partial x^2}$$

$$\xi = \sum\left[A_k \sin\frac{k\pi x}{l} + B_k \frac{\cos k\pi x}{l}\right]\sin\left(\frac{k\pi a t}{l}+\delta_k\right)$$

$$x=0\Big|\quad \xi = \sum_{k=1}^{\infty} B_k \sin\left(\frac{k\pi a t}{l}+\delta_k\right) = M\sin\alpha t$$

$$x=l\Big|\quad \xi = \sum B_k \cos k\pi\left(\sin\frac{k\pi a t}{l}+\delta_k\right) = N\sin\beta t$$

$$B_1\sin\left(\frac{a\pi t}{l}+\delta_1\right) + B_2\sin\left(2\frac{a\pi t}{l}+\delta_2\right) + B_3\sin\left(\frac{3a\pi t}{l}+\delta_3\right) + \dots = M\sin\alpha t$$

$$-B_1\sin\left(\frac{a\pi t}{l}+\delta_1\right) + B_2\sin\left(\frac{2a\pi t}{l}+\delta_2\right) - B_3\sin\left(\frac{3a\pi t}{l}+\delta_3\right) + \dots = N\sin\beta t$$

$$B_2\sin\left(\frac{2a\pi t}{l}+\delta_2\right) + B_4\sin\left(\frac{4a\pi t}{l}+\delta_4\right) + \dots = \frac{1}{2}\left[M\sin\alpha t + N\sin\beta t\right]$$

$$B_1\sin\left(\frac{a\pi t}{l}+\delta_1\right) + B_3\sin\left(\frac{3a\pi t}{l}+\delta_3\right) + \dots = \frac{1}{2}\left[M\sin\alpha t - N\sin\beta t\right]$$

$$\cancel{B_k = \frac{2}{l}\frac{1}{2}\int_0^l \left[M\sin\alpha t + N\sin\beta t\right]\sin\frac{k a\pi t}{l}} \qquad \int_0^{\frac{2\pi}{k\gamma}} \sin m\gamma t \cdot \sin k\gamma t$$

$$\cancel{2\int_0^{} \sin\alpha t \sin k\gamma t\, dt} = \int\left[\cancel{\cos(\alpha+k\gamma)t - \cos(\alpha-k\gamma)t}\right]dt$$

$$= \frac{\sin(\alpha+k\gamma)t}{\alpha+k\gamma}\Bigg|_0^{\frac{2\pi}{k\gamma}} - \frac{\sin(\alpha-k\gamma)t}{\alpha-k\gamma}\Bigg|_0^{\frac{2\pi}{k\gamma}}$$

$$\left.\begin{aligned}&B_1\cos\delta_1\sin\frac{a\pi t}{l}+B_3\cos\delta_3\sin\frac{3a\pi t}{l}+\dots\\&+B_1\sin\delta_1\cos\frac{a\pi t}{l}+B_3\sin\delta_3\cos\frac{3a\pi t}{l}+\dots\end{aligned}\right\}=\underbrace{\tfrac{1}{2}\left[M\sin\alpha t-N\sin\beta t\right]}_{=F_2}$$

$$-2\int_0^{\frac{2l}{a}}\sin\frac{m\pi t a}{l}\sin\frac{n\pi t a}{l}\,dt=\int_0^{\frac{2l}{a}}\left[\cos\frac{(m+n)a\pi t}{l}-\cos\frac{(m-n)a\pi t}{l}\right]dt=$$

$$=\frac{\sin\frac{(m+n)\pi t a}{l}}{\frac{(m+n)\pi}{l}}\Bigg|_0^{\frac{2l}{a}}-\frac{\sin\frac{(m-n)\pi t a}{l}}{\frac{(m-n)\pi}{l}}\Bigg|_0^{\frac{2l}{a}}=0$$

$$2\int_0^{\frac{2l}{a}}\sin^2\frac{m\pi a t}{l}\,dt=\int_0^{\frac{2l}{a}}\left[1-\cos\frac{2ma\pi t}{l}\right]dt=\frac{2l}{a}$$

$$2\int_0^{\frac{2l}{a}}\cos\frac{m\pi t a}{l}\sin\frac{n\pi a t}{l}\,dt=\int_0^{\frac{2l}{a}}\left[\sin\frac{(m+n)a\pi t}{l}-\sin\frac{(m-n)a\pi t}{l}\right]dt=$$

$$=\frac{\cos\frac{(m+n)a\pi t}{l}}{\frac{(m+n)a\pi}{l}}\Bigg|_0^{\frac{2l}{a}}-\frac{\cos\frac{(m-n)a\pi t}{l}}{-}\Bigg|_0^{\frac{2l}{a}}=0$$

$$B_k\cos\delta_k=\frac{a}{l}\int_0^{\frac{2l}{a}}F_2\sin\frac{k\pi a t}{l}\,dt\quad\Bigg\|\quad B_k\sin\delta_k=\frac{a}{l}\int_0^{\frac{2l}{a}}F_2\cos\frac{k\pi a t}{l}\,dt$$

$$-2\int_0^{\frac{2l}{a}}\sin\alpha t\,\sin\frac{k\pi a t}{l}\,dt=\frac{\sin\left(\alpha+\frac{ka\pi}{l}\right)t}{\alpha+\frac{ka\pi}{l}}\Bigg|_0^{\frac{2l}{a}}-\frac{\sin\left(\alpha-\frac{ka\pi}{l}\right)t}{\alpha-\frac{ka\pi}{l}}\Bigg|_0^{\frac{2l}{a}}$$

$$=\frac{\sin\frac{2\alpha l}{a}}{\alpha+\frac{ka\pi}{l}}-\frac{\sin\frac{2\alpha l}{a}}{\alpha-\frac{ka\pi}{l}}=\sin\frac{2\alpha l}{a}\cdot\frac{-2\frac{ka\pi}{l}}{\alpha^2-\left(\frac{ka\pi}{l}\right)^2}$$

Jaka będzie praca wykonana na punkt 0 ?

$$X = E\frac{\partial \xi}{\partial x}$$

$$\int \frac{\partial \xi}{\partial x}\, d\xi_0 = \int \left(\frac{\partial \xi}{\partial x}\right)_0 \left(\frac{\partial \xi}{\partial t}\right)_0 dt$$

$$\left(\frac{\partial \xi}{\partial x}\right)_0 = \sum A_k \frac{k\pi}{l} \sin\left(\frac{k\pi a t}{l} + \delta_k\right)$$

$$\left(\frac{\partial \xi}{\partial t}\right)_{x=0} = \cancel{\sum A_k \, a \left(\frac{k\pi a t}{l} + \delta_k\right)}$$

$$\cancel{= \sum \left[A_k \sin\frac{k\pi x}{l} + B_k \cos\frac{k\pi x}{l}\right] \frac{k\pi a}{l} \cos\delta_k}$$

$$= \sum B_k \frac{k\pi a}{l} \cos\left(\frac{k\pi a t}{l} + \delta_k\right)$$

$$\frac{\partial p}{\partial x} = \mu \nabla^2 u \qquad \frac{\partial p_1}{\partial x} = \mu \nabla^2 u_1 \qquad \frac{\partial (p-p_1)}{\partial x} = \mu \nabla^2 (u-u_1)$$

$$\frac{\partial p}{\partial y} = \mu \nabla^2 v \qquad \vdots \qquad \frac{\partial (p-p_1)}{\partial y} = \mu \nabla^2 (v-v_1)$$

$$\frac{\partial p}{\partial z} = \mu \nabla^2 w \qquad \vdots \qquad \frac{\partial (p-p_1)}{\partial z} = \mu \nabla^2 (w-w_1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\Downarrow$$

$$\nabla^2 p = 0 \qquad\qquad \nabla^2 (p-p_1) = 0$$

$$\nabla p = \mu \nabla^2 v = -\mu \operatorname{curl}^2 v \qquad \operatorname{curl}^2 = \nabla \operatorname{div} - \nabla^2$$

$$\operatorname{div} v = 0$$

$$v = \nabla A + \operatorname{curl} \mathfrak{A}$$

$$\cancel{\operatorname{curl} \nabla p = 0 = \mu \operatorname{curl} \nabla^2 v = \mu \nabla^2 \operatorname{curl} v = -\mu \operatorname{curl}^3 v = 0}$$

$$\cancel{v = \nabla \int \frac{\nabla^2 v}{r} dv} \quad \alpha = \nabla \int \frac{\operatorname{div} \alpha}{r} dv + \operatorname{curl} \int \frac{\operatorname{curl} \alpha}{r} dv$$

$$v = \nabla A + \operatorname{curl} \int \frac{\operatorname{curl} v}{r} dv \qquad \nabla p = \operatorname{curl}^3 \mathfrak{A}$$

We warstwach elektr. podw.

Ciśnienie mechan. $p_{xx} = -\int_0^{\infty} \varepsilon \frac{\partial U}{\partial x}\, dx$

$$= +\frac{1}{4\pi}\int \frac{\partial^2 U}{\partial x^2}\frac{\partial U}{\partial x}\, dx =$$

$$= \frac{1}{8\pi}\left(\frac{\partial U}{\partial x}\right)^2\Big|_0^{\infty} = -\frac{1}{8\pi}\left(\frac{\partial U}{\partial x}\right)_0^2$$

Zastępując przez warstwę podw.

$$\frac{\partial U}{\partial x} = \frac{U_2 - U_1}{\delta} = -4\pi\sigma \qquad p_{xx} = 2\pi\sigma^2$$

Przy polaryzacji według Kohlrauscha

pojemność pro $1\,mm^2$

$0{\cdot}13$ Mikrofarad

pro $1\,cm^2$: 13 Mikrof. $= 13\cdot 10^{-6}\ \frac{\text{Coulomb}}{\text{Volt}}$

<u>1 Volt</u> $\sigma = 13\cdot 10^{-6}$ Coul. $= 13\cdot 10^{-6}\cdot 3\cdot 10^{9}$ (elst) $= 39\cdot 10^{3} \approx 4\cdot 10^{2}$

$p_{xx} = 2\pi(4\cdot 10^2)^2 = 2\cdot 3{,}2\cdot 16\cdot 10^4 = 10^6$ Dyn

$= 1$ Atmosf.!

Dwa walce ~~....~~ spółosiowe ; wewnętrzny poruszający się w kierunku X

Jaki opór ?

$$\mu 2\pi r\frac{\partial u}{\partial r} = \text{const}$$

$$du = \frac{c\,dr}{r}$$

$$u-u_1 = c\log\frac{r}{r_1}$$

$u_2=0$

$$\frac{u-u_1}{-u_1} = \frac{\log\frac{r}{r_1}}{\log\frac{r_2}{r_1}}$$

$\cancel{\mu\frac{\partial u}{\partial x}=}$

$$\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}$$

Na swobodnej powierzchni Y powinno być:

$$X_y = Y_y = Z_y = 0$$

zatem: $p - 2\mu\left(\frac{\partial v}{\partial y}\right)=0$

v musi być $=0$ w otoczeniu zatem $\frac{\partial v}{\partial y}=0$, $p=0$

$$\left(\frac{\partial u}{\partial y}\right)+\left(\frac{\partial v}{\partial x}\right)=0 \qquad \left(\frac{\partial u}{\partial y}\right)=0$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

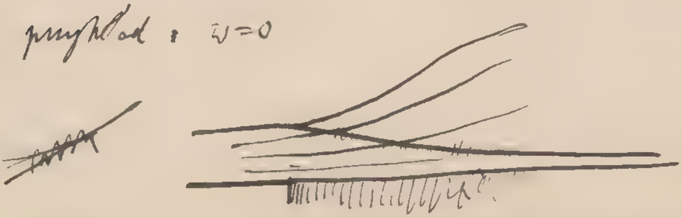
więc $\frac{\partial u}{\partial x}=0$

$u=\text{const}$

wzdłuż

$$\left(\frac{\partial w}{\partial y}\right)+\left(\frac{\partial v}{\partial z}\right)=0 \qquad \left(\frac{\partial w}{\partial y}\right)=0$$

Dwuwymiarowy przykład : $w=0$



Przybliżona teorya: uwzględniając tylko energię kinet. w kierunku X

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} - \mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial x} = -\frac{\partial p}{\partial y} - \mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

} ~~zaniedbać i wziąć tylko $p = f(x)$~~ nie można!

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

~~warunek na powierzchni: $p = 0$ czyli w ogóle wszędzie $p = 0$~~

$$\cancel{\frac{\partial u}{\partial x}=0}$$

~~Uproszczenie przybliżenia $u = a$~~

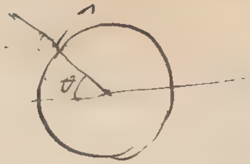
v w ogóle małe zatem v^2 zaniedbać

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} = v\frac{\partial u}{\partial y}-u\frac{\partial v}{\partial y} = v^2\frac{\partial}{\partial y}\left(\frac{u}{v}\right)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y} = u\frac{\partial v}{\partial x}-v\frac{\partial u}{\partial x} = u^2\frac{\partial}{\partial x}\left(\frac{v}{u}\right) = -u^2\frac{\partial}{\partial x}\left(\frac{u}{v}\right)$$

$\Delta^2 p =$

Ruch kuli w cieczy z ślizganiem



$$\cancel{\frac{M}{2\mu}\left(\frac{1}{r}+\frac{3x^2}{r^3}\right)-N\left(\frac{1}{r^3}-\frac{15x^2}{r^5}\right)}$$

$p_{\theta r} = \lambda\, v_\theta$

$p_{rx} = \lambda$

$= p_{rx}\cos\theta + p_{ry}$

Jak po przejściu kuli pozostanie rozmieszczenie elektr.?

$$\psi = -\frac{c}{2}\left(1-\frac{3}{2}\frac{a}{r}+\frac{1}{2}\frac{a^3}{r^3}\right)\omega^2$$

w płaszczyźnie $x=0$:

$$r = a+\delta \qquad \omega = a+\delta$$

$$\psi = -\frac{ca^2}{2}\underbrace{\left[1-\frac{3}{2}\left(1-\frac{\delta}{a}\right)+\frac{1}{2}\left(1-\frac{3\delta}{a}\right)\right]}_{=0}\left(1+2\frac{\delta}{a}\right)$$

$$= -\frac{ca^2}{2}\left[1-\frac{3}{2}\left(1+\frac{\delta}{a}\right)^{-1}+\frac{1}{2}\left(1+\frac{\delta}{a}\right)^{-3}\right]\left(1+\frac{\delta}{a}\right)^2$$

$$= -\frac{ca^2}{2}\left[\left(1+\frac{\delta}{a}\right)^2-\frac{3}{2}\left(1+\frac{\delta}{a}\right)+\frac{1}{2}\left(1+\frac{\delta}{a}\right)^{-1}\right]$$

$$= -\frac{ca^2}{2}\left[1+\frac{2\delta}{a}+\frac{\delta^2}{a^2}-\frac{3}{2}-\frac{3}{2}\frac{\delta}{a}+\frac{1}{2}\left(1-\frac{\delta}{a}-\frac{\delta^2}{a^2}\right)\right]$$

$$\psi = -\frac{ca^2}{2}\frac{\delta^2}{2a^2} = -\frac{c\delta^2}{4}$$

W wielkiej odległości $r=\infty$: $\psi = -\frac{c}{2}\omega^2 = -c\frac{\delta^2}{4}$

zatem: $\omega = \frac{\delta}{\sqrt{2}}$

Dotychczasowe obliczenia ładunku powstającego przez ruch polegają na założeniu że równanie $\nabla^2 u = -4\pi\varepsilon$ ważne tylko w samej warstwie, a natychmiast przy wyjściu z jej obrębu wszystkie otrzymujemy $\varepsilon = 0$

Jeżeli jednak przewodn. małe, a ε duże, to musi to być zastąpione równaniem:

$$-\frac{D\varepsilon}{Dt} = \frac{\partial i_x}{\partial x} + \frac{\partial i_y}{\partial y} + \frac{\partial i_z}{\partial z} = -\lambda\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}\right)$$

$$= 4\pi\lambda\varepsilon$$

$$\downarrow$$

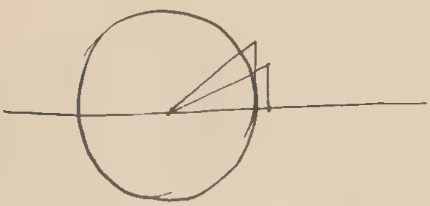
$$\cancel{\frac{\partial\varepsilon}{\partial t}} + u\frac{\partial\varepsilon}{\partial x} + v\frac{\partial\varepsilon}{\partial y} + w\frac{\partial\varepsilon}{\partial z}$$

Jeżeli się położy x w kierunek linii prądu s:

$$-u\frac{\partial\varepsilon}{\partial s} = 4\pi\lambda\varepsilon$$

$$\frac{1}{\varepsilon}\frac{\partial\varepsilon}{\partial s} = -\frac{4\pi\lambda}{u}$$

$$\lg\varepsilon = -4\pi\lambda\int\frac{ds}{u}$$



$w = r$

$$-c\frac{\delta_0^2}{4} = -\frac{c(\delta_\theta \sin\theta)^2}{4}$$

$$\delta_\theta = \frac{\delta_0}{\sin\theta}$$

---

odpowiadająca wartość δ w punkcie $\theta_1 : \theta$

$$\delta_0 = \delta_1 \sin\theta_1 = \rho\sin\theta$$

zatem w każdej warstwie

$\varepsilon = \text{const}$

$$\int_{\theta_1}^{\theta} \frac{a\,d\theta}{\frac{3}{2}\frac{c}{a}\sin\theta_1 . \delta_1} = \frac{2}{3}\frac{a^2}{c\delta_1}\;\frac{\theta_1 - \theta}{\sin\theta_1}$$

$$\varepsilon = \varepsilon_0^{\,-4\pi\lambda\,\frac{2}{3}\,\frac{a^2}{c\delta}\,\frac{\theta_1-\theta}{\sin\theta_1}}$$

$$-4\pi\lambda\,\frac{2}{3}\,\frac{a^2}{c\delta}\,\frac{\theta_1-\theta}{\sin\vartheta_1} = \lg\frac{\varepsilon}{\varepsilon_0}$$

N.p. $\frac{\varepsilon}{\varepsilon_0} = \frac{1}{2}$ $\qquad \theta_1 = \frac{\pi}{2}$ $\qquad a = 10^{-5}$

$\delta = 10^{-6}$

$\lambda = \frac{1}{5}.10^{8}$

Wstawmy do a

$$\frac{2}{3}\,4.\pi.\frac{1}{5}.10^{8}.\frac{10^{-5}\left(\frac{\pi}{2}-\theta\right)}{10^{-6}.\,c} = 0{\cdot}3.\ \ 2{\cdot}3$$

Jeśli u stałe $\qquad \lg\frac{\varepsilon}{\varepsilon_0} = -4\pi\lambda\frac{s}{u}$

u potrzebne tu mniejsze niż s

N.p. $s = 10^{-6}$

$$0{\cdot}3.\ 2{\cdot}3. = -4\pi\,\frac{\frac{1}{5}.10^{8}.10^{-6}}{u}$$

$u =$ cca $200\ \frac{cm}{sec}$!

zatem to wyjść wcale nie

może wyjść !

~~Następ~~ W wodzie dest. (najczyst.) te drobne ilości elektr. natychmiast się rozproszą

ale w lepszych izolatorach nastąpi znaczna deformacya warstwy elektryczn.ej

Jaki ruch cieczy koło baniek powietrza przechodzących przez wodę?

Zdaje się że to będzie podobnie jak ruch kul w cieczy idealnej!

bo tarcie na powierzchni można zaniedbać

Czy nie możnaby utworzyć kulkę z błonki n.p. kollodium, napełnić H_2 aby równy ciężar jako powietrze i puścić do wody, porównując z powietrzem?

Zatem będzie siły ruch jego przystający

Podobnie dla cieczy lżejszej od wody!

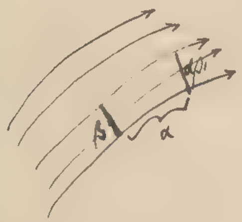
Bańki H_2 będą powolniej przechodzić (z powodu znacznego tarcia) niż O_2.

Analogia do „Spherical Vortex" Lamb. p 265

Naczynie w którym prąd cieczy z ~~at.~~ tarciem

W bliskości powierzchni prędkość prop. do odstępu; kierunek: system krzyw.

$$v = \left(\frac{\partial v}{\partial n}\right)_0 N$$



to będzie zatem także kierunkiem linij prądu elektr.

prąd $$\frac{i}{d\beta} = \int \left(\frac{\partial v}{\partial n}\right)_0 N \, z \, dN = \left(\frac{\partial v}{\partial n}\right)_0 \frac{(\varphi_0 - \varphi_i)}{4\pi}$$

Miejscami wrzenia będzie $(\operatorname{div} i) = \lambda \frac{\partial U}{\partial n}$

$$i\,d\beta - i'\,d\beta' = i(d\beta - d\beta') + (i - i')\,d\beta' \quad \Big\} \quad \lambda \frac{\partial U}{\partial n} = \frac{\partial i_\alpha}{\partial \alpha} + \frac{\partial i_\beta}{\partial \beta}$$

$$= \left[i \frac{\partial d\beta}{\partial \alpha} + d\beta \frac{\partial i}{\partial \alpha}\right] d\alpha = \lambda \frac{\partial U}{\partial n} d\beta\, d\alpha$$

$\cancel{\int_{\alpha_1}^{\alpha_2} \frac{\partial v}{\partial x}\, d\beta\, d\alpha =}$ Czy te konwekcyjne można zastąpić przez potencyj fikcyjny taki jakby je powodował?

$$\frac{i}{d\beta} = \cancel{\left(\frac{\partial v}{\partial n_0}\right)\frac{\varphi_a-\varphi_i}{4\pi} = \lambda}\; \delta' \frac{\partial U'}{\partial \alpha}$$

$$\cancel{\frac{\partial p}{\partial x} = \mu \nabla^2 u}$$

$$\int d\alpha\, d\beta$$

$$\cancel{= \iiint \partial}$$

$$\iint \left(\frac{\partial u}{\partial N}\right)_0 dF = \iiint \nabla^2 u \; dx\,dy\,dz = \frac{1}{\mu}\iiint \frac{\partial p}{\partial x}\, dx\,dy\,dz = \frac{1}{\mu}\iint p\; dy\,dz$$

$$= \frac{1}{\mu}\iint p \cos nx \,.\, dF$$

$$\iint\left[\cos nx \cdot \left(\frac{\partial u}{\partial N}\right) + \cos ny \left(\frac{\partial v}{\partial N}\right) + \cos nz \left(\frac{\partial w}{\partial N}\right)\right] dF = \frac{1}{\mu}\iint p\; dF$$

że $U = \text{const}$ i $p = \text{const}$ są identyczne (?)

$\Delta^2 U = 0$ $\quad\Delta^2 p = 0$ więc jeśli identyczne warunki graniczne to wystarczy

więc $\frac{\partial U}{\partial n}$ powinno być $\sim \frac{\partial p}{\partial n}$

$$\lambda \frac{\partial U}{\partial n} = \frac{\partial i_\alpha}{\partial \alpha} + \frac{\partial i_\beta}{\partial \beta} = \frac{\varphi_a - \varphi_i}{4\pi}\left[\underbrace{\frac{\partial}{\partial \alpha}\left(\frac{\partial v_\alpha}{\partial n_0}\right) + \frac{\partial}{\partial \beta}\left(\frac{\partial v_\beta}{\partial n_0}\right)}\right]$$

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial z}\right)_0 + \frac{\partial}{\partial y}\left(\frac{\partial v}{\partial z}\right)_0 = -\left(\frac{\partial^2 w}{\partial z^2}\right)_0 = \nabla_0^2 w = \frac{1}{\mu}\frac{\partial p}{\partial n}$$

Więc byłoby wogóle:

$$U_1 - U_2 = \frac{\varphi_a - \varphi_i}{4\pi}\frac{1}{\partial \mu}(p_1 - p_2) \quad (?)$$

Jaki procent elektrycznej pracy przy wytwarzaniu Strömungsström ?

$$W_e = \frac{P^2(\varphi_i-\varphi_a)^2}{(4\pi\mu)^2}\,\frac{\varphi\sigma}{L} \qquad W_f = \frac{\pi P^2 R^4}{8\mu L}$$

$$\frac{W_e}{W_f} = \frac{(\varphi_i-\varphi_a)^2\sigma}{2\mu\pi\varphi} = \frac{\left(\frac{4}{300}\right)^2\cdot 4\cdot 6\cdot 10^{-8}}{2\cdot 3.14\cdot 0.010\cdot 10^{-4}} = 10^{-6}$$

$\varphi = 10^{-4}$

powiększyć można: podwyższając $\sigma = \frac{1}{\lambda}$

zmniejszając φ

Np. używając diafragmy i dobry izolator

Jeżeli ruch Browna spowodowany przez zewnętrzne przyczyny, jaka energia przez nie rozproszona ?

Np. $2r = 0.001$ mm $= 10^{-4}$ cm

$v = 0.003\,\frac{mm}{sec} = 3\cdot 10^{-4}$

opór $= 6\pi a\mu v \qquad \mu = 0.018$

praca po sec: $6\pi a\mu v^2 = \frac{\cancel{6}\,3\cdot 3.14\cdot 10^{-4}\cdot 0.018\cdot 9\cdot 10^{-8}}{\cancel{2}}$

$= 10^{-12}\cdot\frac{0.17}{1.5}\cdot 9 \qquad = 1.5\cdot 10^{-12}$ Erg

objętość takiej kulki: $\frac{4\pi}{3}\,\frac{1}{8}\,10^{-12} = \frac{1}{2}\,10^{-12}$ cm^3

Zatem praca po 1 cm^3 substancji: $3\,\frac{Erg}{sec}$

Przez dzień: $60\cdot 60\cdot 24 = 86400$ sec $\qquad 260.000$ Erg $= \frac{260.000}{42\cdot 10^6}$ gcal $= \frac{1}{160}$ gcal

Energja powierzchniowa kulki: $\alpha . O = 4\pi r^2 \alpha$

$$= 3{\cdot}14 . 80 . 10^{-8} \text{ (Erg)}$$

$$= 25 . 10^{-7}$$

Zatem Energja [illegible] [n.p. wody] wystarczyłaby do ogrzania ruchu na $\dfrac{25.10^{-7}}{1{\cdot}5.10^{-12}} = 10 . 10^5$ sec.

$= 2$ dni

$\cancel{\frac{16.10^{5}}{[illegible]}}$

Ruch kuli w cieczy; czy nie można uwzględnić „inertia terms"

Dla tarcia:

$$u = \frac{M}{\cancel{2}}\left(\frac{1}{r} + \frac{x^2}{r^3}\right) + \frac{N}{r^3}\left(1 - \frac{3x^2}{r^2}\right) + c$$

$$v = \frac{M}{\cancel{2}} \frac{xy}{r^3} - \frac{3N}{r^5} xy$$

$$w = \frac{M}{\cancel{2}} \frac{xz}{r^3} - \frac{3N}{r^5} xz$$

Bez tarcia:

$$u = \frac{N}{r^3}\left(1 - \frac{3x^2}{r^2}\right) + c$$

$$v = -\frac{3N}{r^5} xy$$

$$w = \frac{3N}{r^5} xz$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x}$$

$$0 = \frac{\partial p}{\partial x} - \mu \nabla^2 u$$

$$0 = \frac{\partial p}{\partial y} - \mu \nabla^2 v$$

$$0 = \frac{\partial p}{\partial z} - \mu \nabla^2 w$$

$$(u_1+u_2)\frac{\partial(u_1+u_2)}{\partial x} + \dots = -\frac{\partial p}{\partial x} + \mu (\nabla^2 u_1 + \nabla^2 u_2)$$

$$u_1\frac{\partial u_1}{\partial x} + u_2\frac{\partial u_1}{\partial x} + u_1\frac{\partial u_2}{\partial x} + \cancel{u_2\frac{\partial u_2}{\partial x}} = -\cancel{\frac{\partial p_1}{\partial x}} + \cancel{\frac{\partial p_2}{\partial x}} + \frac{\partial p_3}{\partial x} + \mu \cancel{(\nabla^2 u_1 + \nabla^2 u_2)}$$

$$= u_1 \frac{\partial(u_1+u_2)}{\partial x} + u_2 \frac{\partial \dots}{\partial x}$$

$$u = \frac{M}{r}\left(\frac{1}{r} + \frac{x^2}{r^3}\right) + \frac{N}{r^3}\left(1 - \frac{3x^2}{r^2}\right) + c$$

La teoria: $\cancel{u = c + B\frac{\partial}{\partial x}\left(\frac{x}{r^3}\right) - \frac{A}{\mu}\frac{r^2}{6}\frac{\partial}{\partial x}\left(\frac{x}{r^3}\right)}$

$$\frac{\partial u}{\partial x} = M\left[-\frac{x}{r^3} + \frac{2x}{r^3} - \frac{3x^3}{r^5}\right] + N\left[-\frac{3x}{r^5} - \frac{6x}{r^5} + \frac{15x^3}{r^7}\right]$$

$$= M\left[\frac{x}{r^3} - \frac{3x^3}{r^5}\right] + N\left[-\frac{9x}{r^5} + \frac{15x^3}{r^7}\right]$$

$$\frac{\partial u}{\partial y} = M\left[\frac{-y}{r^3} - \frac{3x^2y}{r^5}\right] + N\left[\frac{-3y}{r^5} + \frac{15x^2y}{r^7}\right]$$

$$\frac{\partial u}{\partial z} = M\left[-\frac{z}{r^3} - \frac{3x^2z}{r^5}\right] + N\left[-\frac{3z}{r^5} + \frac{15x^2z}{r^7}\right]$$

$\frac{1}{r} + \frac{x^2}{r^3}$	$\frac{N}{r^3}\left(1 - \frac{3x^2}{r^2}\right) + c$
$\frac{xy}{r^3}$	$-\frac{3Nxy}{r^5}$
$\frac{xz}{r^3}$	$-\frac{3Nxz}{r^5}$

$$M\left\{\cancel{\frac{x}{r^4}} - \cancel{\cdots} - \frac{3x^3}{r^6} + \cancel{\frac{x^3}{r^6}} - \cancel{\frac{3x^5}{r^8}} - \cancel{\frac{xy^2}{r^6}} - \cancel{\frac{3x^3y^2}{r^8}} - \cancel{\frac{xz^2}{r^6}} - \cancel{\frac{3x^3z^2}{r^8}}\right\} +$$

$$+N\left\{-\frac{12x}{r^6} + \cancel{\frac{15x^3}{r^8}} - \frac{9x^3}{r^8} + \cancel{\frac{15x^5}{r^{10}}} - \cancel{\frac{3xy^2}{r^8}} + \cancel{\frac{15x^3y^2}{r^{10}}} - \cancel{\frac{3xz^2}{r^8}} + \cancel{\frac{15x^3z^2}{r^{10}}}\right\}$$

$$= \cancel{M\left\{-\frac{5x^3}{r^6} - \frac{3x^5}{r^8}\right\} + N\left\{-\frac{12x}{r^6} + \frac{x^5}{r^6} + \frac{15x^5}{r^{10}}\right\}} \quad -\frac{12x}{r^6} + \frac{27x^3}{r^8}$$

$$v = \frac{xy}{r^3}\left[M - \frac{3N}{r^2}\right]$$

$$\cancel{\frac{\partial v}{\partial x} = \left[\frac{y}{r^3} - \frac{3x^2y}{r^5}\right]\left[M - \frac{3N}{r^2}\right] + \frac{6Nx^2y}{r^7}}$$

$$\frac{\partial v}{\partial x} = M\left[\frac{y}{r^3} - \frac{3x^2y}{r^5}\right] + 3N\left[-\frac{y}{r^5} + \frac{5x^2y}{r^7}\right]$$

$$\frac{\partial v}{\partial y} = M\left[\frac{x}{r^3} - \frac{3xy^2}{r^5}\right] + 3N\left[-\frac{x}{r^5} + \frac{5xy^2}{r^7}\right]$$

$$\frac{\partial v}{\partial z} = M\frac{-3xyz}{r^5} + 3N\frac{+5xyz}{r^7}$$

$\frac{1}{r} + \frac{x^2}{r^3}$	$\frac{N}{r^3}\left(1 - \frac{3x^2}{r^2}\right) + c$
$\frac{xy}{r^3}$	$-\frac{3xy}{r^2}$
$\frac{xz}{r^3}$	$-\frac{3xz}{r^2}$

$$\underline{M}\left\{\frac{y}{r^4}-\frac{3x^2y}{r^6}+\cancel{\frac{x^2y}{r^6}}-\cancel{\frac{3x^4y}{r^8}}+\cancel{\frac{x^2y}{r^6}}-\cancel{\frac{3x^2y^3}{r^8}}-\cancel{\frac{3x^2yz^2}{r^8}}\right\}$$

$$+3N\left\{-\frac{y}{r^6}+\frac{5x^2y}{r^8}-\frac{x^2y}{r^8}+\cancel{\frac{5x^4y}{r^{10}}}-\frac{x^2y}{r^8}+\cancel{\frac{5x^2y^3}{r^{10}}}+\cancel{\frac{5x^2yz^2}{r^{10}}}\right\}=$$

$$II=\frac{M}{r}\left[M\left\{\frac{y}{r^4}-\frac{4x^2y}{r^6}\right\}+3N\left\{\frac{y}{r^6}+\frac{8x^2y}{r^8}\right\}\right]$$

$$\cancel{M\left\{\frac{x}{r^4}-\frac{2x^3}{r^6}\right\}}\quad I=\frac{M}{r}\left[M\left\{-\frac{4x^3}{r^6}\right\}+N\left\{-\frac{12x}{r^6}+\frac{24x^3}{r^8}\right\}\right]$$

Gribs uzy $\frac{\partial I}{\partial y}-\frac{\partial II}{\partial x}=0$

$$-4M\left\{-\frac{6x^3y}{r^8}\right\}+12N\left\{+\frac{6xy}{r^8}-\frac{16x^3y}{r^{10}}\right\}\ \Big\|\ \underline{M}\left\{-\frac{4xy}{r^6}-\frac{8xy}{r^6}+\frac{24x^3y}{r^8}\right\}+$$

$$+3N\left\{-\frac{6xy}{r^8}+\frac{16xy}{r^8}-\frac{64x^3y}{r^{10}}\right\}$$

$72-30$

$$\underline{M}\left\{\frac{12xy}{r^6}\right\}+N\left\{\frac{42xy}{r^8}\right\}=\cancel{\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}}$$

$$u_2\frac{\partial u_1}{\partial x}+v_2\frac{\partial u_1}{\partial y}+w_2\frac{\partial u_1}{\partial z}=Mc\left(\frac{x}{r^3}-\frac{3x^3}{r^5}\right)+\frac{MN}{r^3}\left\{\frac{x}{r^3}-\frac{3x^3}{r^5}-\frac{3x^3}{r^5}+\cancel{\frac{9x^5}{r^7}}\right.$$

$\frac{7x}{r^3}$

$$\left.+\frac{3xy^2}{r^5}+\cancel{\frac{9x^3y^2}{r^7}}+\frac{3xz^2}{r^5}+\cancel{\frac{9x^3z^2}{r^7}}\right\}$$

$$=Mc\left(\frac{x}{r^3}-\frac{3x^3}{r^5}\right)+MN\cdot\frac{4x}{r^6}$$

$$u_2 \frac{\partial v_1}{\partial x} + v_2 \frac{\partial v_1}{\partial y} + w_2 \frac{\partial v_1}{\partial z} = Mc\left(\frac{y}{r^3} - \frac{3x^2y}{r^5}\right) + \frac{MN}{r^3}\left\{\frac{y}{r^3} - \frac{3x^2y}{r^5} - \frac{3x^2y}{r^5} + \frac{9x^4y}{r^7} - \right.$$

$$\left. - \frac{3x^2y}{r^5} + \frac{9x^2y^3}{r^7} + 9\frac{x^2yz^2}{r^7}\right\}$$

$$= Mc\left(\frac{y}{r^3} - \frac{3x^2y}{r^5}\right) + MN\frac{y}{r^6}$$

$$\frac{\partial}{\partial y}\left\{Mc\left(\frac{x}{r^3} - \frac{3xy^2}{r^5}\right) + MN\frac{4x}{r^6}\right\} - \frac{\partial}{\partial x}\left\{Mc\left(\frac{y}{r^3} - \frac{3x^2y}{r^5}\right) + MN\frac{y}{r^6}\right\} =$$

$$Mc\left\{-\frac{3xy}{r^5} + 15\frac{x^3y}{r^7} + \frac{3xy}{r^5} - 15\frac{x^3y}{r^7} - \frac{6xy}{r^5}\right\} + MN\left\{-\frac{24xy}{r^8} + 6\frac{xy}{r^8}\right\}$$

$$= Mc\left(-\frac{6xy}{r^5}\right) + MN\left(\frac{18xy}{r^8}\right)$$

$$M^2\frac{12xy}{r^6} + MN\frac{60xy}{r^8} = \cancel{\ldots} \geq 0$$ zatem nie może być rozwiązaniem

$$0 = \frac{\partial p_1}{\partial x} - \mu \nabla^2(u_1+u_2) \qquad u_2 \frac{\partial u_2}{\partial x} + v_2 \frac{\partial u_2}{\partial y} + w_2 \frac{\partial u_2}{\partial z} = -\frac{\partial p_2}{\partial x}$$

$$0 = \frac{\partial p_1}{\partial y} - \mu \nabla^2(v_1+v_2) \qquad u_2 \frac{\partial v_2}{\partial x} + v_2 \frac{\partial v_2}{\partial y} + w_2 \frac{\partial v_2}{\partial z} = -\frac{\partial p_2}{\partial y}$$

$$0 = \frac{\partial p_1}{\partial z} - \mu \nabla^2(w_1+w_2)$$

Próba czy: $(u_1+u_2)\frac{\partial(u_1+u_2)}{\partial x} + \dots = -\frac{\partial p}{\partial x} + \mu \nabla^2(u_1+u_2)$

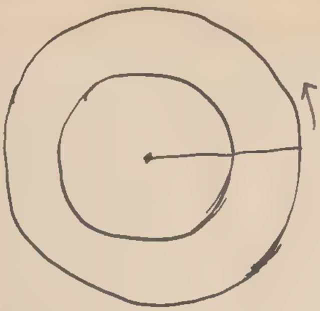
$$u_1 \frac{\partial u_1}{\partial x} + u_2 \frac{\partial u_1}{\partial x} + u_1 \frac{\partial u_2}{\partial x} + u_2 \frac{\partial u_2}{\partial x} - \quad = -\frac{\partial p}{\partial x} + \mu \nabla^2(u_1+u_2)$$

$$= -\frac{\partial(p-p_1)}{\partial x}$$

$$u_1 \frac{\partial u}{\partial x} + u_2 \frac{\partial u_1}{\partial x}$$

$$= \frac{\partial(p_1+p_2-p)}{\partial x}$$

Ruch walcowaty $\qquad \frac{f}{r} = \omega$ = prędkość kątowa $= \varphi(r)$



$$u = -f\frac{y}{r} = -\varphi y$$

$$v = f\frac{x}{r} = \varphi x$$

$$\frac{\partial u}{\partial x} = -\varphi' \frac{xy}{r} \qquad \frac{\partial v}{\partial y} = \varphi' \frac{xy}{r}$$

$$\frac{\partial^2 u}{\partial x^2} = -\varphi'' \frac{x^2 y}{r^2} - \frac{\varphi' y}{r} + \varphi' \frac{x^2 y}{r^3} \qquad \frac{\partial^2 v}{\partial y^2} = \varphi'' \frac{x y^2}{r^2} + \frac{\varphi' x}{r} - \varphi' \frac{x y^2}{r^3}$$

$$\frac{\partial u}{\partial y} = -\varphi - \varphi' \frac{y^2}{r} \qquad \frac{\partial v}{\partial x} = \varphi + \varphi' \frac{x^2}{r}$$

$$\frac{\partial^2 u}{\partial y^2} = -3\varphi' \frac{y}{r} - \varphi'' \frac{y^3}{r^2} + \varphi' \frac{y^3}{r^3} \qquad \frac{\partial^2 v}{\partial x^2} = 3\varphi' \frac{x}{r} + \varphi'' \frac{x^3}{r^2} - \varphi' \frac{x^3}{r^3}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -\varphi'' y - 3\varphi' \frac{y}{r} \qquad \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \varphi'' x + 3\varphi' \frac{x}{r}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \varphi\varphi' \frac{xy^2}{r} - \varphi^2 x - \varphi\varphi' \frac{xy^2}{r} = -\varphi^2 x$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = -\varphi^2 y - \varphi\varphi' \frac{x^2 y}{r} + \varphi\varphi' \frac{x^2 y}{r} = -\varphi^2 y$$

$$\frac{\partial^2 p}{\partial x \partial y} = -2\varphi\varphi' \frac{xy}{r} = 2\varphi\varphi' \frac{xy}{r}$$ nie daje żadnego warunku

I. Bez tarcia:

$$\left.\begin{aligned} -\varphi^2 x &= -\frac{\partial p}{\partial x} \;\Big\|\; \cdot \frac{x}{r} \\ -\varphi^2 y &= -\frac{\partial p}{\partial y} \;\Big\|\; \cdot \frac{y}{r} \end{aligned}\right\} \quad \frac{dp}{dr} = \varphi^2 \frac{x^2+y^2}{r} = r\cdot\varphi^2$$

$$p = \int \varphi^2 \cdot r\, dr$$

φ nieoznaczone, dowolne

mimo że dane mogą być φ na powierzchni

II Powolne tarcie:

$$\frac{1}{\mu}\frac{\partial p}{\partial x} = -\varphi'' y - 3\varphi' \frac{y}{r} \qquad \frac{x}{r}$$

$$\frac{1}{\mu}\frac{\partial p}{\partial y} = \varphi'' x + 3\varphi' \frac{x}{r} \qquad \frac{y}{r}$$

$$\frac{1}{\mu}\frac{dp}{dr} = 0$$

$$p = \text{const}$$

$$\varphi'' + \frac{3\varphi'}{r} = 0$$

$$\frac{d}{dr}(r^3\varphi') = r^3\varphi'' + 3r^2\varphi' = r^3\left(\varphi'' + \frac{3\varphi'}{r}\right) = 0$$

$$\frac{1}{\mu}\frac{\partial^2 p}{\partial x\partial y} = -\varphi'' - \frac{3\varphi'}{r} - \varphi''' \frac{xy}{r} - 3\varphi'' \frac{xy}{r^2} + 3\varphi' \frac{xy}{r^3}$$

$$= \varphi'' + \frac{3\varphi'}{r} + \varphi''' \frac{xy}{r} + 3\varphi'' \frac{xy}{r^2} - 3\varphi' \frac{xy}{r^3} = 0$$

$$\frac{3}{r^4} - \frac{3xy}{r^6} - \frac{3}{r^4} - 9\frac{xy}{r^6} + 12\frac{xy}{r^6} = 0$$

spełnione przez $\varphi'' = -\frac{3c}{r^4}$, $\varphi''' = \frac{12c}{r^5}$

$$r^3\varphi' = c$$

$$d\varphi = \frac{c\,dr}{r^3}$$

$$\varphi = -\frac{c}{2r^2} + b$$

III Szybkie tarcie:

$$-\varphi^2 x = -\frac{\partial p}{\partial x} - \mu\left(\varphi'' y + 3\varphi' \frac{y}{r}\right) \qquad \frac{x}{r} \qquad \frac{\partial}{\partial y}$$

$$-\varphi^2 y = -\frac{\partial p}{\partial y} + \mu\left(\varphi'' x + 3\varphi' \frac{x}{r}\right) \qquad \frac{y}{r} \qquad -\frac{\partial}{\partial x}$$

$$-\varphi^2 \cdot \frac{x^2+y^2}{r} = -\frac{dp}{dr} = -\varphi^2 \cdot r$$

$$\underbrace{-2\varphi\varphi' \frac{xy}{r} + 2\varphi\varphi' \frac{xy}{r}}_{0} = \dots\dots$$

Zatem to samo rozwiązanie co przedtem

tylko że p teraz będzie inne

$$\varphi = -\frac{c}{2r^2} + b$$

$$\varphi' = \frac{c}{r^3}$$

$$\varphi'' = -\frac{3c}{r^4}$$

Mianowicie: $\frac{dp}{dr} = r\varphi^2$

$$p = \int r \underbrace{\left(b - \frac{c}{2r^2}\right)^2}_{b^2 - \frac{bc}{r^2} + \frac{c^2}{4r^4}} dr$$

Więc w całem 3 stałe: b, c, p_0!

$$p = \left[b^2 \frac{r^2}{2} - bc \lg r - \frac{c^2}{8r^2}\right] + p_0$$

Przy tem wszystkiem spółczynnik tarcia μ wcale nie wchodzi w rachunki

więc ten sam układ mógłby się ustanowić (z czasem) dla cieczy z nieskończ. małem tarciem

więc przez warunek: „nieskończ. małe tarcie" równania hydrod. stały by się oznaczone dla danych r

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Wogóle: $\begin{cases} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + \mu \nabla^2 u \\ \\ \end{cases}$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Energia rozproszona: $\Phi = \mu\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left[\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right]\right.$

Dwa przypadki: albo $u\,v\,w$ niezależne wogóle od μ

albo ruch taki że $\Phi = 0$

To ostatnie tylko możliwe jeśli $\frac{\partial u}{\partial x} = \dots = 0$ jeśli μ skończone

albo też μ nieskończ. małe

W ogólnym przypadku rozwiązać i potem μ robić coraz mniejsze

wtedy rozwinąć: w szereg Maclaurina: $u = u_0 + \mu u_1 + \frac{\mu^2}{2} u_2 + \cdots$

jeśli w ogóle możliwe takie rozwinięcie

$$\frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} + \mu \frac{\partial u_1}{\partial x}$$

zaniedbać wszystkie wyższe potęgi z wyjątkiem pierwszej i 0

$$u_0 \frac{\partial u_0}{\partial x} + v_0 \frac{\partial u_0}{\partial y} + w_0 \frac{\partial u_0}{\partial z} = - \frac{\partial p_0}{\partial x}$$

$$u_0 \frac{\partial u_1}{\partial x} + v_0 \frac{\partial u_1}{\partial y} + w_0 \frac{\partial u_1}{\partial z} + u_1 \frac{\partial u_0}{\partial x} + v_1 \frac{\partial u_0}{\partial y} + w_1 \frac{\partial u_0}{\partial z} = - \frac{\partial p_1}{\partial x} + \nabla^2 u_0$$

$$\cancel{u_0 \frac{\partial (u_0+u_1)}{\partial x} + v_0 \frac{\partial (u_0+u_1)}{\partial y} + w_0 \frac{\partial (u_0+u_1)}{\partial z} + (u_0+u_1) \frac{\partial u_0}{\partial x} + (v_0+v_1) \frac{\partial u_0}{\partial y} + (w_0+v_1) \frac{\partial u_0}{\partial z} = - \frac{\partial (p_1 + 2p_0)}{\partial x} + \nabla^2 u_0}$$

$$u_\mu = (u)_0 + \mu \left(\frac{\partial u}{\partial \mu}\right)_0 + \frac{\mu^2}{1.2} \left(\frac{\partial^2 u}{\partial \mu^2}\right)_0 + \cdots$$

$$u_1 = \left(\frac{\partial u}{\partial \mu}\right)_0 \quad \text{itd.}$$

$$\left(\frac{\partial}{\partial \mu}\left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}\right]\right)_0 = - \left(\frac{\partial p}{\partial x \partial \mu}\right)_0 + \nabla^2 u$$

Jeśli ruch niezależny od p to równoczesnie:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{\partial p}{\partial x}$$
$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}$$
$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}$$

$$\Delta^2 u=0 \qquad \Delta^2 v=0 \qquad \Delta^2 w=0$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

$$\underbrace{v\frac{\partial^2 u}{\partial x^2}-u\frac{\partial^2 v}{\partial x^2}}_{\partial}+\underbrace{v\frac{\partial^2 u}{\partial y^2}-u\frac{\partial^2 v}{\partial y^2}}=0$$

Rugując $\frac{\partial^2 p}{\partial x\partial y}$:

$$u\frac{\partial u}{\partial x}+v\underbrace{\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)}+w\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)+v\frac{\partial v}{\partial x}+w\frac{\partial w}{\partial x}$$

$$\frac{1}{2}\frac{\partial}{\partial x}(u^2+v^2+w^2)+v\zeta-w\eta=-\frac{\partial p}{\partial x}$$
$$\frac{1}{2}\frac{\partial}{\partial y}(u^2+v^2+w^2)+w\xi-u\zeta=-\frac{\partial p}{\partial y}$$
$$\frac{1}{2}\frac{\partial}{\partial z}(u^2+v^2+w^2)+u\eta-v\xi=-\frac{\partial p}{\partial z}$$

$$\cancel{u\frac{\partial u}{\partial x}+u\frac{\partial v}{\partial y}+u\frac{\partial w}{\partial z}}$$
$$\cancel{-u\frac{\partial u}{\partial x}-v\frac{\partial v}{\partial x}-w\frac{\partial w}{\partial x}}$$

$$\nabla(v\,.\,\mathrm{curl}\,v)=-\nabla\left(p+\frac{u^2+v^2+w^2}{2}\right)$$

pod jakiemi warunkami mogą równania istnieć?
$$\begin{cases}\mathrm{curl}\,\nabla(v\,.\,\mathrm{curl}\,v)=0\\ \Delta^2 v=0\\ \mathrm{div}\,v=0\end{cases}$$

$$\frac{\partial v}{\partial y}.\zeta-\frac{\partial w}{\partial y}\eta+v\frac{\partial \zeta}{\partial y}-w\frac{\partial \eta}{\partial y}-\frac{\partial v}{\partial x}\xi+\frac{\partial u}{\partial x}\zeta-w\frac{\partial \xi}{\partial x}+u\frac{\partial \zeta}{\partial x}=0 \qquad \pm\frac{\partial w}{\partial z}.\zeta \qquad \pm w\frac{\partial \zeta}{\partial z}$$

$$\zeta\cancel{\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)}-\left(\xi\frac{\partial w}{\partial x}+\eta\frac{\partial w}{\partial y}+\zeta\frac{\partial w}{\partial z}\right)-w\cancel{\left(\frac{\partial \xi}{\partial x}+\frac{\partial \eta}{\partial y}+\frac{\partial \zeta}{\partial z}\right)}+\left(u\frac{\partial \zeta}{\partial x}+v\frac{\partial \zeta}{\partial y}+w\frac{\partial \zeta}{\partial z}\right)=0$$

~~curl (V × curl v) = ... curl~~

$$u\,\Delta^2 u = u\frac{\partial^2 u}{\partial x^2} + u\frac{\partial^2 u}{\partial y^2} + u\frac{\partial^2 u}{\partial z^2} \quad *$$

$$\pm\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = \frac{\partial}{\partial x}\left(u\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(u\frac{\partial u}{\partial z}\right)$$

$$-\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right]$$

$$= \frac{1}{2}\Delta^2(u^2) - \left[\left(\frac{\partial u}{\partial x}\right)^2 + \cdots \right]$$

Dwuwymiarowe zadanie:

$$w = \frac{\partial}{\partial z} = \xi = \eta = 0$$

$$\begin{cases} u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} = -\dfrac{\partial p}{\partial x} \\[2ex] u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y} = -\dfrac{\partial p}{\partial y} \end{cases}$$

$$\Delta^2 u = 0$$

$$\underbrace{\Delta^2 v = 0}_{\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} = 0$$

$$\downarrow$$

$$u\underbrace{\left(\frac{\partial^2 u}{\partial x\partial y} - \frac{\partial^2 v}{\partial x^2}\right)}_{\|} + v\underbrace{\left(\frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 v}{\partial x\partial y}\right)} = 0$$

$$u\left(\frac{\partial^2 u}{\partial x\partial y} + \frac{\partial^2 v}{\partial y^2}\right) - v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x\partial y}\right) = 0$$

$$u\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - v\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

więc to równanie i tak spełnione jest ↑ spełnione

$\Delta^2 u = 0 \qquad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

$\Delta^2 v = 0$

$u = f_1(x+iy) = \varphi_1(xy) + i\psi_1(xy)$

$\frac{\partial u}{\partial x} = f' \qquad \frac{\partial u}{\partial y} = if'$

$\frac{\partial^2 u}{\partial x^2} = f'' \qquad \frac{\partial^2 u}{\partial y^2} = -f''$

$v = if_2(x+iy) = i\varphi_2(xy) - \psi_2(xy)$

$\frac{\partial v}{\partial y} = -f_2'$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$f_1' - f_2' = 0$

$f_1(z) = f_2(z) + c$

$f(x+iy) = \varphi(x,y) + i\psi(x,y)$

$\Delta^2\varphi = 0 \qquad \Delta^2\psi = 0$

$u = \varphi(x,y)$

$$\left.\begin{aligned} f' &= \frac{\partial\varphi}{\partial x} + i\frac{\partial\psi}{\partial x} \\ if' &= \frac{\partial\varphi}{\partial y} + i\frac{\partial\psi}{\partial y}\end{aligned}\right\}$$

$\frac{\partial u}{\partial x} = \frac{\partial\varphi}{\partial x} = \frac{\partial\psi}{\partial y} = -\frac{\partial v}{\partial y}$

$\frac{\partial\varphi}{\partial x} = +\frac{\partial\psi}{\partial y}$

$v = -\psi(x,y) + f_c(x)$

$\frac{\partial\varphi}{\partial y} = -\frac{\partial\psi}{\partial x}$

~~z differenziare rispetto a y~~

$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2\psi}{\partial x\partial y} = -\frac{\partial^2 v}{\partial x\partial y} + \frac{\partial^2 f_c(x)}{\partial x\partial y}$

$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = \frac{\partial^2 f_c(x)}{\partial x\partial y} = 0$

$\frac{\partial^2 u}{\partial x\partial y} = \frac{\partial^2\psi}{\partial y^2} = -\frac{\partial^2\psi}{\partial x^2} = \frac{\partial^2\varphi}{\partial x\partial y} =$

$\frac{\partial v}{\partial x} = \underbrace{-\frac{\partial\psi}{\partial x}}_{-\frac{\partial u}{\partial y}} + \frac{df_c}{dx}$

$\frac{\partial^2 v}{\partial x^2} = -\frac{\partial^2\psi}{\partial x\partial y} + \frac{d^2 f}{dx^2} = \frac{\partial^2\psi}{\partial y^2} + \frac{d^2 f}{dx^2}$

$\frac{\partial^2 v}{\partial y^2} = -\frac{\partial^2\psi}{\partial x^2}$

Pytanie: o ile warunki graniczne mogą być dowolnie obrane?

$r(\cos\varphi + i\sin\varphi) = re^{i\varphi}$ $\qquad$ $\log r(\cos\varphi + i\sin\varphi) = i\varphi + \log r$

$$z^2 = r^2(\cos^2\varphi - \sin^2\varphi + 2i\sin\varphi\cos\varphi)$$
$$= r^2 e^{2i\varphi} = r^2\cos 2\varphi + i\, r^2\sin 2\varphi$$

I. $\log z = \log r + i\varphi$

$u = \log r + b_1 y + c_1$

$v = -\varphi + bx + c = -\operatorname{arctg}\frac{y}{x} + bx + c$

$\frac{\partial u}{\partial x} = \frac{x}{r^2}$ $\qquad$ $\frac{\partial^2 u}{\partial x^2} = \frac{1}{r^2} - \frac{2x^2}{r^4}$

$\frac{\partial u}{\partial y} = \frac{y}{r^2} + b_1$ $\qquad$ $\underbrace{\frac{\partial^2 u}{\partial y^2} = \frac{1}{r^2} - \frac{2y^2}{r^4}}_{\Sigma = 0}$

$\frac{\partial v}{\partial x} = + \frac{\frac{y}{x^2}}{1+\frac{y^2}{x^2}} + b = \frac{y}{y^2+x^2} + b = \frac{y}{r^2} + b$ $\qquad$ $\frac{\partial v}{\partial y} = \frac{-\frac{1}{x}}{1+\frac{y^2}{x^2}} = -\frac{x}{y^2+x^2} = -\frac{x}{r^2}$

$\frac{\partial^2 v}{\partial x^2} = -2\frac{yx}{r^4}$ $\qquad$ $\frac{\partial^2 v}{\partial y^2} = + \frac{2yx}{r^4}$ $\qquad$ $\Sigma = 0$

Jeśli kierunek linij prądu niezależny od μ

$$\frac{\partial}{\partial\mu}\left(\frac{u}{v}\right)=0 \qquad \frac{1}{v}\frac{\partial u}{\partial\mu} - \frac{u}{v^2}\frac{\partial v}{\partial\mu}=0$$

$$\frac{1}{u}\frac{\partial u}{\partial\mu} = \frac{1}{v}\frac{\partial v}{\partial\mu} = \frac{1}{w}\frac{\partial w}{\partial\mu}$$

$$\frac{\partial}{\partial\mu}\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right] = -\frac{\partial^2 p}{\partial\mu\,\partial x}+\cancel{\mu}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\mu$$

$$\frac{\partial}{\partial\mu}\left[u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right] = -\frac{\partial^2 p}{\partial\mu\,\partial y}+\cancel{\mu}\,\partial^2 u$$

$$\begin{aligned} Du &= -\frac{\partial p}{\partial x}+\mu\nabla^2 u \\ Dv &= -\frac{\partial p}{\partial y}+\mu\nabla^2 v \\ Dw &= -\frac{\partial p}{\partial z}+\mu\nabla^2 w \end{aligned} \qquad \frac{\partial}{\partial y}Du - \frac{\partial}{\partial x}Dv = \mu\nabla^2\overbrace{\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)}^{\zeta}$$

$$\operatorname{curl} V(v \operatorname{curl} v) = \mu\nabla^2 \operatorname{curl} v = -\mu \operatorname{curl}^3 v$$

więc niezależnie od μ pozostaje tylko warunek:

$$U \operatorname{curl} V(v\operatorname{curl} v) = U\nabla^2 \operatorname{curl} v$$

$$\frac{\partial}{\partial y}(v\zeta - w\eta) - \frac{\partial}{\partial x}(w\xi-u\zeta) : \frac{\partial}{\partial z}(w\xi - u\zeta) - \frac{\partial}{\partial y}(u\eta - v\xi) : \dots$$

$$= \frac{\partial^2\zeta}{\partial x^2}+\frac{\partial^2\zeta}{\partial y^2}+\frac{\partial^2\zeta}{\partial z^2} : \frac{\partial^2\xi}{\partial x^2}+\frac{\partial^2\xi}{\partial y^2}+\frac{\partial^2\xi}{\partial z^2} : -$$

ponieważ obie strony kształtu $\frac{0}{0}$ zatem $\frac{\partial}{\partial\mu}$

$$\frac{\frac{\partial}{\partial\mu}\left[\frac{\partial}{\partial y}(v\zeta-w\eta)-\frac{\partial}{\partial x}(w\xi-u\zeta)\right]}{\frac{\partial}{\partial\mu}\left[\frac{\partial}{\partial z}(w\xi-u\zeta)-\frac{\partial}{\partial y}(u\eta-v\xi)\right]} = \frac{\Delta^2\zeta}{\Delta^2\xi}$$

$$\Delta^2 \xi \left[\frac{\partial^2}{\partial \mu \partial y}(\mu \nabla^2 u) - \frac{\partial^2}{\partial \mu \partial x}(\mu \nabla^2 v) \right] =$$

$$\nabla^2 \xi + \mu$$

Niemal bezpośrednio:

$$\frac{\partial}{\partial \mu}(v\zeta - w\eta) = -\frac{\partial^2 p}{\partial x \partial \mu} + \nabla^2 u \quad \cancel{+ \mu \nabla^2 \frac{\partial u}{\partial \mu}}$$

$$\nabla^2 u = \frac{\partial}{\partial \mu}\left[\frac{\partial p}{\partial x} - (v\zeta - w\eta)\right]$$

Powierzchnia, gdzie dwie ciecze z tarciem graniczą ze sobą:

$$\frac{D}{Dt}u_1 = X - \frac{\partial p_1}{\partial x} + \mu_1 \nabla^2 u_1 \qquad D u_2 = -\frac{\partial p_2}{\partial x} + \mu_2 \nabla^2 u_2$$

$$\frac{D}{Dt}v_1 = -\frac{\partial p_1}{\partial y} + \mu_1 \nabla^2 v_1 \qquad D v_2 = -\frac{\partial p_2}{\partial y} + \mu_2 \nabla^2 v_2$$

$$\frac{D}{Dt}w_1 = -\frac{\partial p_1}{\partial z} + \mu_1 \nabla^2 w_1 \qquad D w_2 = -\frac{\partial p_2}{\partial z} + \mu_2 \nabla^2 w_2$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \qquad \frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial w_2}{\partial z} = 0$$

Dla powierzchni ~~[illegible]~~ ~~się wznosić z czasem~~ $f(x,y,z) = 0$

$\cancel{u_1 = u_2}$ po upływie czasu Δt będzie

$\cancel{v_1 = v_2}$ $\cancel{w_1 = w_2}$ $$\frac{\partial f}{\partial t} + u_1 \frac{\partial f}{\partial x} + v_1 \frac{\partial f}{\partial y} + w_1 \frac{\partial f}{\partial z} = 0 \quad \Big| \quad \frac{\partial f}{\partial t} + u_2 \frac{\partial f}{\partial x} + \quad = 0$$

co tylko możliwe jeśli

$u_1 = u_2 \;\|\; v_1 = v_2 \;\|\; w_1 = w_2$ na powierzchni

opór tyż tak samo:

$$l\, p_{1xx} + m\, p_{1xy} + n\, p_{1xz} = l\, p_{2xx} + m\, p_{2xy} + n\, p_{2xz}$$

itd.

Dzięki porditura w oleczy

Tak samo jak gdyby parcie było na równe jako − 2

$$\frac{Du_1}{Dt} = -\frac{\partial p}{\partial x} + \mu_1 \nabla^2 u \qquad \Bigg| \qquad \frac{Du_2}{Dt} = \cancel{g}\!\!/ -\frac{1}{\rho_2}\frac{\partial p}{\partial x} + \frac{\mu_2}{\rho_2}\nabla^2 u$$

$\mu_1 = \cancel{0.017}$

$= 0.017$

$$\left.\begin{aligned}\rho_2 &= 0.0013\\ \mu_2 &= 0.00017\end{aligned}\right\} \quad \frac{\mu_2}{\rho_2} = \frac{0.17}{1.3} = 0.13$$

dla wodoru
$$\left.\begin{aligned}\rho_2 &= 0.00009\\ \mu_2 &= 0.00009\end{aligned}\right\} \frac{\mu_2}{\rho_2} = 1.0$$

Wewnątrz:

więc gdy powietrze wody wcale pimu się nie wydostanie to już bardzo u H_2

Na powierzchni:

$$\begin{array}{ll|l} \text{Układając oś} & OX \text{ w } n & n = a \\ & & u_1 = u_2 = 0 \\ & OY \text{ w } n\perp & v_1 = v_2 \\ & OZ \text{ w } n\parallel & w_1 = w_2 = 0 \end{array}$$

$$-p + 2\mu_1 \frac{\partial u_1}{\partial x} = -p + 2\mu_2 \frac{\partial u_2}{\partial x}$$

$$\mu_1\left(\frac{\partial u_1}{\partial y} + \underline{\underline{\frac{\partial v_1}{\partial x}}}\right) = \mu_2\left(\frac{\partial u_2}{\partial y} + \underline{\underline{\frac{\partial v_2}{\partial x}}}\right)$$

$$\mu_1\left(\frac{\partial u_1}{\partial z} + \frac{\partial w_1}{\partial x}\right) = \mu_2\left(\frac{\partial u_2}{\partial z} + \frac{\partial w_2}{\partial x}\right) \qquad \Big|\quad w = 0$$

zatem: $\mu_1 \frac{\partial v_1}{\partial x} = \mu_2 \frac{\partial v_2}{\partial x}$

$\mu_1 \Delta^2 u_1$ i $\mu_2 \Delta^2 u_2$ będą zatem tego samego rzędu wielkości

$\frac{\partial p_1}{\partial x} \quad \dots \dots \quad \frac{\partial p_2}{\partial x}$

zatem uwzględniając $\frac{Du_1}{Dt}$ można pominąć zawsze $\rho_2 \frac{Du_2}{Dt}$

(bo są tego samego rzędu wielkości

Stierujemy dla wody:

$$u = c\left[1 + \frac{a^3}{2r^3}\left(1 - \frac{3x^2}{r^2}\right)\right] = \frac{\partial\varphi}{\partial x}$$

$$v_\omega = -\frac{3ca^3}{2r^5}\, x\,\omega$$

$$\varphi = c\left(x + \frac{a^3 x}{2r^3}\right) = cx\left[1 + \frac{a^3}{2r^3}\right]$$

$$\cancel{= cx[\cos\theta]} = c\, r\cos\theta\left[1 + \frac{a^3}{2r^3}\right]$$

H_2 wzdłuż południków: $v_\theta = \left(\frac{\partial\varphi}{\partial\theta}\right)_{r=\text{const}} = -\,c\,r\sin\theta\left[1 + \frac{a^3}{2r^3}\right]$

$$\left.\frac{\partial v_\theta}{\partial r}\right|_{r=a} = -c\sin\theta\left[1 + \frac{a^3}{2r^3} - \frac{a^3}{r^3}\right]_{r=a} = -\frac{c}{2}\sin\theta$$

$$v_\theta\Big|_{r=a} = -\frac{3}{2}ac\sin\theta$$

Dla gazów bardzo rozrzedzonych przy zwykłej temp.

Np. $p = 1$ mm $\quad \rho = 1{\cdot}5 \cdot 10^{-6}$

$\mu = 0{\cdot}00017 \qquad \frac{\mu}{\rho} = 100$!

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \mu \nabla^2 u$$ tak że ρ bardzo małe

zatem przybliżenie: $\left.\begin{array}{l}\nabla^2 u = 0\\ \nabla^2 v = 0\\ \nabla^2 w = 0\end{array}\right\}$ $\nabla^2 v = 0$ ale to nie można twierdzić = $\mathrm{curl}^2 v$

~~[illegible]~~ bo $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

Nie zawierają $\frac{\partial}{\partial t}$

zatem przy danych warunkach granicznych nie zmienia się z czasem

Więc ruch ustanawia się natychmiastowo | zatem nie ma zjawisk akustycznych

Patrz Lamb p. 526 | dorzucić warunek $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

Np. kula dla prędkości $u = c$, $v = w = 0$

$r = a \qquad u = v = w = 0$

Z tego wynika, że $\begin{cases} v = w = 0 \text{ wszędzie} \\ u = c\left[1 - \frac{a}{r}\right] \end{cases}$

Wogóle jeżeli ruch powstaje wskutek przesunięcia ciał to zawsze

warunki graniczne kształtu $\begin{array}{l} u = c_1 \\ v = c_2 \\ w = c_3 \end{array}$ dla pewnych ~~[illegible]~~ powierzchni

zupełnie analogicznie jak: U w elektrostatyce między konduktorami na i... potencjał

Jeśli ~~prawie~~ w spoczynku, to wszystko w spoczynku.

Przepływ przez rurkę ... zaś nie następuje ... jeśli ... przetłoczony tłokiem

---

W tamtym ~~przykładzie~~:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=\frac{u_0\,a\,x}{r^3}$$

$$c + AR = C$$

$$1+\frac{AR}{c}=k$$

$$\frac{AR}{c}=k-1$$

$$\frac{c}{A}\rho\frac{D\theta}{Dt}+p\frac{\partial u}{\partial x}=2\mu\left[+\frac{2}{3}\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\right]$$

z przewodnictwem ciepła $+\kappa\Delta^2\theta$

$$\frac{D\theta}{Dt}=\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}$$

$$\frac{c}{A}\rho\,u_0\left[1-\frac{a}{r}\right]\frac{\partial\theta}{\partial x}+\rho R\theta\,u_0\frac{ax}{r^3}=2\mu\frac{u_0^2a^2}{r^6}\Big[\underbrace{y^2+z^2+\tfrac{2}{3}x^2}_{r^2-\frac{x^2}{3}}\Big]$$

$$\frac{\partial}{\partial x}(\rho u)=0 \qquad \cancel{u\frac{\partial\rho}{\partial x}+\rho\frac{\partial u}{\partial x}=0}$$

$$\rho u=\text{const}=\rho_0u_0 \qquad \rho=\frac{\rho_0}{1-\frac{a}{r}}$$

[wtedy na powierzchni warstwa zgęszczona] uwzględnić też tam zachodzi powolny ruch v w

$$\frac{c}{A}\rho_0u_0\frac{\partial\theta}{\partial x}+\theta.R.\rho_0u_0\frac{ax}{r^3(1-\frac{a}{r})}=2\mu\frac{u_0^2a^2}{r^6}\left(r^2-\frac{x^2}{3}\right)$$

$$\frac{\partial\theta}{\partial x}+\theta(k-1)\frac{a}{r^2-ar}\frac{x}{r}=2\mu\frac{k-1}{R}\frac{u_0a^2}{\rho_0}\frac{r^2-\frac{x^2}{3}}{r^6}$$

$$\frac{a}{r^2-ar}=\underbrace{\frac{-1}{r}+\frac{1}{r-a}}\qquad \frac{\partial}{\partial x}\log\left(\frac{r-a}{r}\right)$$

$$\frac{\partial\theta}{\partial x} + \theta X_1 = X_2 \qquad \cancel{\theta = a e^{\int}} \quad \theta = zy$$

$$y\frac{dz}{dx} + X_1 z y + z\frac{dy}{dx} = X_2 \qquad X_1 y + \frac{dy}{dx} = 0$$

$$y = e^{-\int X_1 dx}$$

$$\theta = e^{-\int X_1 dx}\left[c + \int X_2 e^{+\int X_1 dx} dx\right]$$

$$X_1 = (k-1)\frac{\partial}{\partial x}\log\frac{r-a}{r}$$

$$\int X_1 dx = (k-1)\log\frac{r-a}{r} + \varphi(y,z) = \log\left(\frac{r-a}{r}\right)^{k-1}$$

$$e^{\int} = \left(\frac{r-a}{r}\right)^{k-1} \qquad X_2 = \frac{2\mu(k-1)}{R}\frac{u_0 a^2}{\rho_0}\frac{r^2-\frac{x^2}{3}}{r^6}$$

$$\theta = \left(\frac{r}{r-a}\right)^{k-1}\left[\overset{f(y,z)}{c} + \frac{2\mu(k-1)u_0 a^2}{R\rho_0}\int_0^x \frac{r^2-\frac{x^2}{3}}{r^6}\left(\frac{r-a}{r}\right)^{k-1} dx\right]$$

~~c oznacza się z tego że temp. dla $x=\infty$ w wielkiej odległości nie zmieniona θ_0~~

Do oznaczenia stałych:

Dla $x = -\infty$: $\theta = \theta_0$ niezal. od y, z

$$\int_0^\infty \frac{r^2-\frac{x^2}{3}}{r^6}\left(\frac{r-a}{r}\right)^{k-1} dx = \varphi(y,z)$$

$$\theta_{-\infty} = f(y,z) - \frac{2\mu(k-1)u_0 a^2}{R\rho_0}\varphi(y,z)$$

$$f(y,z) = \theta_{-\infty} + \frac{2\mu(k-1)u_0 a^2}{R}\varphi$$

$$\theta = \left(\frac{r}{r-a}\right)^{k-1}\left[\theta_{-\infty} + \frac{2\mu(k-1)u_0 a^2}{R\, p_0} \int_{-\infty}^{x} \frac{r^2 - \frac{x^2}{3}}{r^6}\left(\frac{r-a}{r}\right)^{k-1} dx\right]$$

więc indukcja wyżej zgrzana po przejściu

$$p_{xx} = \frac{4}{3}\mu\frac{\partial u}{\partial x} \qquad p_{yz} = 0$$

$$p_{yy} = p_{zz} = -\frac{2}{3}\mu\frac{\partial u}{\partial x} \qquad p_{xy} = \mu\frac{\partial u}{\partial y} \qquad p_{xz} = \mu\frac{\partial u}{\partial z}$$

$$\frac{\partial u}{\partial x} = \frac{u_0 a x}{r^3} \qquad \frac{\partial u}{\partial y} = \frac{u_0 a y}{r^3} \qquad \frac{\partial u}{\partial z} = \frac{u_0 a z}{r^3}$$

$$p_{rx} = \frac{u_0 a \mu}{r^4}\left(\frac{4}{3}x^2 + y^2 + z^2\right) = \frac{u_0 a \mu}{r^4}\left(r^2 + \frac{x^2}{3}\right)$$

$$p_{ry} = \frac{u_0 a \mu}{r^4}\left(xy - \frac{2}{3}yx\right) = \frac{u_0 a \mu}{r^4}\frac{xy}{3}$$

$$p_{rz} = \frac{u_0 a \mu}{r^4}\left(xz - \frac{2}{3}xz\right) = \frac{u_0 a \mu}{r^4}\frac{xz}{3}$$

$$\Sigma p_{ry} = 0 \qquad \Sigma p_{rz} = 0$$

$$\Sigma p_{rx} = \frac{u_0 a \mu}{a^4} a^2 \int_0^{\pi}\left(1 + \frac{\cos^2\theta}{3}\right) 2\pi a^2 \sin\theta\, d\theta$$

$$= u_0 a \mu \cdot 2\pi \left(-\cos\theta - \frac{\cos^3\theta}{9}\right)\Big|_0^{\pi} = 2a\pi\, u_0 \mu \cdot \frac{20}{9} = \frac{40}{9} a\pi u_0 \mu$$

Więc np. końcowa prędkość pod wpływem grawitacji:

$$\frac{4}{3}a^3\pi\, s\, g = \frac{40}{9} a\pi u_0 \mu$$

$$u_0 = \frac{3}{10}\frac{a^2 s g}{\mu}$$

Przy tamtem przybliżeniu więc w ogóle nie nastąpi żaden ruch jeśli $u\,v\,w$

na powierzchniach $=0$.

Ażeby w zamkniętym naczyniu otrzymać ruch, należy uwzględnić: $\frac{\partial\mu}{\partial x}$ ~~...~~

wtedy jednak jeśli ruch powolny można zaniedbać jeszcze $\mu\frac{\partial u}{\partial x}$ etc.

$$\left.\begin{aligned}\rho\frac{\partial u}{\partial t} &= -\frac{\partial p}{\partial x}+\mu\nabla^2 u\\ \rho\frac{\partial v}{\partial t} &= -\frac{\partial p}{\partial y}+\mu\nabla^2 v\\ \rho\frac{\partial w}{\partial t} &= -\frac{\partial p}{\partial z}+\mu\nabla^2 w\end{aligned}\right\}$$

$p = \cancel{\ }\,R\rho\theta$

zaniedbując zmienność μ:

$$\underbrace{\frac{\partial}{\partial x}\left(\rho\frac{\partial u}{\partial t}\right)+\frac{\partial}{\partial y}\left(\rho\frac{\partial v}{\partial t}\right)+\frac{\partial}{\partial z}\left(\rho\frac{\partial w}{\partial t}\right)}_{} = \overset{\nabla^2 p+}{\mu\nabla^2\left(\frac{\partial u}{\partial x}\cdots\right)} = \cancel{\ }$$

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0$$

$$\rho\frac{\partial}{\partial t}\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\cdots\right]+\left[\frac{\partial\rho}{\partial x}\frac{\partial u}{\partial t}+\frac{\partial\rho}{\partial y}\frac{\partial v}{\partial t}\cdots\right]=$$

$$\frac{\partial\rho}{\partial t}+\rho\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\cdots\right]+u\frac{\partial\rho}{\partial x}+v\frac{\partial\rho}{\partial y}+\cdots=0$$

$$\frac{\partial^2\rho}{\partial t^2}+\frac{\partial\rho}{\partial t}\left[\frac{\partial u}{\partial x}+\cdots\right]+\rho\frac{\partial}{\partial t}\left[\frac{\partial v}{\partial y}\cdots\right]+\frac{\partial u}{\partial t}\frac{\partial\rho}{\partial x}\cdots+u\frac{\partial^2\rho}{\partial x\,\partial t}\cdots=0$$

Dla stacjonarnego stanu:

$$\mu\nabla^2 u=\frac{\partial p}{\partial x}\qquad\qquad \frac{c}{A}\rho\frac{D\theta}{\partial t}+p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=2\mu\left[\cdots\right)$$

$$\mu\nabla^2 v=\frac{\partial p}{\partial y}$$

$$\mu\nabla^2 w=\frac{\partial p}{\partial z}$$

$$\underbrace{\frac{1}{\rho}\frac{\partial\rho}{\partial t}}_{\frac{\partial}{\partial t}(\log\rho)} + \left[\frac{\partial u}{\partial x} + \qquad\right] + u\frac{\partial}{\partial x}(\log\rho) + v\frac{\partial}{\partial y}(\log\rho) + \qquad = 0$$

$$\log\rho = \log p - \log\theta - \log R$$

Jak zmienia się p w ruchu w rurkach w których wszystkie wyrazy drugiego rzędu równania termicznego? Zaniedbujmy : inercję termiczną:

$v = w = 0$

$$\mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \frac{\partial p}{\partial x}$$

$$\frac{\partial}{\partial x}(\rho u) = 0 \qquad \rho u = f(y,z)$$

$$\frac{c}{A}\rho\, u\frac{\partial\theta}{\partial x} + R\rho\theta\frac{\partial u}{\partial x} = 2\mu\left[-\frac{1}{3}\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right]$$

Albo przez p:

$$\mu\nabla^2 u = \frac{\partial p}{\partial x} \qquad p = \varphi(x)$$

$$\frac{pu}{\theta} = f(y,z)$$

$$\frac{c}{AR}u\frac{1}{\theta}\frac{\partial\theta}{\partial x} + p\frac{\partial u}{\partial x} = 2\mu\left[\frac{2}{3}\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right]$$

$$\varphi(x)\frac{u}{\theta} = f(y,z) \qquad p = \frac{\theta}{u}f(y,z)$$

$$\frac{u}{\theta}\frac{\partial p}{\partial x} + p\frac{\partial}{\partial x}\left(\frac{u}{\theta}\right) = 0 \qquad \mu\nabla^2 u = \frac{\partial p}{\partial x} = f(y,z)\frac{\partial}{\partial x}\left(\frac{\theta}{u}\right)$$

$$\mu\nabla^2 u = R\left(\theta\frac{\partial\rho}{\partial x} + \rho\frac{\partial\theta}{\partial x}\right)$$

$$\frac{c}{A}\rho u\frac{\partial\theta}{\partial x} + R\rho\theta\frac{\partial u}{\partial x} = 2\mu\left[\frac{2}{3}\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right]$$

u | $\rho u = f(y,z)$

$\rho\theta = \varphi(x)$

$$\left(\frac{c}{A} + R\right)\rho u\frac{\partial\theta}{\partial x} + R\left(\theta u\frac{\partial\rho}{\partial x} + \theta\rho\frac{\partial u}{\partial x}\right) = \mu\left[u\nabla^2 u + 2\left(\frac{2}{3}\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right)\right]$$

Czy możliwe rozwiązanie formy $\theta = f(p)$?

$$\text{zatem } \rho = \varphi(p) = \frac{p}{R f(p)}$$

$$\left.\begin{aligned} \mu \nabla^2 u &= \frac{\partial p}{\partial x} \\ \mu \nabla^2 v &= \frac{\partial p}{\partial y} \\ \mu \nabla^2 w &= \frac{\partial p}{\partial z} \end{aligned}\right\}$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

$$\downarrow$$

$$\text{I).}\ \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} = 0$$

$$\text{II).}\ \frac{c}{A}\rho\left(u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z}\right) + p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 2\mu\ \ldots$$

$$\text{II).}\ \frac{c}{A}\varphi(p) f'(p)\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right) + p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) =$$

$$\text{I).}\ \varphi(p)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + \varphi'(p)\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right) = 0 \qquad f(p) = \frac{p}{R\varphi(p)}$$

$$\text{I).}\ u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z} = \mu\left(u\nabla^2 u + v\nabla^2 v + w\nabla^2 w\right) \qquad f'(p) = \frac{1}{R}\left[\frac{1}{\varphi(p)} - \frac{p\varphi'(p)}{\varphi(p)}\right]$$

$$\frac{\varphi'(p)}{\varphi(p)} = -\frac{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}}{\mu\left(u\nabla^2 u + v\nabla^2 v + w\nabla^2 w\right)}$$

$$\frac{c}{A}\varphi(p) f'(p)\,\mu\left(u\nabla^2 u + v\Delta^2 v + w\Delta^2 w\right) \pm p\,\mu\left[u\Delta^2 u + \ldots\right]\frac{\varphi'(p)}{\varphi(p)} = 2\mu\left[\ldots\right]$$

$$\left(u\nabla^2 u + v\nabla^2 v + w\nabla^2 w\right)\underbrace{\left[\frac{c}{A}\varphi(p) f'(p) - p\frac{\varphi'(p)}{\varphi(p)}\right]}_{\left[\frac{c}{AR}\left(1 - p\frac{\varphi'(p)}{\varphi(p)}\right) - p\frac{\varphi'(p)}{\varphi(p)}\right]} = 2\mu\left[-\frac{1}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial x}\right)^2 \ldots\right]$$

$$AR = C - c \qquad \frac{c}{C-c} = \frac{1}{k-1} \qquad \left[\frac{1}{k-1} - \frac{k}{k-1}\,\frac{p\varphi'(p)}{\varphi(p)}\right]$$

Założenie które się robi ~~...~~ w teorii Joule-Thomsona: $\theta = \text{const}$
$f = \theta_0$

$$\varphi = \frac{p}{R\theta_0}$$

$$\varphi' = \frac{1}{R\theta_0} \qquad \frac{p\varphi'}{\varphi} = 1$$

Wtedy:

$$-(u\nabla^2 u + v\Delta^2 v + w\Delta^2 w) = 2\left[-\frac{1}{3}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)^2 + \dots\right]$$

$$\therefore\; 3(u\nabla^2 u + v\nabla^2 v + w\nabla^2 w) - 2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)^2 + 6\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right] +$$

$$+ 3\left[\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)^2+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right] = 0$$

W ~~jednowymiarowym~~ przykładzie $v = w = 0$:

$$3u\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) - 2\left(\frac{\partial u}{\partial x}\right)^2 + 6\left(\frac{\partial u}{\partial x}\right)^2 + 3\left(\frac{\partial u}{\partial z}\right)^2 + 3\left(\frac{\partial u}{\partial y}\right)^2 = 0$$

$$4\left(\frac{\partial u}{\partial x}\right)^2 + 3u\frac{\partial^2 u}{\partial x^2} + 3\Big[\underbrace{u\frac{\partial^2 u}{\partial y^2}+\left(\frac{\partial u}{\partial y}\right)^2}_{\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial y}\right)} + \underbrace{u\frac{\partial^2 u}{\partial z^2}+\left(\frac{\partial u}{\partial z}\right)^2}_{\frac{\partial}{\partial z}\left(u\frac{\partial u}{\partial z}\right)}\Big] = 0$$

Jednowymiarowy przykład:

$$4\left(\frac{\partial u}{\partial x}\right)^2 + 3u\frac{\partial^2 u}{\partial x^2} = 0$$

$$4\,\frac{\frac{\partial u}{\partial x}}{u} + 3\,\frac{\frac{\partial^2 u}{\partial x^2}}{\frac{\partial u}{\partial x}} = 0$$

$$\lg\left[u^4\cdot\left(\frac{\partial u}{\partial x}\right)^3\right] = \text{const}$$

$$\frac{du}{dx}\,u^{\frac{4}{3}} = c$$

$$\frac{3}{7}u^{\frac{7}{3}} = cx + c_0$$

~~Praca w ogóle nie jest oznaczona przez~~ $\iint \frac{\partial p}{\partial n} p \, dS = \mu \int u \nabla^2 \ldots + \nabla v \, dy + \ldots$ (podczas 1 sec)

$\frac{\partial p}{\partial x} = \mu \nabla^2 u$ tylko przez $\iint p_{xx} \, u \, dS$ etc.

$$\iiint \left(u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} + w \frac{\partial p}{\partial z} \right) dv = \iint u p \, dy\, dz + v p \, dx\, dz + w \, p \, dx\, dy -$$

$$- \iiint p \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) dx\, dy\, dz$$

$$= \int p \left(u \cos nx + v \cos ny + w \cos nz \right) dS - \iiint p \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) dv$$

zatem:

$$\rho \left(\frac{\partial u}{\partial x} + \ldots \right) + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} \ldots = 0$$

$$\mu \iiint \left(u \Delta^2 u + \ldots \right) dv = W_p -$$

~~Ale~~ jeśli jednak granica ~~...~~ cieczy w 1 sek w niektórych miejscach gdzie ruch staje się niezmiennym, to całkowita praca wewnętrzna byłaby oznaczona przez

$$W_{p_I} = \mu \iiint \left(u \Delta^2 u + \ldots \right) dv + \iiint p \left(\frac{\partial u}{\partial x} + \ldots \right) dv$$

$$- \int p \left(u l + v m + w n \right) dS$$

albo też przez funkcyę Φ:

$$W_{p_{II}} = \iiint \Phi \, dv$$ to ostatnie zawsze ważne

W_{p_I} jeśli (powierzchnia) przez izopiezy $= p_2 \int_2 [u l + v m + w n] \, dS - p_1 \int_1 (u l \ldots) \, dS$

~~jeśli~~

Równanie ciągłości daje dla dowolnego przekroju poprzecznego wszystkich linii prądu:

$$\int \rho(u\,l + v\,m + w\,n)\,dS = \text{const} = m$$

jeśli ρ tylko $fcn(p)$ [$\rho = f(p)$], wtedy ~~przekładając~~ dla tych samych powierzchni (izopiest.):

$$\rho_1 \int_1 \quad = \rho_2 \int_2$$

$$\rho_1 \int_1 \; - \rho_2 \int_2 \; = 0 \qquad f(p_1) \int_1 - f(p_2) \int_2 = 0$$

ale z tego jeszcze nie wynika: $p_1 \int_1 - p_2 \int_2 = 0$

t. j. to tylko jeśli $f(p_1) \sim p_1$ to znaczy w układzie izotermicznym

Inaczej równanie to może być spełnione [$W_{p_3} = 0$] tylko w specyalnych

przypadkach.

$T_p = \int$ [illegible]

$= \int$ [illegible]

$= $ [illegible] θ [illegible]

$= $ [illegible] $+ \theta$ [illegible]

$= \int$ [illegible] $\,dx$

$= \int \Phi +$ [illegible]

$= \int$ [illegible] $\int$ [illegible] $\theta\,dx$ [illegible] $-$

$-$ [illegible] $\int$ [illegible]

$= \int$ [illegible] $\int$ [illegible]

$= \int \Phi\,dx + (1 -$ [illegible] $) \underbrace{\int \text{[illegible]}}_{= T_p\,!}$

$\therefore T_p = \int \Phi\,dx$ [illegible]

T_p [illegible]

$$\int \Phi\, d\omega = \mu \int dx\,dy\,dz \Big\{ -\frac{2}{3}\Big[\Big(\frac{\partial u}{\partial x}\Big)^2 + \Big(\frac{\partial v}{\partial y}\Big)^2 + \Big(\frac{\partial w}{\partial z}\Big)^2 + 2\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} + 2\frac{\partial u}{\partial x}\frac{\partial w}{\partial z} + 2\frac{\partial v}{\partial y}\frac{\partial w}{\partial z}\Big] +$$

$$2\Big(\frac{\partial u}{\partial x}\Big)^2 + 2\Big(\frac{\partial v}{\partial y}\Big)^2 + 2\Big(\frac{\partial w}{\partial z}\Big)^2 + \Big(\frac{\partial u}{\partial y}\Big)^2 + 2\frac{\partial u}{\partial y}\frac{\partial v}{\partial x} + \Big(\frac{\partial v}{\partial x}\Big)^2$$

$$+ \frac{1}{3}\Big[\Big(\frac{\partial u}{\partial x}\Big)^2 + \Big(\frac{\partial v}{\partial y}\Big)^2 + \Big(\frac{\partial w}{\partial z}\Big)^2\Big] + \Big(\frac{\partial u}{\partial x}\Big)^2 + \Big(\frac{\partial u}{\partial y}\Big)^2 + \Big(\frac{\partial u}{\partial z}\Big)^2 + \cdots \quad + \cdots$$

$$+ 2\Big[\frac{\partial u}{\partial y}\frac{\partial v}{\partial x} + \frac{\partial u}{\partial z}\frac{\partial w}{\partial x} + \frac{\partial v}{\partial z}\frac{\partial w}{\partial y} - \frac{2}{3}\Big(\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} + \frac{\partial v}{\partial y}\frac{\partial w}{\partial z} + \frac{\partial w}{\partial z}\frac{\partial u}{\partial x}\Big)\Big]$$

$$= \iint \frac{1}{3}\Big(u\frac{\partial u}{\partial x}\,l + v\frac{\partial v}{\partial y}\,m + w\frac{\partial w}{\partial z}\,n\Big)dS + \underbrace{\iint u\Big(\frac{\partial u}{\partial x}\,l + \frac{\partial u}{\partial y}\,m + \frac{\partial u}{\partial z}\,n\Big) + \cdots}_{=\iint \frac{1}{2}\frac{\partial(u^2+v^2+w^2)}{\partial n}\,dS}$$

$$+2\iint\Big[u\frac{\partial v}{\partial x}m + v\frac{\partial w}{\partial y}n + w\frac{\partial u}{\partial z}l - \frac{2}{3}\Big(u\frac{\partial v}{\partial y}l + v\frac{\partial w}{\partial z}m + w\frac{\partial u}{\partial x}n\Big)\Big]dS$$

$$-\iiint \frac{1}{3}\Big(u\frac{\partial^2 u}{\partial x^2} + v\frac{\partial^2 v}{\partial y^2} + w\frac{\partial^2 w}{\partial z^2}\Big)d\omega - \iiint (u\nabla^2 u + v\nabla^2 v + w\nabla^2 w)\,d\omega\ -$$

$$-2\iiint\Big[u\frac{\partial^2 v}{\partial x\partial y} + v\frac{\partial^2 w}{\partial y\partial z} + w\frac{\partial^2 u}{\partial x\partial z} - \frac{2}{3}\Big(u\frac{\partial^2 v}{\partial x\partial y} + v\frac{\partial^2 w}{\partial y\partial z} + w\frac{\partial^2 u}{\partial x\partial z}\Big)\Big]d\omega \quad =$$

$$= \frac{1}{3}\iint \left(u\frac{\partial u}{\partial x}l + v\frac{\partial v}{\partial y}m + w\frac{\partial w}{\partial z}n\right) dS \;+\frac{1}{2}\iint \frac{\partial}{\partial n}(u^2+v^2+w^2)\, dS \;+$$

$$\left.\begin{aligned} &\iint u\frac{\partial v}{\partial x}m + v\frac{\partial w}{\partial y}m + w\frac{\partial u}{\partial z}l + v\frac{\partial u}{\partial y}l + w\frac{\partial v}{\partial z}m + u\frac{\partial w}{\partial x}n \;- \\ &\iint -\frac{2}{3}\left[u\frac{\partial v}{\partial y}l + v\frac{\partial w}{\partial z}m + w\frac{\partial u}{\partial x}n + v\frac{\partial u}{\partial x}m + w\frac{\partial v}{\partial y}n + u\frac{\partial w}{\partial z}l\right]\end{aligned}\right\} dS$$

$$-\frac{1}{3}\iiint\left(u\frac{\partial^2 u}{\partial x^2} + v\frac{\partial^2 v}{\partial y^2} + w\frac{\partial^2 w}{\partial z^2}\right)d\omega - \iiint\left(u\nabla^2 u + v\nabla^2 v + w\nabla^2 w\right)d\omega \;-$$

$$-\iiint u\frac{\partial^2 v}{\partial x\partial y} + v\frac{\partial^2 w}{\partial y\partial z} + w\frac{\partial^2 u}{\partial x\partial z} + v\frac{\partial^2 u}{\partial x\partial y} + w\frac{\partial^2 v}{\partial y\partial z} + u\frac{\partial^2 w}{\partial x\partial z} \;-$$

$$-\frac{2}{3}\left[u\frac{\partial^2 v}{\partial x\partial y} + v\frac{\partial^2 w}{\partial y\partial z} + w\frac{\partial^2 u}{\partial x\partial z} + v\frac{\partial^2 u}{\partial x\partial y} + w\frac{\partial^2 v}{\partial y\partial z} + u\frac{\partial^2 w}{\partial x\partial z}\right] d\omega$$

$$\iiint = -\frac{1}{3}\iiint u\left(\frac{\partial^2 u}{\partial x^2} + \cancel{\frac{\partial^2 v}{\partial x\partial y}} + \frac{\partial^2 w}{\partial x\partial z}\right) + v\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial x\partial y} + \frac{\partial^2 w}{\partial y\partial z}\right) + w\left(\;\cdots\;\right)$$

$$-\iiint\left(u\nabla^2 u + v\nabla^2 v + w\nabla^2 w\right)d\omega$$

$$\iiint = -\frac{1}{3}\iiint\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) d\omega$$

$$-\iiint\left(u\nabla^2 u + v\nabla^2 v + w\nabla^2 w\right)d\omega$$

$$\iint = \frac{1}{2}\iint\frac{\partial}{\partial n}(u^2+v^2+w^2)\,dS + \iint u\left[l\frac{\partial u}{\partial x} + m\frac{\partial v}{\partial x} + n\frac{\partial w}{\partial x}\right] + v\left[l\frac{\partial u}{\partial y} + m\frac{\partial v}{\partial y} + n\frac{\partial w}{\partial y}\right] + w\left[\;\right.$$

$$-\frac{2}{3}\iint\left[u\, l\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + v\, m\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + w\, n\left(\partial\;\cdots\right)\right] dS$$

$$\int \Phi\, d\omega = \frac{1}{2}\iint \frac{\partial}{\partial n}(u^2+v^2+w^2)\, dS + \iint \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)(\underbrace{ul+vm+wn}_{v_n})\, dS -$$

$$-\frac{2}{3}\iint (ul+vm+wn)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right) dS \quad -$$

$$-\iiint (u\nabla^2 u + v\nabla^2 v + w\nabla^2 w)\, d\omega - \frac{1}{3}\iiint \left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}\right)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right) d\omega$$

~~Dla cieczy nieściśliwych zatem~~: $w_{pI} = W_{pII}$ ~~jest~~ pod tymi warunkami

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

$$u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} + \rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right) = 0$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} = -\left[u\frac{\partial(\log\rho)}{\partial x} + v\frac{\partial(\log\rho)}{\partial y} + w\frac{\partial(\log\rho)}{\partial z}\right]$$

$$\log\rho = \log p - \log R - \log\theta$$

$$\int \Phi\, d\omega = \int \cdots \quad -\iiint (u\nabla^2 u + v\nabla^2 v + w\nabla^2 w)\, d\omega -$$

$$\cancel{+\frac{1}{3}\cdots} \quad \cancel{+\frac{1}{3}\iiint \frac{1}{p}\left(u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}\right) - \frac{1}{\theta}\, u\frac{\partial\theta}{\partial x}\cdots}$$

$$+\frac{1}{3}\iiint \left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}\right)\underbrace{\left(u\frac{\partial(\log p - \log\theta)}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)}_{\frac{1}{p}\left(u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}\right) - \frac{1}{\theta}\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}\right)} d\omega$$

$$\underbrace{u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}}_{=-\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)}$$

$$\iiint p\left(\frac{\partial u}{\partial x}+\ \ \right) d\omega = -\iiint \left(u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}\right) d\omega + R\iiint \rho\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}\right) d\omega$$

$$W_{pI} = R\iiint \rho\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right)dv =$$

$$= R\iint \rho(u\theta l + v\theta m + w\theta n)\,dS - R\iiint \theta\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right]dv$$

$$= R\iint \rho\theta(ul + vm + wn)\,dS = \iint(p\,u\,l + \ldots\ldots)$$

$$p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + \left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + \ldots\right) = R\rho\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right)$$

III. $$p\left(\frac{\partial u}{\partial x} + \ldots\right) + \frac{c}{A}\rho\left(u\frac{\partial\theta}{\partial x} + \ldots\right) = \Phi$$

$$\underbrace{\iiint p\left(\frac{\partial u}{\partial x} + \ldots\right)dv} + \underbrace{\frac{c}{A}\iiint \rho\left(u\frac{\partial\theta}{\partial x} + \ldots\right)dv} = \iiint \Phi\,dv$$

$$= \iint p(ul + vm + wn)\,dS + \frac{c}{A}\iint \rho\theta(ul + vm + wn)\,dS$$

$$- \iiint\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right)dv - \frac{c}{A}\iiint \theta\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \ldots\right]dv$$

$$\left(1 + \frac{c}{AR}\right)\iint p(ul + vm + wn)\,dS = \iiint(u\Delta^2 u + v\Delta^2 v\ \ldots)\,dv + \Phi\,dv$$

$$\cancel{u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}} = \cancel{\ldots}$$

$$\frac{k}{k-1}\,W_{pI} = \iiint(u\Delta^2 u + v\Delta^2 v\ \ldots) + \Phi\,dv$$

$$\frac{1}{k-1}\,W_{pI} = \qquad\qquad \Phi_r$$

$$W_{12} = -\iiint \left(u\nabla^2 u + v\nabla^2 v + w\nabla^2 w\right) d\omega \qquad \| \; ?$$

$$\iiint \Phi\, d\omega = \frac{1}{2}\iint \frac{\partial}{\partial n}(u^2+v^2+w^2)\, dS + \iint \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right) v_n\, dS -$$

$$- \iint (ul + vm + wn)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) dS -$$

$$- \iiint \left(u\nabla^2 u + v\nabla^2 v + w\nabla^2 w\right) d\omega + \frac{1}{2}\iiint \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2 d\omega$$

$$\nabla(v \cdot \operatorname{curl} v) = -\nabla\left(p + \frac{u^2+v^2+w^2}{2}\right) + \underbrace{\mu \nabla^2 v}_{= -\mu \operatorname{curl}^2 v}$$

$$\operatorname{curl} \nabla(v \cdot \operatorname{curl} v) = -\mu \operatorname{curl}^3 v = \cancel{\mu\, \operatorname{curl}} = \mu \nabla^2 \operatorname{curl} v$$

$$\operatorname{div} \nabla(v \cdot \operatorname{curl} v) = (\operatorname{curl} v)^2 - \int v \cdot \operatorname{curl}^2 v = -\nabla^2\left(p + \frac{v^2}{2}\right)$$

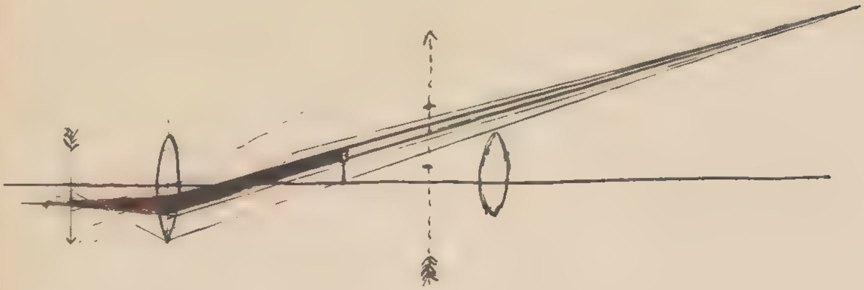
Ale teraz: $\operatorname{curl} \nabla(v \cdot \operatorname{curl} v) = 0$

$$\nabla(v \cdot \operatorname{curl} v) = \nabla U$$

Czy to jest jednoznacznie oznaczone przez v na powierzchni?

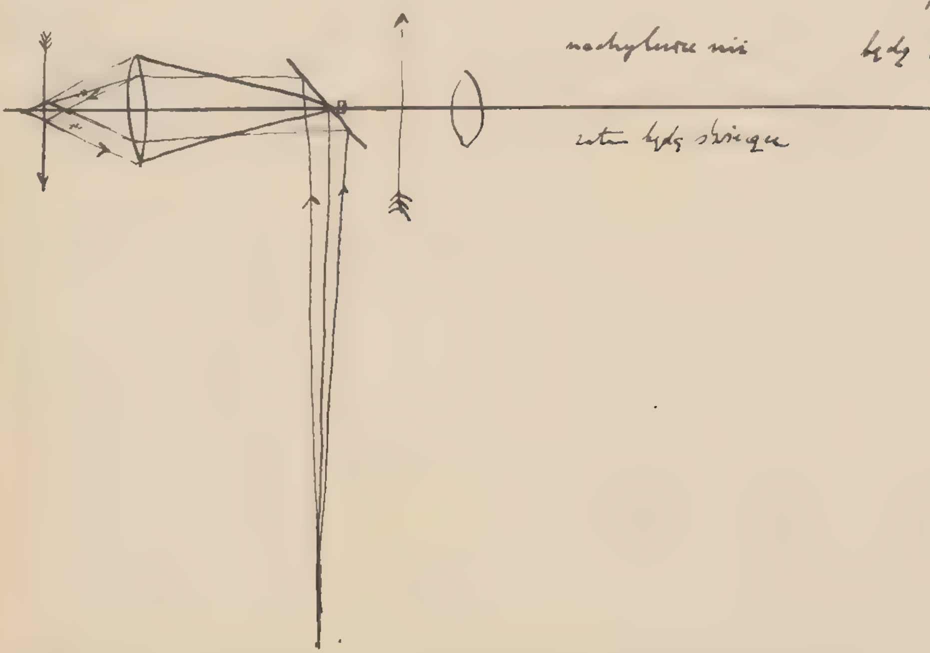
Na powierzchni $\operatorname{curl} v$ leży w powierzchni samej, $v = 0$, ale w kierunku stycznym

więc ∇U musi się stać $= 0$ na powierzchni nieruchomej



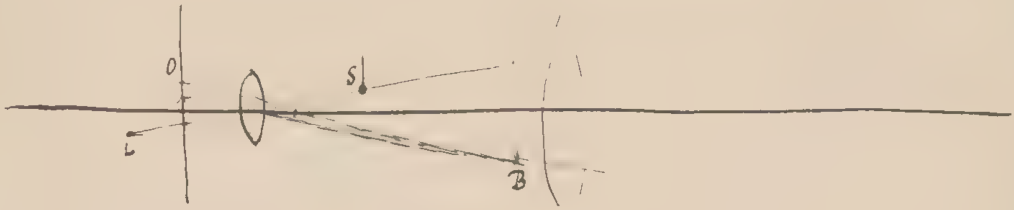
Dogodniej jednak tak:

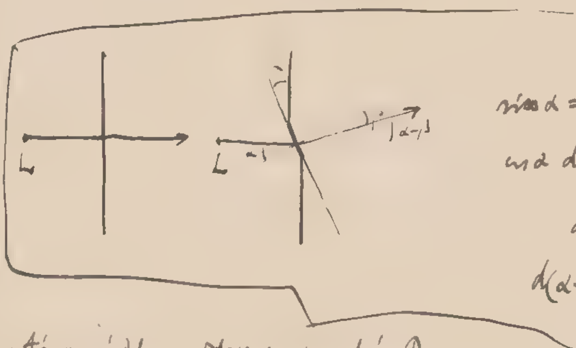
Te części zwierciadła które mają większe
nachylenie niż   będą wysyłać prom



zatem będą świecące

obraz światła L w S zakryty ekranem ; obraz O powstanie w O tylko o tyle o ile powierzchnia O jest nieregularna.





$$\sin\alpha = n \sin\beta$$

$$\cos\alpha \, d\alpha = n \cos\beta \, d\beta$$

$$d\alpha = n \, d\beta$$

$$d(\alpha-\beta) = (n-1)\, d\beta$$

To da się zastosować do mierzenia ruchów Browna

2). do obserw. tłumik ciekłych cieczy [wykrycie wyjątkowo zgęszczeń miejscowych

Czy odbijające powierzchnie (czy nie) wprowadzą światło zamiast z L wprost z S ?

zatem taki plan:

U gazów przy niskim ciśnieniu!

Np. zmniejszając p_1 i p_2 100 krotnie

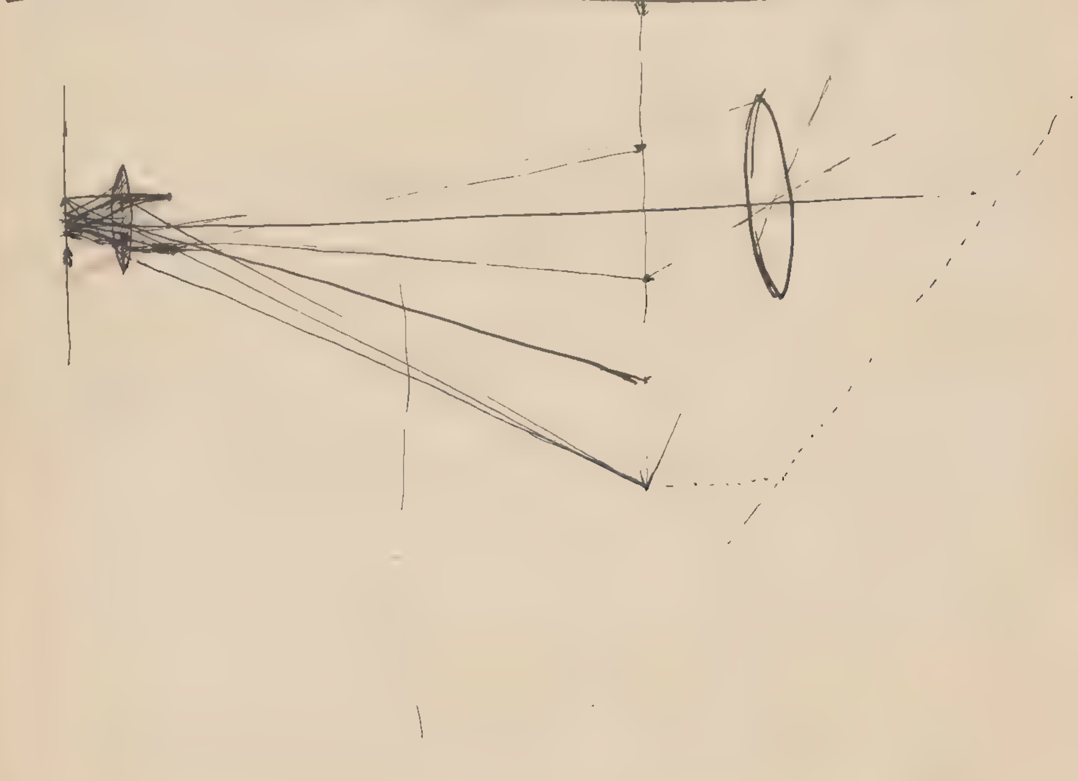
k będzie powiększone 100 razy, tak samo u

$u = 63{\cdot}4$ cm

Opór $\eta \Delta^2 u$ w rurce $\frac{r}{2} = \varepsilon = 0{\cdot}01$ cm

$= 204$ co wobec $\frac{\partial p_1}{\partial x} = \frac{10^6}{100}$ wynosi 2%

Więc wiele mniejszy ~~p~~ wpływ z drugiej strony tutaj „Gleitung”



N.p. jeśli $\varepsilon =$ ~~0·1 cm~~ $0{\cdot}01$ cm

$\eta = 0{\cdot}00017$

$l = 10$

$u_0 = a = 0{\cdot}6$ cm

$\Delta^2 u = 2c = \frac{2a}{\varepsilon^2} = \frac{1{\cdot}2}{10^{-4}}$

$\eta \Delta^2 u = 0{\cdot}00017 \cdot 1{\cdot}2 \cdot 10^4 = 2{\cdot}04$

wobec $\frac{\partial p}{\partial x} = 10^6$ więc niedostrzegalnie mało

$p_1 - p_2 = \frac{2\eta l u_0}{\varepsilon^2} = \frac{2.10.\,0{\cdot}00017.\,0{\cdot}6}{10^{-4}}$

$= \underline{17_{34}} = 20$

$= 20.10^{-6}$ atm.

Jeżeli $\varepsilon = 0{\cdot}001$ cm

zwiększa się $\eta \Delta^2 u = 204$ wobec 10^6

Jeżeli $\varepsilon = 0{\cdot}0001\text{ cm} = 1\mu$ $\quad \eta \Delta^2 u = 20400 = 2\%$

~~Więc mogłoby się to uwydatnić tylko przy dyfuzji przez błony, gips etc.~~

Znaczniejszy będzie ~~...~~ wpływ w zadaniu: rurki (ale niewielka różnica!)

Więc uwydatnić mogłoby się to tylko przy dyfuzji przez błony, gips etc. ~~...~~

N.p. przy gipsie $r = 5.10^{-5}$: 8%

W cieczy: N.p. w zadaniu pierwszym $u = 4.10^{-6} \left(\frac{cm}{sec}\right)$

Opór w rurce długości 1 cm, o przekroju $r = 0{\cdot}001$ cm

$P = \frac{4\eta .4.10^{-6}}{10^{-6}} = 16.\eta = 0{\cdot}18.16 = 0{\cdot}3$

podczas gdy $\frac{\partial p}{\partial x}$ ciśnienia osmotycznego: bezporównania większe

w gipsie drobnoz. według Stanuja $r = 0{\cdot}0005$ mm (Aurich $0{\cdot}0001$)

opór w rurce długości 1 cm $P = \frac{4.10^{-6}.\,4.\,0{\cdot}018}{10^{-10}} = 3000$ więc więcej bardzo

$$\frac{\partial p}{\partial x} - \eta\, \Delta^2 u = 0$$

$$\frac{\partial(\rho u)}{\partial x} = 0$$

$$\begin{cases} \dfrac{\partial p}{\partial x} = \eta\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}\right) \\ \dfrac{\partial(pu)}{\partial x} = 0 \end{cases}$$

$$p = R\theta\rho \qquad \rho = \frac{p}{R\theta}$$

$$\cancel{u\frac{\partial p}{\partial x} + p\frac{\partial u}{\partial x} = 0}$$

$$pu = f_0(y)$$

$$u = f_0(y) \qquad \frac{\partial u}{\partial x} = 0 \qquad \frac{\partial^2 u}{\partial x^2} = 0$$

$$\frac{\partial p}{\partial x} = \eta \frac{\partial^2 u}{\partial y^2}$$

$$p = f_0(x)$$

$$\frac{\partial^2 p}{\partial x \partial y} = 0 = \frac{\partial^3 u}{\partial y^3} = 0 \qquad \frac{\partial^2 u}{\partial y^2} = \text{const} = 2c$$

$$u = a + by + cy^2$$

$$\left.\begin{aligned} p &= 2c\eta\, x + \text{const} \\ p_1 &= \text{const} \\ p_2 &= 2c\eta l + \text{const} \end{aligned}\right\} c = \frac{p_2 - p_1}{2\eta l}$$

$$u = 0 \quad y = \pm\varepsilon$$

$$\left.\begin{aligned} 0 &= a + b\varepsilon + c\varepsilon^2 \\ 0 &= a - b\varepsilon + c\varepsilon^2 \end{aligned}\right\} \begin{aligned} b &= 0 \\ a &= -c\varepsilon^2 \end{aligned}$$

2 osten $u = c(y^2 - \varepsilon^2) = \dfrac{p_1 - p_2}{2\eta l}(\varepsilon^2 - y^2)$

$$\int_{-\varepsilon}^{+\varepsilon} u\, dy = \frac{p_1 - p_2}{2\eta l}\left(\varepsilon^2 y - \frac{y^3}{3}\right)\Bigg|_{-\varepsilon}^{+\varepsilon} = \frac{p_1 - p_2}{2\eta l}\,\frac{4}{3}\varepsilon^3 = \frac{2}{3}\,\frac{p_1 - p_2}{l}\,\frac{\varepsilon^3}{\eta}$$

$$u_0 = a = -c\varepsilon^2 = \frac{p_1 - p_2}{2\eta l}\varepsilon^2 \qquad = \frac{4}{3}\varepsilon u_0$$

Dyfuzya w bardzo wązkich rurkach musi się wreszcie zmniejszać. Gdyby tak wązka że przypada tylko 1 drobina toby dyfuzya zupełnie ustała.

(uwięzionego)

Więc wogóle musiał wchodzić w rachuby opór tarcia drobin uderzanych o siebie także tarcie o ~~przeciwległe~~ ściany.

To się więcej wydaje w dyfuzyi gazów

$$(u_1 - u_2) A_{12} \rho_1 \rho_2 + \frac{\partial p}{\partial x} = 0$$

$$\frac{\partial p_1}{\partial t} = k \frac{\partial^2 p_1}{\partial x^2} \qquad k = \frac{p_0^2 T^2}{A_{12}\, d_1 d_2 T_0^2} \frac{1}{p} \neq \frac{p}{A_{12} \rho_1 \rho_2}$$

$$u_1 - u_2 = - \frac{\frac{\partial p_1}{\partial x}}{p} k$$

N.p. Na odległości 1cm przypada różnica concentracyi $\begin{array}{c} p_1 = p \\ p_2 = 0 \end{array} \Bigg| \begin{array}{c} p_1 = 0 \\ p_2 = p \end{array}$

wtedy $u_1 - u_2 = -k$

N.p. H_2 – powietrze: $k = 0{\cdot}634 \frac{cm^2}{sec}$

Ale w jaki sposób się rozkłada to na H_2 i pow.? warunek $p = p_1 + p_2 = const$

W skutek tarcia warunek oporu typu: $u_1 = u_2 = 0$ na ściance; właściwie zatem uwzględniając opływ ścian także w tym zadaniu należałoby wprowadzić $\eta \frac{\partial u}{\partial y}$ etc: zadanie dwuwymiarowe

przybliżenie: $\frac{\partial p_1}{\partial x} + (u_1 - u_2) A_{12} \rho_1 \rho_2 - \eta \Delta^2 u = 0$

N.p. gaz przepływ. między równoległemi ścianami $u = a + by + cy^2$

$$\eta \Delta^2 u = 2c\,\eta = \frac{\partial p}{\partial x}$$

$$u_0 = a$$

Biorę $d = 0{\cdot}3$ mm
$l = 3$ mm

$\frac{7}{5}\frac{d}{l} = 8\%$

$q = \frac{0{\cdot}3^2}{4}\pi = \frac{3{\cdot}14 \cdot 9}{28{\cdot}26} = 0{\cdot}07$

$\cancel{\frac{2}{\alpha\delta_{3\,\text{dni}}} = \frac{9}{10} = 9\%}$

$\frac{2}{\alpha\delta_{\text{sąsiedni}}} = 4\%$

$n = 3000$

---

lepiej: $d = 0{\cdot}3$ mm
$l = 2$ mm

$\cancel{\frac{4}{5}\frac{d}{l} = 12\%}$

$\cancel{\frac{2}{\alpha\delta_{3\,\text{dni}}} = 5\%}$

$\cancel{n = \frac{1000}{5 \cdot 0{\cdot}07} = 3000}$

Inny sposób mierzenia stałej dyfuzyi bardzo szybko za pomocą ~~...~~ ~~...~~ których przetłum...

Concentration unterschiede in verschieden ...

Inny sposób bardzo szybki: Kombinacya z ruchem wody. Np. przepływ czystej wody przez rurki z substancyi.

$u_0 = 0$

H_2O

90% H_2O + 10% $C_{12}H_{22}O_{11}$

$u_1 = 0{\cdot}1$

Metoda do mierzenia dyfuzyi

Wiele mg. cukru przejdzie pro godzinę, jeśli długość rurki $= 1$ cm przekrój $= 1\ mm^2$

$k = 40 \cdot 10^{-7}$

$$dS = k\, q \frac{\partial u}{\partial x}\, dt \qquad \frac{\partial u}{\partial x} = \frac{u_1 - u_0}{x} = \frac{0{\cdot}1}{1} = 0{\cdot}1$$

$$dS = 0{\cdot}01 \cdot 0{\cdot}1 \cdot 40 \cdot 10^{-7} \cdot 60 \cdot 60 = 4 \cdot 6 \cdot 6 \cdot 10^{-7} = \frac{36{\cdot}4}{144} \cdot 10^{-7} g.$$
$$= 1{\cdot}4 \cdot 10^{-5} g.$$

dS pro dzień $\quad \begin{array}{l} 1{\cdot}44 \cdot 24 \\ \overline{\ \ 168} \\ \ \ 576 \\ \overline{3{\cdot}4\,56} \end{array} \quad = 3{\cdot}5 \cdot 10^{-4} g = 0{\cdot}35$ mg. pro 3 dni: $1{\cdot}05$ mg

$z =$ zawartość rurki: $0{\cdot}01 \cdot 0{\cdot}1 = 1$ mg.! więc znaczny błąd możliwy przy napełnianiu

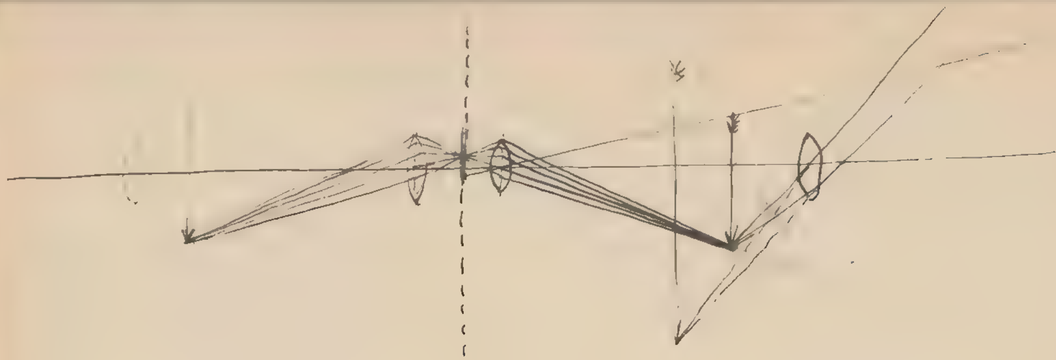
Ten sam stosunek $\left(\frac{z}{dS} = 100\%\right)$ będzie przy zwiększaniu przekroju

Ale ten stosunkowy możliwy błąd zmniejsza się w ~~st~~ kwadratowym stosunku skrócenia długości rurki; jeśli długość $= 1$ mm to $\frac{z}{dS}$ już tylko 1%

Ażeby correkcya z powodu końców nie była znaczniejszą musiałby też przekrój odpowiednio się zmniejszyć [Według Rayleigh'a reduk. długości pragnąć $x + \frac{4}{5} d$

więc jeśli $d = 0{\cdot}1$ mm: $\frac{4}{5} d = 8\%$]
ażeby zmniejszyć ... 0

Ażeby jednak ~~po~~ całkowita ilość $dS_{3\,dni}$ wynosiła 1 g musiałoby być aż tyle
$10^3 \cdot 10 = 10^4$ takich rurek!



Na odległości 27 cm : $1^\circ = 5$ mm

$1' = 0{\cdot}08$ mm

Powiększenie: Obj. 507 Ok. 1 500 / Ok. ~~VIII~~ 1_σ 1000 $\Big\}\ \delta = 0{\cdot}9$ mm $\alpha = 0{\cdot}36$ mm, $\alpha =$

z tego powiększenie Ok. 1_σ : 10

$a = 280$

$b = 102{\cdot}3$

$c = 9{\cdot}3$

$$\begin{array}{r} 1023 \\ 2046 \\ 3069 \\ \hline 10.46529 \\ 10416 \\ \hline \sqrt{49{\cdot}29} \end{array} = 7{\cdot}0|2 \qquad \begin{array}{r} 186 \\ 744 \\ \hline 2604 \\ 10416 \end{array}$$

$$\begin{array}{r} 560 \\ 102{\cdot}3 \\ \hline 662{\cdot}3 \end{array}$$

$$\frac{1}{7}\log\left(\frac{669{\cdot}3}{655{\cdot}3}\right) \qquad \begin{array}{r} 82562 \\ -81644 \\ \hline 0{\cdot}00918 \end{array} \qquad \begin{array}{r} 96284-3 \\ +36222 \\ \hline 0{\cdot}32506-2 \\ -0{\cdot}84510 \\ \hline 0{\cdot}47996-3 \end{array}$$

$$\begin{array}{r} p = 2{\cdot}52004 \\ -2{\cdot}51322 \\ \hline 0{\cdot}00682 \end{array} \; p_0 \; = \; 1{\cdot}016$$

Więc effekt zmienności κ wyniósłby : 1·6 % podczas gdy bez zmienności κ

(dla spadku temperatury liniowego) otrzymałby się : − 1·33 %

Do przybliżonego obliczenia skutku:

N. p. $\gamma = 0{\cdot}002$ stanowiące przeciętną wartość $1-\frac{x}{h} = \frac{1}{2}$

$$1+\gamma \frac{\theta_1}{2}\left(1-\frac{x}{h}\right) = 1 + \frac{0{\cdot}002 . 100}{2} \frac{1}{2} = 1 + 0{\cdot}05$$

więc zamiast θ_1 : 93 ... 98

$$\frac{98}{326{\cdot}5 \log \frac{378}{280}}$$

$$\begin{array}{r} 57749 \\ -\ 44716 \\ \hline 13033 \end{array} \qquad \begin{array}{r} 0{\cdot}11504 - \\ 36222 \\ 2{\cdot}5\cancel{4}38\cancel{8} \\ \hline 1{\cdot}99114 \end{array} \qquad \begin{array}{r} 1{\cdot}99123 \\ -\ 1{\cdot}99114 \\ \hline 0{\cdot}00009 \end{array}$$

Dokładniej:

$$\int_0^h \frac{dx}{280 + \frac{x}{h}\,\theta_1\left[1+\gamma\frac{\theta_1}{2}\left(1-\frac{x}{h}\right)\right]} \neq \int_0^h \frac{dx}{280+\frac{x}{h}\,\theta_1\left[1+0{\cdot}1\left(1-\frac{x}{h}\right)\right]}$$

$$= \int_0^h \frac{dx}{280 + 1{\cdot}1 . \frac{\theta_1}{h} x - \frac{0{\cdot}1 . \theta_1}{h^2} x^2} = h \int_0^1 \frac{dz}{280 + 1{\cdot}1 . \theta_1 z - 0{\cdot}1 . \theta_1 z^2}$$

93_{93}

$$\int \frac{dx}{a+bx-cx^2} = \frac{1}{\sqrt{b^2-4ac}} \log \frac{b+2cx-\sqrt{b^2-4ac}}{b+2cx+\sqrt{b^2-4ac}}$$

$$\int_0^1 = \frac{1}{\sqrt{b^2-4ac}} \log \frac{b+2c-\sqrt{b^2-4ac}}{b+2c+\sqrt{b^2-4ac}} \cdot \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}$$

$$\frac{\cancel{b^2}+2bc+2c\sqrt{} - \cancel{b^2}+4ac}{\cancel{b^2}+2bc-2c\sqrt{} - \cancel{b^2}+4ac} = \frac{b+\sqrt{b^2-4ac}+2a}{b-\sqrt{}+2a}$$

$$\theta = \frac{x}{h}\,\frac{\theta_1\left[1+\gamma\frac{\theta_1}{2}\right]}{\left[1+\gamma\frac{\theta}{2}\right]} = \frac{x}{h}\,\theta_1\left[1+\gamma\frac{\theta_1}{2}\right]\left[1-\frac{\gamma}{2}\frac{x}{h}\theta_1\right]$$

$$= \frac{x}{h}\,\theta_1\left[1+\gamma\frac{\theta_1}{2}\left(1-\frac{x}{h}\right)\right]$$

$$\int_0^h p\,dx = \cancel{\frac{p}{R}\int\frac{dx}{280+\frac{x}{h}\theta_1\left[1+\gamma\frac{\theta_1}{2}\left(1-\frac{x}{h}\right)\right]} = \frac{p}{R}\frac{h}{\theta_1}\int\frac{dx}{x}\left[1-\gamma\frac{\theta_1}{2}\left(1-\frac{x}{h}\right)\right]}$$

$$= \frac{p}{R}\int_0^h \frac{dx}{280+\frac{x}{h}\,\theta_1\left[1+\gamma\frac{\theta_1}{2}\left(1-\frac{x}{h}\right)\right]} = \frac{p_0\,h}{R\,(280+50)}$$

Tymczasem stawiając $\gamma=0$:

$$\frac{p}{R}\int\frac{dx}{280+\frac{x}{h}\theta_1} = \frac{p}{R}\,\frac{h}{\theta_1}\,\log\left(280+\frac{x\theta_1}{h}\right)\Big|_0^h = \frac{p}{R}\,\frac{h}{\theta_1}\,\log\left(\frac{280+\theta_1}{280}\right) = \frac{p_0\,h}{R(280+50)}$$

$$p = p_0\,\frac{\frac{\theta_1}{280+50}}{\underbrace{\log\left(\frac{280+\theta_1}{280}\right)}_{\log\left(1+\frac{\theta_1}{280}\right) = \frac{\theta_1}{280}-\frac{1}{2}\frac{\theta_1^2}{280^2}+\cdots}}$$

$$= p_0\,\frac{\frac{1}{280+50}}{\frac{1}{280}-\frac{1}{2}\frac{\theta_1}{280^2}+-} = p_0\,\frac{1}{(280+50)\left[\frac{1}{280}-\frac{1}{2}\frac{\theta_1}{280^2}+-\right]}$$

$\theta_1 = 93$

$\frac{93}{2} = 46.5$
280
326.5

$$\frac{93}{326.5\,\log\frac{373}{280}}$$

57171	0.09534 − 1	1.96848
44716	0.36222	−1.97144
0.12455	2.51388	0.99704
	1.97144	

$$p = 0.9932\,p_0$$

To byłoby ważne przy stałej objętości

$$\kappa_0 (1+\gamma\theta) \frac{d\theta}{dx} = c\, dx$$

dla $x=0$: $\theta=0$

$$\kappa_0\left(\theta+\gamma\frac{\theta^2}{2}\right) = cx+b$$

$$\cancel{\kappa_0\left(\theta_0+\gamma\frac{\theta_0^2}{2}\right)}\, 0 = +b$$

$$\kappa_0\left(\theta_1+\gamma\frac{\theta_1^2}{2}\right) = ch$$

$$\cancel{\kappa_0\frac{\theta_1-\theta_0}{h}\left[1+\gamma\frac{\theta_0+\theta_1}{2}\right]=c}$$

$$c = \frac{\kappa_0}{h}\left(\theta_1+\gamma\frac{\theta_1^2}{2}\right)$$

$$\frac{p}{\rho} = R\left(\theta + 280\right)$$

$1\alpha)$.: $\frac{p_0}{\rho_0} = R(\theta_{50}+280)$ $\int\rho\, dx = \rho_0 h$

$$\rho = \frac{p}{R\theta} \qquad \int\rho\, dx = \frac{p}{R}\int\frac{dx}{(\theta+280)}$$

$\frac{\gamma\theta}{6}$ $\frac{100}{1000}=5\%$

$\frac{p}{\rho_1} = R.280$

$\frac{p}{\rho_2} = R.373$

$$\kappa_0(\theta-\theta_0)\left[1+\frac{\gamma}{2}(\theta+\theta_0)\right] = c\cdot x$$

$\rho_0 = \frac{\rho_1+\rho_2}{2}$

$$\theta-\theta_0 = \frac{cx}{\kappa_0\left[1+\frac{\gamma}{2}(\theta+\theta_0)\right]} = \frac{cx}{\kappa_0}\left[1-\frac{\gamma}{2}\cancel{(\theta+\theta_0)}\right]$$

$$\rho_0 = \frac{p}{2R}\left(\frac{1}{280}+\frac{1}{373}\right) = \frac{p_0}{R\, 326.5}$$

przybliżenie: $\theta = \theta_0 + \frac{cx}{\kappa_0} = \theta_0 + (\theta_1-\theta_0)\frac{x}{h}$

$$p = p_0\frac{2}{326.5\left(\frac{1}{280}+\frac{1}{373}\right)}$$

$$\theta-\theta_0 = \frac{cx}{\kappa_0}\left[1-\frac{\gamma}{2}\left(2\theta_0+(\theta_1-\theta_0)\frac{x}{h}\right)\right]$$

57171	44716	
42829	55284	0.79605−3
0.0026810	0.0035714	2.51388
	26810	~~3.~~
	0.0062524	~~0.9~~
		0.30993

$$\kappa_0\left(\theta+\gamma\frac{\theta^2}{2}\right) = \kappa_0\frac{x}{h}\left(\theta_1+\gamma\frac{\theta_1^2}{2}\right)$$

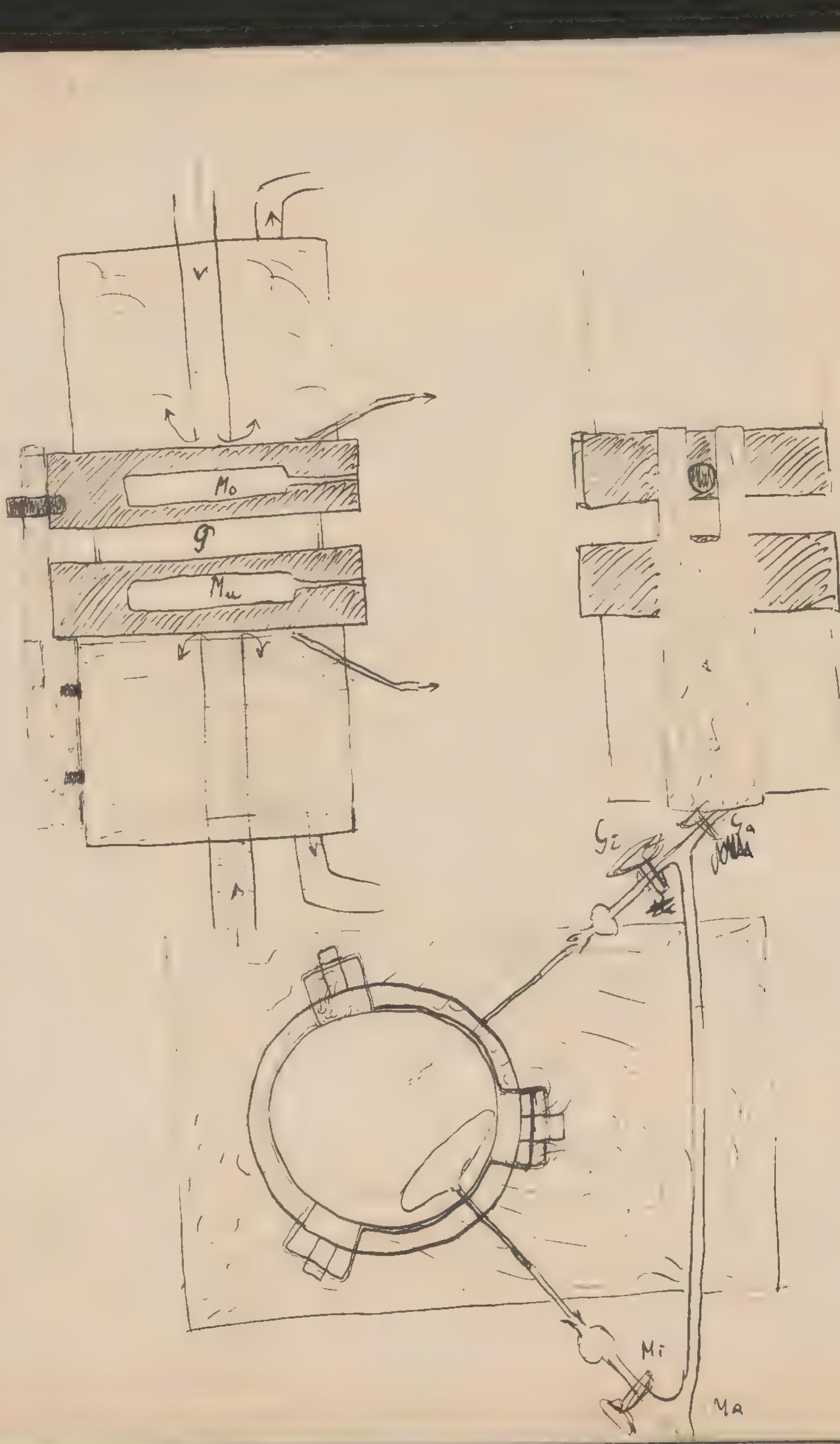
$$\theta\left[1+\gamma\frac{\theta}{2}\right] = \frac{x}{h}\theta_1\left[1+\gamma\frac{\theta_1}{2}\right]$$

0.30103
~~0.97881~~
0.36522

0.30103
− 0.30993
0.99110

$(x = 0.97872\,10)$

64



Mo
G
Mu
Gi
Ga
Mi
Ma

Porządek operacyj:

1). Przepuścić wodę lub parę 50° u górze i na dole

α). Otworzyć kurki Gi i Mi [i także Ga lub Ma na chwilę ażeby ciśnienie wewnątrz nie było wyższe od atmosf. !

β). zamknąć obydwa Gi i Mi

γ). otworzyć Ga i Ma (kolejno i wypraktykować kroplę rtęci, lub gliceryny?

do niedużej części rurki ostudzić i zaznaczyć miejsce na podziałce

δ. zamknąć Ga i Ma

2). Przepuścić u górze parę 100° na dole wodę zimną 7°

α. Otworzyć kurki Gi i Mi (ale równocześnie obydwa na raz

ostudzić i zaznaczyć miejsce kropli na podziałce

β). zamknąć Gi i Mi,

3). Aparat odwrócić, u górze parę, na dole zimną wodę

α). Otworzyć kurki Gi i Mi

ostudzić i zaznaczyć miejsce kropli na podziałce

β. zamknąć Gi Mi

4). Przepuścić wodę lub parę 50° u górze i na dole

α). Otworzyć kurki Gi Mi

ostudzić i zaznaczyć miejsce kropli na podziałce

(v = 0.02 – 0.14 cm/sec)

the case of waterdrops as found in J J Thomson's and C T R Wilson's condensation experiments the range of the order of several centimetres, in Perrin's experiments on the validity of Stokes' law for particles of emulsions it would amount even to hundreds of meters; ~~it would be easy to find suitable~~ in the same way it is sufficiently great for practical purposes, when highly viscous liquids are used, as Ladenburg did in his research. ~~Of course it excludes the Ordinary hydrodynamic experiments~~ ~~out of question~~

~~The~~ ~~inertia~~ method of approximation (correction for inertia might be applied also to the other cases treated before. For movements in closed vessels besides however it will be of importance than in liquids extending to infinity; it will imply rather cumbersome calculations. ~~...~~ we may point to the Arnold's experiments, who found ~~the ... ...~~ (influence of cylindric tube ~~wall~~ to ~~the motion of a sphere~~] to ... sufficiently exact ... Ladenburgs correction to ... dius would make the

$\frac{\varrho v b}{2\mu}$ equal to

~~out of question~~ excluded of course ...

Zulärdn & Komarski

$r > \frac{ga^2}{r^2}\sin\varphi = 2\cdot 2\gamma\,\omega^2$

$r^3 \quad \frac{ga^2}{\omega^2}$

$r^3 = 289\,r^2$

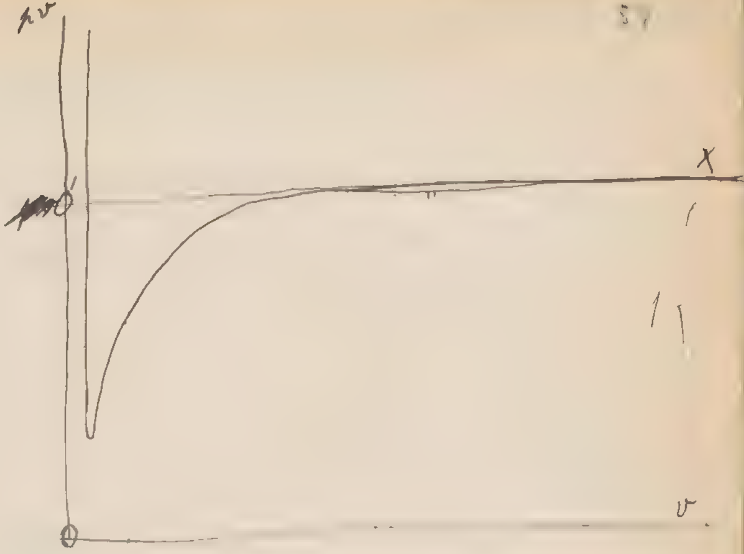
1
1
50
75
100
15
2[illegible]
30
50
10[illegible]

CO_2

$t = 40^\circ$

p	pv	v
1	1·000	1·000
50	0·8500	0·0170
75	0·6200	0·00827
100	0·3090	0·0031
150	0·3770	0·00251
200	0·4075	0·00231
300	0·6485	0·00216
500	0·9900	0·00198
1000	1·7800	0·00178

$\frac{1}{2}\sum m c^2 = \frac{3}{2}(pv) + \sum\sum R r$



Wenn man die Sache auf $O'X$ bezieht so hat man direct den Wert von $\sum\sum Rr = \sum R\sqrt[3]{v}$

Diese Curven werden in allgem. bei verschiedenen Temperaturen verschieden ..., denn bei gleichem v wird 1) die Anzahl der Summanden verschieden sein (wie $\frac{1}{T}$ daher $\sum\sum$ wie $\frac{1}{T^2}$) 2) die mittlere Distanzen ändern wegen verschieden sein wie $T^{\frac{1}{3}}$

Wenn also $R = f(r) = f(\sqrt[3]{v})$ gesetzt wird so ist $\sum\sum Rr = \frac{1}{T^2}\, T^{\frac{1}{3}} f(T^{\frac{1}{3}})$

Spez. Wärme einatomiger Gase.

Wie wenn sie nur unendlich wenig von Kugelgestalt abweichen? Dann wird k doch 1.66 sein

Was aber wenn man voraussetzt dass eine Rotation überhaupt nicht eintreten kann, dadurch dass z.B. Erhaltung der Rotations Ebene

Wie wenn als Wirbel angesehen?

Strahlung?

Wenn ein Kreisel in eine Kugel eingeschlossen wird, so kann man doch auf ~~die~~ seine Existenz schliessen infolge des eigenthümlichen Verhaltens der Kugel. Wenn aber zwei in entgegengesetzter Richtung rotierende Kreisel von gleichem Trägheitsmoment in einander gesetzt werden so kann sich deren Wirkung nach aussen nicht bemerkbar machen

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Methode zur Messung der Wärmeleitungs-Zunahme mit der Temperatur.

2 unendliche Platten $h\left\{\begin{array}{l}\overline{\qquad\qquad\qquad}\ \vartheta_1\\ \overline{\qquad\qquad\qquad}\ \vartheta_0\end{array}\right.$

I). Falls κ konstant wäre, so wäre Gasdruck entsprechend $\frac{\vartheta_0+\vartheta_1}{2}$

II) Dagegen wenn $\kappa = \kappa_0(1+\beta\vartheta)$:

$$\cancel{\kappa_0}(1+\beta\vartheta)\frac{d\vartheta}{dx} = \text{const.} = a$$

$$\vartheta + \beta\frac{\vartheta^2}{2} = ax+b$$

$$b = \vartheta_0 + \frac{\beta}{2}\vartheta_0^2$$

$$a = \frac{\vartheta_1-\vartheta_0+\beta\frac{\vartheta_1^2-\vartheta_0^2}{2}}{h}$$

Oder wenn $\vartheta_0 = 0$ gesetzt wird:

$$\vartheta+\beta\frac{\vartheta^2}{2} = \frac{x}{h}\left(\vartheta_1+\frac{\beta}{2}\vartheta_1^2\right)$$

Mittlere Temperatur $\Theta = \frac{1}{h}\int_0^h \vartheta\, dx = \frac{1}{h}\int_0^1 \vartheta\,\frac{d\vartheta(1+\beta\vartheta)}{a} = \cancel{\frac{1}{h}\frac{\vartheta_1+\frac{\beta}{2}\vartheta_1^2}{a}}$

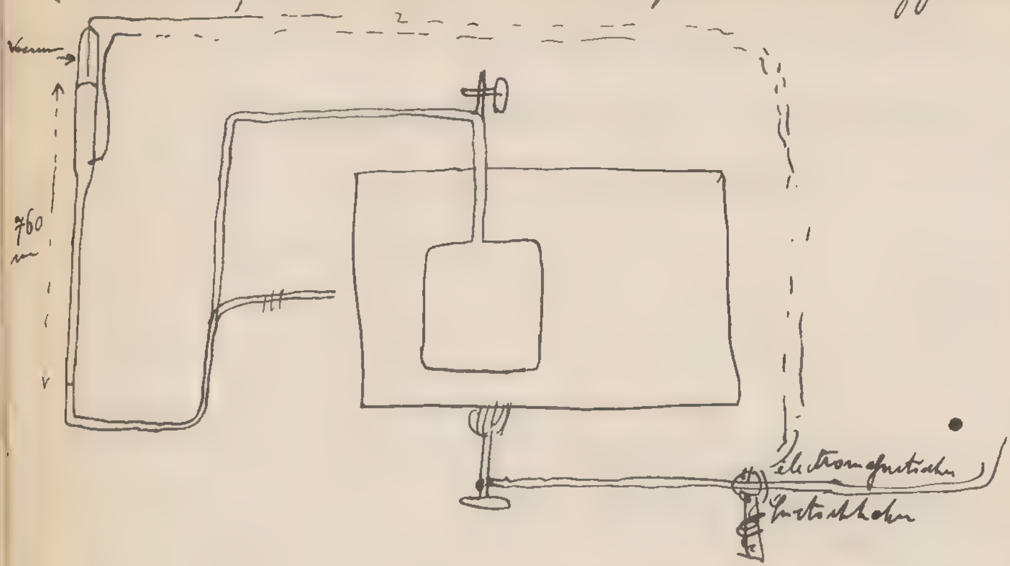
$$\Theta = \frac{1}{ah}\left(\frac{\vartheta_1^2}{2}+\beta\frac{\vartheta_1^3}{3}\right) = \frac{\frac{\vartheta_1^2}{2}+\beta\frac{\vartheta_1^3}{3}}{\vartheta_1+\beta\frac{\vartheta_1^2}{2}} = \frac{\vartheta_1}{2}\,\frac{1+\frac{2}{3}\beta\vartheta_1}{1+\frac{1}{2}\beta\vartheta_1} \doteq \frac{\vartheta_1}{2}\left[1+\frac{1}{6}\beta\vartheta_1\right]$$

Wenn also etwa $\vartheta_0=0$, $\vartheta_1 = 100$, $\beta = 0.0018$

$$\Theta = 50\left[1+0.03\right] = 50^\circ + 1.5^\circ$$

und zwar unabhängig von Distanz der Platten, unabhängig von Strahlung

Thermostat, welcher vom äusseren Luftdruck unabhängig ist:



Correctionsformel für Drahtende Schliessmeth.

$$w i^2 \cancel{dx} = \frac{2\pi \cancel{dx}}{\lg \frac{R}{r}} \kappa \lg \frac{\vartheta}{\theta} + q \kappa' \frac{\partial^2 \vartheta}{\partial x^2}$$

$$C \cancel{} = \vartheta \frac{\partial^3 \vartheta}{\partial x^3} \qquad = \frac{\partial}{\partial x}\left(\vartheta \frac{\partial^2 \vartheta}{\partial x^2}\right) - \frac{\partial \vartheta}{\partial x}\frac{\partial^2 \vartheta}{\partial x^2}$$

$$= \frac{\partial}{\partial x}\left[\vartheta \frac{\partial^2 \vartheta}{\partial x^2} - \frac{1}{2}\left(\frac{\partial \vartheta}{\partial x}\right)^2\right]$$

$$Cx + C_1 = \vartheta \frac{\partial^2 \vartheta}{\partial x^2} - \frac{1}{2}\left(\frac{\partial \vartheta}{\partial x}\right)^2$$

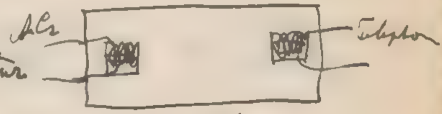
$$= \frac{\partial}{\partial x}\left(\vartheta \frac{\partial \vartheta}{\partial x}\right) - \left(\frac{\partial \vartheta}{\partial x}\right)^2$$

Scheinbare Widerstands Vermehrung von Wechselströmen in Umgebung von O_2; wegen Arbeit die zur Ozonisirung verbraucht wird.

Bei andern Gasen nichts? Wie bei Evacuirung?

Hydromechanik verdünnter Gase; Planetenatmosphären, Reibungsdrehungsmoment, Mischung verschiedener Gase, Diffusion, Stoney's Hypothese; Sternschnuppen; ~~Bei~~ Druckunterschiede in Geissler-Röhren; Druck im Funkenfelde; Schallfortpflanzung; Reibungsströme;

Strahlenbildung von Gasen



Hydromechanik von Gasen bei kritischer Temperatur

Anwendung zur Ortsbestimmung derselben dadurch dass keine Schallfortpflanzung.

Thermal diffusion und ~~Jou~~ Thomson-Joule Effect ~~[illegible]~~ sind inverse Phänomene? Gibt es thermal diffusion in Lösungen?

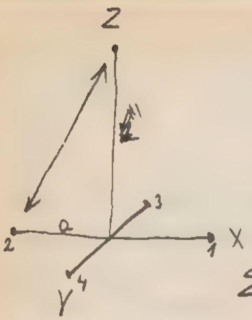
~~[illegible]~~ Convection von suspendirten Teilchen durch Strom; entsteht infolge Electrisirung infolge Flüssigkeitsreibung? ~~[illegible]~~ Reibungselectr. entsteht bei jedem Geschwindigkeitsgefälle? Wie wenn verschiedene Pulver gemischt und im electrischen Felde geschüttelt werden? Ist Electrolyse nicht ein identischer Vorgang?

Selbstinductionscoefficienten bei Entladung von Kugeln etc. Schwingungen die dadurch entstehen. ~~[illegible]~~

Scheinbare dielectrische Hysteresis, infolge elastischer Nachwirkung der Deformation im electrost. Felde

Quantitative Messungen über Absorption, Emission etc. von Dämpfen z.B. Joddampf, Na-Dampf

Leduc & Sacerdote: Dalton's Gesetz anzugreifen: dass Volumina sich addiren. Warum? Kein Gesetz!



$$V = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4}$$

$$r_1^2 = z^2 + (a-x)^2 + y^2$$
$$r_2^2 = z^2 + (a+x)^2 + y^2$$
$$r_3^2 = z^2 + (a+y)^2 + x^2$$
$$r_4^2 = z^2 + (a-y)^2 + x^2$$

$$\frac{\partial V}{\partial x} = -\frac{1}{r_1}\frac{\partial r_1}{\partial x} \; \cdots$$

$$\Sigma X = \frac{1}{r_1}\frac{\partial r_1}{\partial x} + \cdots$$

$$\frac{\partial}{\partial x}\Sigma X = -\frac{\partial^2 V}{\partial x^2} = -\frac{1}{r_1^2}\left(\frac{\partial r_1}{\partial x}\right)^2 + \frac{1}{r_1}\frac{\partial^2 r_1}{\partial x^2} \; \cdots \qquad \frac{a-x}{z} \qquad -\frac{x^2}{z^3} - \frac{1}{z}$$

$$\cancel{= -\frac{(a-x)^2}{[z^2+(a-x)^2]^2} - \frac{(a+x)^2}{[z^2+(a+x)^2]^2} - \frac{x}{[z^2+a^2+x^2]^2} - \frac{x}{[z^2+a^2+x^2]^2}} + \cdot$$

$$\Sigma X = \frac{-(a-x)}{[z^2+(a-x)^2]^{3/2}} + \frac{a+x}{[z^2+(a+x)^2]^{3/2}} + \frac{2x}{[z^2+a^2+x^2]^{3/2}}$$

$$-\frac{\partial^2 V}{\partial x^2} = -3\frac{(a-x)^2}{[z^2+(a-x)^2]^{5/2}} - \frac{3(a+x)^2}{[z^2+(a+x)^2]^{5/2}} - \frac{6x^2}{[z^2+a^2+x^2]^{5/2}}$$

$$+ \frac{1}{[z^2+(a-x)^2]^{3/2}} + \frac{1}{[z^2+(a+x)^2]^{3/2}} + \frac{2}{[z^2+a^2+x^2]^{3/2}}$$

$$Z = -\frac{\partial V}{\partial z} = \frac{z}{[z^2+(a-x)^2]^{3/2}} + \frac{z}{[z^2+(a+x)^2]^{3/2}} + \frac{2z}{[z^2+a^2+x^2]^{3/2}}$$

$$\frac{\partial Z}{\partial z} = -\frac{\partial^2 V}{\partial z^2} = -\frac{3z^2}{[z^2+(a-x)^2]^{5/2}} - \frac{3z^2}{[z^2+(a+x)^2]^{5/2}} - \frac{6z^2}{[z^2+a^2+x^2]^{5/2}} +$$

$$+ \frac{1}{[z^2+(a-x)^2]^{3/2}} + \frac{1}{[z^2+(a+x)^2]^{3/2}} + \frac{2}{[z^2+a^2+x^2]^{3/2}}$$

$$-\cancel{\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial z^2}\right) = \frac{-3}{[z^2+(a-x)^2]^{3/2}} + \dots + \frac{2}{[z^2+(a-x)^2]^{3/2}}}$$

$$-\frac{\partial^2 V}{\partial x^2} = \cancel{-3 \dots a^2 - 2ax + x^2}$$

$$= \frac{-3}{[z^2+a^2]^{5/2}} \left\{ \frac{(a-x)^2}{\left[1 + \frac{-2ax+x^2}{a^2+z^2}\right]^{5/2}} + \frac{(a+x)^2}{\left[1+\frac{2ax+x^2}{a^2+z^2}\right]^{5/2}} + \frac{2x^2}{\left[1+\frac{x^2}{a^2+z^2}\right]^{5/2}} \right\} +$$

$$+ \frac{1}{[a^2+z^2]^{3/2}} \left\{ \frac{1}{\left[1 + \frac{-2ax+x^2}{a^2+z^2}\right]^{3/2}} + \text{-------} \right\}$$

für lim $x = 0$: $-\dfrac{\partial^2 V}{\partial x^2} = \dfrac{-6a^2}{(a^2+z^2)^{5/2}} + \dfrac{4}{(a^2+z^2)^{3/2}}$

$$-\frac{\partial^2 V}{\partial z^2} = \frac{-12 z^2}{(a^2+z^2)^{5/2}} + \frac{4}{(a^2+z^2)^{3/2}}$$

Damit <0: $\dfrac{\partial^2 V}{\partial x^2}$

$-\dfrac{6a^2}{a^2+z^2} + 4 < 0$	$\dfrac{-12 z^2}{a^2+z^2} + 4 < 0$
$-2a^2 + 4z^2 < 0$	$-8z^2 + 4a^2 < 0$
$2z^2 < a^2$	$a^2 < 2z^2$ daher höchstens:

$$\frac{\partial^2 V}{\partial x^2} = 0 \quad \text{und} \quad \frac{\partial^2 V}{\partial z^2} = 0 \quad \text{für} \quad z^2 = \frac{a^2}{2}$$

$$-\frac{\partial^3 V}{\partial x^3} = -15 \frac{(a-x)^3}{[z^2+(a-x)^2]^{7/2}} + 15 \frac{(a+x)^3}{[z^2+(a+x)^2]^{7/2}} + \frac{30 x^3}{[z^2+a^2+x^2]^{7/2}}$$

$$+ 9 \frac{(a-x)}{[z^2+(a-x)^2]^{5/2}} - 9 \frac{(a+x)}{[z^2+(a-x)^2]^{5/2}} - \frac{18 x}{[z^2+a^2+x^2]^{5/2}} \Bigg|_{x=0} = 0$$

$$-\frac{\partial^4 V}{\partial x^4} = -3.5.7\frac{(a-x)^4}{\left[z^2+(a-x)^2\right]^{9/2}} - 3.5.7\frac{(a+x)^4}{\cdots} - 2.3.5.7\frac{x^4}{\cdots}$$

$$+45\frac{(a-x)^2}{\left[z^2+(a-x)^2\right]^{7/2}} + 45\frac{(a+x)^2}{\cdots} + 2.45\frac{x^2}{\cdots}$$

$$+45\frac{(a-x)^2}{\left[z^2+(a-x)^2\right]^{7/2}} + 45\frac{(a+x)^2}{\cdots} + 2.45\frac{x^2}{\cdots}$$

$$-9\frac{1}{\left[z^2+(a-x)^2\right]^{5/2}} - 9\frac{1}{\cdots} - 2.9\frac{1}{\cdots}$$

für $x=0$: $$-\frac{\partial^4 V}{\partial x^4} = -\frac{4.3.5.7\,a^4}{(z^2+a^2)^{9/2}} + 180\frac{a^2}{(a^2+z^2)^{7/2}} - \frac{36}{(a^2+z^2)^{5/2}}$$

$$= \frac{1}{(a^2+z^2)^{9/2}}\left\{ -\underset{456}{420}\,a^4 + 180(a^4+a^2z^2) - 36(a^4+2a^2z^2+z^4)\right\}$$

$$= \frac{1}{(a^2+z^2)^{9/2}}\left\{-276\,a^4 + 108\,a^2z^2 - 36z^4\right\}$$

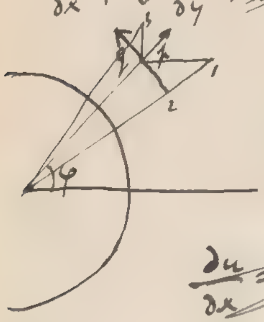
$$= \frac{12}{(a^2+z^2)^{9/2}}\left\{-23\,a^4 + 9\,a^2z^2 - 3z^4\right\} < 0$$

Wenn $a^2 = 2z^2$ $\underbrace{}_{=}$ $-92 + \overset{9\,5}{18} - 3 = -77\,z^2$

Zweidimensional, incompressibel

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \cancel{w\frac{\partial u}{\partial z}} = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}\right) - \frac{1}{\rho}\frac{\partial P}{\partial x} \qquad \Big|\ \frac{\partial}{\partial y}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \cancel{w\frac{\partial v}{\partial z}} = \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \cancel{\frac{\partial^2 v}{\partial z^2}}\right) - \frac{1}{\rho}\frac{\partial P}{\partial y} \qquad \Big|\ \frac{\partial}{\partial x}$$



$$p = u\cos\varphi + v\sin\varphi \qquad \Big\|\ u = p\cos\varphi - q\sin\varphi$$

$$q = -u\sin\varphi + v\cos\varphi \qquad \Big\|\ v = p\sin\varphi + q\cos\varphi$$

$$x = r\cos\varphi$$

$$\cancel{\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial u}{\partial \varphi}\frac{\partial \varphi}{\partial x}}$$

$$\cancel{= \frac{\partial u}{\partial r}\frac{1}{\cos\varphi} - \frac{\partial u}{\partial \varphi}\frac{1}{r\sin\varphi}}$$

$$\frac{\partial u}{\partial x} = \frac{u_1 - u_0}{\Delta x} = \frac{u_1 - u_2}{\Delta x} + \frac{u_2 - u_0}{\Delta x} = \frac{u_1 - u_2}{\Delta r}\cos\varphi + \frac{u_2 - u_0}{r\Delta\varphi}\sin\varphi$$

$$= \frac{\partial u}{\partial r}\cos\varphi - \frac{\partial u}{\partial \varphi}\frac{\sin\varphi}{r} =$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial r^2}\cos^2\varphi - \frac{\partial^2 u}{\partial r\partial\varphi}\frac{\cos\varphi\sin\varphi}{r} + \cancel{\frac{\partial u}{\partial\varphi}\frac{\sin\varphi\cos\varphi}{r^2}} - \frac{\partial^2 u}{\partial r\partial\varphi}\frac{\cos\varphi\sin\varphi}{r} -$$

$$+ \frac{\partial u}{\partial r}\frac{\sin^2\varphi}{r} + \frac{\partial^2 u}{\partial\varphi^2}\frac{\sin^2\varphi}{r^2} + \cancel{\frac{\partial u}{\partial\varphi}\frac{\cos\varphi\sin\varphi}{r^2}}$$

$$= \frac{\partial^2 u}{\partial r^2}\cos^2\varphi - 2\frac{\partial^2 u}{\partial r\partial\varphi}\frac{\sin\varphi\cos\varphi}{r} + \frac{\partial^2 u}{\partial\varphi^2}\frac{\sin^2\varphi}{r^2} + \frac{\partial u}{\partial r}\frac{\sin^2\varphi}{r} + 2\frac{\partial u}{\partial\varphi}\frac{\sin\varphi\cos\varphi}{r^2}$$

$$\frac{\partial u}{\partial y} = \frac{u_3 - u_0}{\Delta y} = \frac{u_3 - u_4}{\Delta y} + \frac{u_4 - u_0}{\Delta y} = \frac{\partial u}{\partial r}\sin\varphi + \frac{\partial u}{\partial\varphi}\frac{\cos\varphi}{r}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial r^2}\sin^2\varphi + \frac{\partial^2 u}{\partial r\partial\varphi}\frac{\sin\cos}{r} - \frac{\partial u}{\partial\varphi}\frac{\sin\cos}{r^2} + \cancel{\frac{\partial^2 u}{\partial r\partial\varphi}\frac{\sin\cos}{r}} + \frac{\partial u}{\partial r}\frac{\cos^2}{r} + \frac{\partial^2 u}{\partial\varphi^2}\frac{\cos^2}{r^2} - \frac{\partial u}{\partial\varphi}\frac{\sin\cos}{r^2}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 u}{\partial\varphi^2} + \frac{1}{r}\frac{\partial u}{\partial r}$$

$$\frac{\partial u}{\partial r} = \frac{\partial p}{\partial r}\cos\varphi - \frac{\partial q}{\partial r}\sin\varphi \qquad \Big|\ \cos\varphi$$

$$\frac{\partial u}{\partial \varphi} = \frac{\partial p}{\partial \varphi}\cos\varphi - \frac{\partial q}{\partial \varphi}\sin\varphi - p\sin\varphi - q\cos\varphi \qquad \Big|\ -\frac{\sin\varphi}{r}$$

$$\frac{\partial u}{\partial x} =$$

$\nabla 6^2 - V 6 \operatorname{curl} 6$

$\operatorname{curl}\left(V 6 \operatorname{curl} 6 - \mu \nabla^2 6\right) = 0$

Aussichtslos!

~~$\mathfrak{GTL}$~~

$$\cancel{\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}} + u\frac{\partial^2 u}{\partial x \partial y} + \cancel{\frac{\partial v}{\partial y}\frac{\partial u}{\partial y}} + v\frac{\partial^2 u}{\partial y^2} - \cancel{\frac{\partial u}{\partial x}\frac{\partial v}{\partial x}} - u\frac{\partial^2 v}{\partial x^2} - \cancel{\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}} - v\frac{\partial^2 v}{\partial x \partial y}$$

$$= \mu\left(\frac{\partial^3 u}{\partial x^2 \partial y} + \frac{\partial^3 u}{\partial y^3} - \frac{\partial^3 v}{\partial x^3} - \frac{\partial^3 v}{\partial x\, \partial y^2}\right)$$

$$\zeta = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \qquad \frac{\partial \zeta}{\partial x} = \frac{\partial^2 u}{\partial x \partial y} - \frac{\partial^2 v}{\partial x^2} = -\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + v\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) = \mu\left(\frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}\right)\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)$$

$$u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} = \mu\left(\frac{\partial \zeta}{\partial x^2} + \frac{\partial \zeta}{\partial y^2}\right)$$

$$\frac{\partial u}{\partial x}\frac{\partial \zeta}{\partial x} + \frac{\partial v}{\partial x}\frac{\partial \zeta}{\partial y} + u\frac{\partial^2 \zeta}{\partial x^2} + v\frac{\partial^2 \zeta}{\partial x \partial y} =$$

$$\frac{\partial(u\zeta)}{\partial x} + \frac{\partial(v\zeta)}{\partial y} = \nearrow$$

$$\frac{\partial}{\partial x}\left[u\zeta - \mu\frac{\partial \zeta}{\partial x}\right] + \frac{\partial}{\partial y}\left[v\zeta - \mu\frac{\partial \zeta}{\partial y}\right] = 0$$

~~$\operatorname{div} 6\zeta - \mu\nabla\zeta$~~

$\mathfrak{a} = \nabla V + \mathrm{curl}\, \mathfrak{A}$

$$u = \frac{\partial V}{\partial x} + -\frac{\partial A_2}{\partial z} + \frac{\partial A_3}{\partial y}$$

$$v = \frac{\partial V}{\partial y} + -\frac{\partial A_3}{\partial x} + \frac{\partial A_1}{\partial z}$$

$$w = \frac{\partial V}{\partial z} + -\frac{\partial A_1}{\partial y} + \frac{\partial A_2}{\partial x}$$

Also zweidimensional:

$$u = \frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y}$$

$$v = \frac{\partial \varphi}{\partial y} + \frac{\partial \psi}{\partial x}$$

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0 \qquad \zeta = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}$$

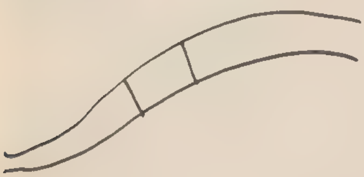
φ und ψ ganz unabhängig

Wenn jetzt z.B. ψ willkürlich gewählt, wie muss φ gewählt werden?

$$\left(\frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y}\right)\left(\frac{\partial^3 \psi}{\partial x^3} + \frac{\partial^3 \psi}{\partial x \partial y^2}\right) + \left(\frac{\partial \varphi}{\partial y} - \frac{\partial \psi}{\partial x}\right)\left(\frac{\partial^3 \psi}{\partial x^2 \partial y} + \frac{\partial^3 \psi}{\partial y^3}\right) = \mu \frac{\partial^4 \psi}{\partial x^4} + \dots$$

Flüssigkeits Gleichungen bezogen auf ~~Bahnen der~~ Stromlinien (ohne Reibung)

In zwei Dimensionen:



$$m \frac{dv}{dt} = F_t = m \frac{dv}{ds}\frac{ds}{dt} = m v \frac{dv}{ds} = \frac{1}{2}\frac{d}{ds}(mv^2) = \frac{dE}{ds}$$

$$m \frac{v^2}{R} = F_n$$

$$mv^2 = 2\int F_t\, ds$$

$$F_t = \frac{1}{q}\frac{\partial(qp)}{\partial s}$$

$$R = 2\frac{\int F_t\, ds}{F_n} = 2\frac{E}{F_n}$$

$$F_n = \frac{\partial P}{\partial R} + \frac{P}{R}$$

$$F_n = \frac{1}{R}\frac{\partial (RP)}{\partial R}$$

$$2E = P + R\frac{\partial P}{\partial R} \qquad \frac{\partial P}{\partial R} = \frac{2E - P}{R}$$

$$(\mathfrak{S}\nabla)\mathfrak{S} = \nabla P + \mu\nabla^2\mathfrak{S} \quad = \nabla\mathfrak{S}^2 + V\mathfrak{S}\,\mathrm{curl}\,\mathfrak{S}$$

$$\mathrm{curl} \quad = \quad \mu\nabla^2\,\mathrm{curl}\,\mathfrak{S} \;=\; \mathrm{curl}\,V\mathfrak{S}\,\mathrm{curl}\,\mathfrak{S}$$

$$\nabla\frac{\partial P}{\partial\mu} + \nabla^2\mathfrak{S} + \mu\nabla^2\frac{\partial\mathfrak{S}}{\partial\mu} = \nabla\frac{\partial(\mathfrak{S}^2)}{\partial\mu} + \frac{\partial}{\partial\mu}V\mathfrak{S}\,\mathrm{curl}\,\mathfrak{S}$$

Stationäre Strömung ohne Kräfte, mit Reibg

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{1}{\rho}\frac{\partial P}{\partial x}$$

$$\frac{x}{\mu} = \xi \qquad \frac{\partial u}{\partial x} = \frac{1}{\mu}\frac{\partial u}{\partial \xi} \qquad \frac{\partial^2 u}{\partial x^2} = \frac{1}{\mu^2}\frac{\partial^2 u}{\partial \xi^2}$$

$$u\frac{\partial u}{\partial \xi} + v\frac{\partial u}{\partial \eta} + w\frac{\partial u}{\partial \zeta} = \frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2} + \frac{\partial^2 u}{\partial \zeta^2} - \frac{1}{\rho}\frac{\partial P}{\partial \xi} \quad \text{etc.}$$

Wenn man also diese Gleichungen auflösen kann, so hat man auch die Lösung für ein beliebiges μ und umgekehrt

---

Wenn die Änderung von μ auf u, v, w ohne Einfluss sein soll, so muss

$$\nabla\frac{\partial P}{\partial\mu} + \nabla^2\mathfrak{S} = 0 \quad \text{sein.}$$

Also: $\mathrm{curl}\,\nabla^2\mathfrak{S} = 0$

$$(\mathfrak{S}\nabla)\mathfrak{S} = \underbrace{\nabla P - \mu\nabla\frac{\partial P}{\partial\mu}} = \nabla\mathfrak{S}^2 + V\mathfrak{S}\,\mathrm{curl}\,\mathfrak{S}$$

$$= \nabla\left[P - \mu\frac{\partial P}{\partial\mu}\right] = -\nabla\mu^2\frac{\partial}{\partial\mu}\left(\frac{P}{\mu}\right) = \nabla\Pi$$

Also nur Druck wird geändert

$$f(x,y,z,\mu) - \mu\,\frac{\partial f(x,y,z,\mu)}{\partial\mu} = F(x,y,z)$$

$$\frac{\partial f}{\partial\mu} - \frac{\partial f}{\partial\mu} - \mu\,\frac{\partial^2 f}{\partial\mu^2} = 0$$

$$\frac{\partial f}{\partial\mu} = \varphi(x,y,z)$$

$$f = \mu\,\varphi(x,y,z) + \psi(x,y,z)$$

$$\cancel{\mu\,\varphi(x,y,z)} + \psi(x\,y\,z) - \cancel{\mu\,\varphi(x,y,z)} \quad = F(x\,y\,z)$$

$$P = \mu\,\varphi(x,y,z) + \Pi$$

$$\nabla P = \mu\,\nabla\varphi + \nabla\Pi$$

$$\left.\begin{aligned}(\mathfrak{v}_*\nabla)\mathfrak{v}_* &= \nabla P + \mu\,\nabla^2\mathfrak{v}_* = \mu\nabla\varphi + \nabla\Pi + \mu\,\nabla^2\mathfrak{v}\\ (\mathfrak{v}\nabla)\mathfrak{v} &= \nabla\Pi\end{aligned}\right\}$$

$$\left[-\nabla\varphi = \nabla^2\mathfrak{v}\right. \qquad \varphi = \text{willkürliche}\ \overset{\text{scalare}}{\text{Function}}$$

$$\left.\begin{aligned}-\nabla^2\mathfrak{v} &= \operatorname{curl}^2\mathfrak{v}\\ \operatorname{curl}\nabla^2\mathfrak{v} &= -\operatorname{curl}^3\mathfrak{v} = 0\\ &\cancel{\operatorname{curl}^3\mathfrak{v} = 0}\\ \operatorname{curl} V\mathfrak{v}\operatorname{curl}\mathfrak{v} &= 0\end{aligned}\right\} \quad !$$

$$\operatorname{curl} V a b = a \operatorname{div} b - b \operatorname{div} a + \underbrace{(b\nabla)a - (a\nabla)b}$$

$$(b\nabla)a = \nabla_a ab + V\operatorname{curl} a \,.\, b \,\Big\| = V(\operatorname{curl} a\,.\,b - \operatorname{curl} b\,.\,a) + \underbrace{+\nabla_a ab - \nabla_b ab}$$

$$\sigma = \nabla u + \operatorname{curl} v$$

$$\operatorname{curl}\sigma = \operatorname{curl}^2 v$$

$$\operatorname{curl}^2\sigma = \operatorname{curl}^3 v = +\nabla\varphi \quad \text{Soit}: \ \sigma = \int \frac{\nabla\varphi\, dv}{4\pi r}$$

$$\nabla\sigma \operatorname{curl}\sigma = \nabla(\nabla u + \operatorname{curl}(v)) \operatorname{curl}^2 v = \nabla\Phi$$

$$\operatorname{curl}\sigma = \operatorname{curl}\int \frac{\nabla\varphi\, dv}{4\pi r} = \int \frac{\operatorname{curl}\nabla\varphi\, dv}{4\pi r}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{\partial\varphi}{\partial x} \qquad u = \int \frac{\frac{\partial\varphi}{\partial x}}{4\pi r}\, dv$$

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = \frac{\partial\varphi}{\partial y} \qquad v =$$

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = \frac{\partial\varphi}{\partial z} \qquad w =$$

$$\operatorname{curl}^4 v = \operatorname{curl}^3\sigma = 0$$

$$\left.\begin{aligned} \operatorname{curl}\sigma &= iA_1 + jA_2 + kA_3 \\ \nabla^2 A_1 &= 0 = \nabla^2 A_2 = \nabla^2 A_3 \end{aligned}\right| = \operatorname{curl}^2 v = i\nabla^2 B_1 + \cdots$$

$$\nabla(iA_1 + jA_2 + kA_3)$$

wenn $\operatorname{curl} a = \nabla b \qquad \nabla^2 b = 0$ ~~ANR~~

$$\operatorname{curl}^2 a = 0 = \nabla \operatorname{div} a - \nabla^2 a$$

$$a = \nabla \int \frac{\operatorname{div} a}{r} dv + \underbrace{\operatorname{curl} \int \frac{\operatorname{curl} a}{r} dv}_{} + \cdots \qquad \nabla^2 A = 0$$

$$= \operatorname{curl} \int \frac{\nabla b}{r} dv$$

ist div $= 0$?

dann muss $\nabla b = 0 = \operatorname{curl} a = 0$

Somit im obigen Falle: $\operatorname{curl}^2 6 = 0$

$$\operatorname{curl} 6 = \nabla \operatorname{pot} = 0$$

$$6 = \nabla \operatorname{pot}$$ also nur Potentialbewegung

Dann auch Druck derselbe

$$\nabla 6^2 = \nabla P + \mu \nabla^2 6 \qquad \nabla^2 u = 0 = \nabla^2 v = \nabla^2 w$$

$$\nabla 6^2 = \nabla P \qquad u = \frac{\partial \varphi}{\partial x} \qquad \nabla^3 \varphi = 0 \text{ und } \nabla^2 \varphi = 0$$

$\nabla^2 6 = 0$ } von selbst erfüllt

Somit besteht jede stationäre Potentialbewegung auch bei beliebiger innerer Reibung ?

z.B. $u = \cancel{\quad} - \frac{ay}{r^2} \qquad \frac{\partial u}{\partial x} = 2\frac{axy}{r^4} \qquad \frac{\partial v}{\partial y} = -\frac{2axy}{r^4}$

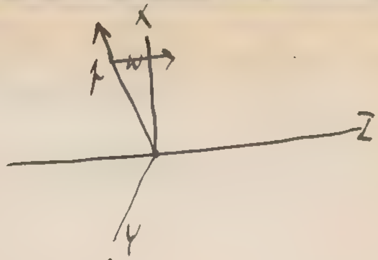
$$v = \frac{ax}{r^2} \qquad \frac{\partial u}{\partial y} = -\frac{a}{r^2} + \frac{2ay^2}{r^4} \qquad \frac{\partial v}{\partial x} = \frac{a}{r^2} - \frac{2ax^2}{r^4}$$

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{2ay}{r^4} - \frac{8ax^2y}{r^6}$$

$$\frac{\partial^2 u}{\partial y^2} = +\frac{2ay}{r^4} + \frac{4ay}{r^4} - \frac{8ay^3}{r^6} \qquad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{8ay}{r^4} - \frac{8ay(x^2+y^2)}{r^6} = 0$$

z = Rotationsaxe … w



$u = p\cos(px) = p\cos\varphi$

$v = p\sin(py) = p\sin\varphi$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\cos\varphi - \cancel{\frac{\partial u}{\partial \varphi}\frac{\sin\varphi}{r}} = \frac{\partial u}{\partial r}\cos\varphi + p\frac{\sin^2\varphi}{r}$$

$$\frac{\partial u}{\partial x} = \frac{\partial p}{\partial r}\cos^2\varphi + \frac{p}{r}\sin^2\varphi \qquad \Big\| \qquad \frac{\partial v}{\partial x} = \frac{\partial p}{\partial r}\sin\varphi\cos\varphi - \frac{p}{r}\sin\varphi\cos\varphi$$

$$\cancel{\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial r^2}\cos^2\varphi + \frac{\partial u}{\partial r}\frac{\sin^2\varphi}{r^2} = \frac{\partial^2 p}{\partial r^2}\cos^3\varphi + \frac{\partial p}{\partial r}\frac{\sin^2\varphi\cos\varphi}{r}}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}\sin\varphi = \frac{\partial p}{\partial r}\sin\varphi\cos\varphi - \frac{p}{r}\sin\varphi\cos\varphi$$

$$\cancel{\frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial r^2}\sin^2\varphi + \frac{\partial u}{\partial r}\frac{\cos^2\varphi}{r} = \frac{\partial^2 p}{\partial r^2}\sin^2\varphi\cos\varphi + \frac{\partial p}{\partial r}\frac{\cos^3\varphi}{r}}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 p}{\partial r^2}\cos\varphi + \frac{\partial p}{\partial r}\frac{\cos\varphi}{r} - \frac{p}{r^2}\cos\varphi \qquad \Big\| \qquad \frac{\partial v}{\partial y} = \frac{\partial p}{\partial r}\sin^2\varphi + \frac{p}{r}\cos^2\varphi$$

$$\cancel{+\frac{p^2}{r}\sin\varphi\cos\varphi - \frac{p^2}{r}\sin^2\varphi\cos\varphi}$$

$$p\frac{\partial p}{\partial r}\cos^3\varphi + p\frac{\partial p}{\partial r}\sin^2\varphi\cos\varphi + w\frac{\partial p}{\partial z}\cos\varphi = \mu\left[\frac{\partial^2 p}{\partial r^2}\cos\varphi + \frac{\partial p}{\partial r}\frac{\cos\varphi}{r} - \frac{p}{r^2}\cos\varphi + \frac{\partial^2 p}{\partial z^2}\cos\varphi\right]$$

$$- \frac{1}{\rho}\frac{\partial P}{\partial r}\cos\varphi$$

$$p\frac{\partial p}{\partial r} + w\frac{\partial p}{\partial z} = \mu\left[\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} - \frac{p}{r^2} + \frac{\partial^2 p}{\partial z^2}\right] - \frac{1}{\rho}\frac{\partial P}{\partial r} \qquad \Big| \qquad \frac{\partial}{\partial z}$$

$$u\frac{\partial w}{\partial r}\cos\varphi + v\frac{\partial w}{\partial r}\sin\varphi + w\frac{\partial w}{\partial z} =$$

$$p\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = \mu\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right] - \frac{1}{\rho}\frac{\partial P}{\partial z} \qquad \Big| \qquad \frac{\partial}{\partial r}$$

$$\cancel{\frac{\partial p}{\partial r}\frac{\partial p}{\partial r} + p\frac{\partial^2 p}{\partial r\,\partial z} - \frac{\partial p}{\partial r}\frac{\partial w}{\partial r} - p\frac{\partial w}{\partial r^2} + \frac{\partial w}{\partial z}\frac{\partial p}{\partial z} + w\frac{\partial^2 p}{\partial z^2} - \frac{\partial w}{\partial r}\frac{\partial w}{\partial z} - w\frac{\partial^2 w}{\partial r\,\partial z}} =$$

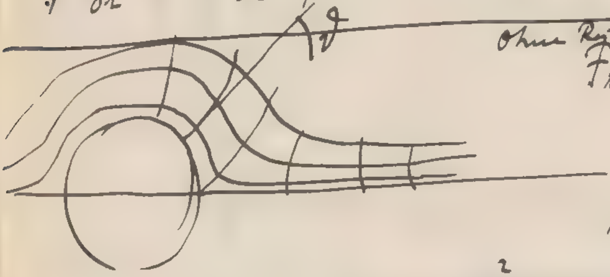
$= \mu$ Continuitätsbedingung: $\cancel{\frac{\partial p}{\partial r}\cos\varphi + \frac{\partial p}{\partial z}\sin\varphi + \frac{\partial w}{\partial z} = 0}$

$$\cancel{\frac{\partial p}{\partial r} + \frac{\partial w}{\partial z} = 0} \qquad \underbrace{\frac{\partial p}{\partial r} + \frac{p}{r}}_{= \frac{1}{r}\frac{\partial (pr)}{\partial r}} + \frac{\partial w}{\partial z} = 0 \qquad \frac{\partial}{\partial r}(rp) + \frac{\partial}{\partial z}(rw) = 0$$

$$\frac{\partial}{\partial z}\frac{1}{r}\frac{\partial(pr)}{\partial r} = -\frac{\partial^2 w}{\partial z\,\partial r}$$

$$p\frac{\partial p}{\partial r} + w\frac{\partial p}{\partial z} = \mu\left[\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} - \frac{p}{r^2} + \frac{\partial^2 p}{\partial z^2}\right] - \frac{1}{\rho}\frac{\partial P}{\partial r}$$

$$p\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = \mu\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right] - \frac{1}{\rho}\frac{\partial P}{\partial z}$$



Ohne Reibung:

$$F_t = \frac{\partial}{\partial s}(Pq) \qquad \text{Continuität:} \quad \frac{\partial}{\partial s}(vq) = 0$$

$$= q\,\delta s\,\frac{\partial v}{\partial t} = \frac{\partial v}{\partial s}\cdot v = \frac{1}{2}\frac{\partial (v^2)}{\partial s} \qquad v\frac{\partial q}{\partial s} + q\frac{\partial v}{\partial s} = 0$$

$$\cancel{v_1 - v_2 = \int F_t\,dt =}$$

$$\cancel{v_2^2 - v_1^2 = 2\int_1^2 F_t\,ds = 2(P_2 q_2 - P_1 q_1)}$$

Daraus folgt: $P = P_0 - \frac{v^2}{2}$ für bestimmte Stromlinie

$q = r\,\delta\varphi\,\delta\Delta$

$$q\,\delta s\,\frac{v^2}{R} = F_n = \frac{\partial}{\partial \Delta}(P r)\,\delta\varphi\,\delta s\,\delta\Delta$$

$$\frac{\partial (Pr)}{\partial \Delta} = \frac{\partial (Pr)}{\partial r}\sin\vartheta \qquad \Delta s = \frac{\Delta r}{\sin\vartheta}$$

$$r\frac{v^2}{R} = \frac{\partial (Pr)}{\partial \Delta} = \frac{\partial (v r)}{\cancel{\quad}}$$

$$R = \frac{\partial s}{\partial \vartheta} = \frac{\partial r}{\partial \vartheta}\frac{1}{\sin\vartheta}$$

$$r v^2 \frac{\partial \vartheta}{\partial s} = \frac{\partial (Pr)}{\partial r}\sin\vartheta = r v^2\frac{\partial \vartheta}{\partial r}\sin\vartheta$$

$$r v^2\frac{\partial \vartheta}{\partial r} = \frac{\partial (Pr)}{\partial r}$$

Wenn die Richtung der Stromlinien gegeben ist:

$$\frac{u}{w} = f(r,z) \qquad \frac{\partial w}{\partial r} f + w\frac{\partial f}{\partial r} + \frac{w}{r} f + \frac{\partial w}{\partial z} = 0$$

$$u = w\, f(r,z) \qquad \frac{\partial w}{\partial r} f + \frac{\partial w}{\partial z} + w\frac{\partial f}{\partial r} + \frac{w}{r} f = 0$$

$$wf\left(w\frac{\partial f}{\partial r} + \frac{\partial w}{\partial r} f\right) + w\left(\frac{\partial w}{\partial z} f + w\frac{\partial f}{\partial z}\right) = -\frac{1}{\rho}\frac{\partial P}{\partial r} +$$

$$+\mu\left[2\frac{\partial w}{\partial r}\frac{\partial f}{\partial r} + w\frac{\partial^2 f}{\partial r^2} + \frac{\partial^2 w}{\partial r^2} f + \frac{w}{r}\frac{\partial f}{\partial r} + \frac{f}{r}\frac{\partial w}{\partial r} - \frac{f}{r^2} w + \frac{\partial^2 w}{\partial z^2} f + 2\frac{\partial w}{\partial z}\frac{\partial f}{\partial z} + w\frac{\partial^2 f}{\partial z^2}\right]$$

$$wf\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = \mu\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right] - \frac{1}{\rho}\frac{\partial P}{\partial z} \qquad \| f$$

$$wf\frac{\partial f}{\partial r} + w^2\frac{\partial f}{\partial z} = \mu\left[w\frac{\partial^2 f}{\partial r^2} + 2\frac{\partial w}{\partial r}\frac{\partial f}{\partial r} + \frac{w}{r}\frac{\partial f}{\partial r} - f\frac{w}{r^2} + 2\frac{\partial w}{\partial z}\frac{\partial f}{\partial z} + w\frac{\partial^2 f}{\partial z^2}\right]$$

$$-\frac{1}{\rho}\left(\frac{\partial P}{\partial r} + f\frac{\partial P}{\partial z}\right)$$

$$\frac{\partial^2 w}{\partial r^2} f + 2\frac{\partial w}{\partial r}\frac{\partial f}{\partial r} + w\frac{\partial^2 f}{\partial r^2} + \frac{\partial w}{\partial r}\frac{f}{r} + \frac{w}{r}\frac{\partial f}{\partial r} - \frac{wf}{r^2} + \frac{\partial^2 w}{\partial r\,\partial z} = 0$$

Folglich

$$= \mu\left[\frac{\partial^2 w}{\partial z^2} f + 2\frac{\partial w}{\partial z}\frac{\partial f}{\partial z} - \frac{\partial^2 w}{\partial r\,\partial z} + w\frac{\partial^2 f}{\partial z^2}\right] - \frac{1}{\rho}\frac{\partial P}{\partial r} =$$

$$= w^2\frac{\partial f}{\partial z} - \frac{w}{r} f^2$$

[illegible]

$\frac{\cdot}{\cdot} \cdot \frac{\cdot}{\cdot} =)$ [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

σ sei eine Lösung

$$(\sigma\nabla)\sigma = \frac{1}{2}\nabla\sigma^2 + \nabla\sigma \operatorname{curl}\sigma = \mu\nabla^2\sigma - \frac{1}{\rho}\nabla P$$

Dann ist $\sigma_1 = c\sigma$ auch eine Lösung für denselben Fall, aber bei $c\mu$ und Pc^2 ~~kleineren~~ größeren Druck

$\sigma_1 = \sigma c$ $\quad \mu_1 = \mu c$ $\quad P_1 = Pc^2$

$$c^2\left(\frac{1}{2}\nabla\sigma^2 + \nabla\sigma \operatorname{curl}\sigma\right) = c\mu_1\nabla^2\sigma - \frac{1}{\rho}\nabla P_1$$

Angenommen $u_1\ v_1\ w_1$ sei eine Lösung, unter welchen Umständen wird u_1+u etc. eine Lösung sein?

$$\cancel{u_1\frac{\partial u_1}{\partial x}+v_1\frac{\partial u_1}{\partial y}+w_1\frac{\partial u_1}{\partial z}=\frac{\partial^2 u_1}{\partial x^2}+\frac{\partial^2 u_1}{\partial y^2}+\frac{\partial^2 u_1}{\partial z^2}-\frac{1}{\rho}\frac{\partial P_1}{\partial x}}$$

$$u\frac{\partial u_1}{\partial x}+v\frac{\partial u_1}{\partial y}+w\frac{\partial u_1}{\partial z}+u_1\frac{\partial u}{\partial x}+v_1\frac{\partial u}{\partial y}+w_1\frac{\partial u}{\partial z}=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}-\frac{1}{\rho}\frac{\partial(P-P_1)}{\partial x}$$

$$+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}$$

Wenn z.B. $\left.u,\ v,\ w=\text{const}\right\}a$

$$-\frac{1}{\rho}\frac{\partial(P-P_1)}{\partial x}=\sum a\nabla u_1$$

etc. $\qquad -\frac{1}{\rho}\nabla(P-P_1)=(a\nabla)\sigma$

$$u\left(\frac{\partial^2 u_1}{\partial x\partial y}-\frac{\partial^2 v_1}{\partial x^2}\right)+v\left(\frac{\partial^2 u_1}{\partial y^2}-\frac{\partial^2 v_1}{\partial y\partial x}\right)+w\left(\frac{\partial^2 u_1}{\partial y\partial z}-\frac{\partial^2 v_1}{\partial x\partial z}\right)=0 \quad \text{etc.}$$

$$u\frac{\partial}{\partial x}\left(\frac{\partial u_1}{\partial y}-\frac{\partial v_1}{\partial x}\right)+v\frac{\partial}{\partial y}\left(\frac{\partial u_1}{\partial y}-\frac{\partial v_1}{\partial x}\right)+w\frac{\partial}{\partial z}\left(\frac{\partial u_1}{\partial y}-\frac{\partial v_1}{\partial x}\right)=0$$

$$\left\{\begin{aligned} &u\frac{\partial\xi}{\partial x}+v\frac{\partial\xi}{\partial y}+w\frac{\partial\xi}{\partial z}=0\\ &u\frac{\partial\xi}{\partial x}+\cdots=0\\ &u\frac{\partial\eta}{\partial x}+\cdots=0\end{aligned}\right.$$

$\cancel{\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0}$

Bedingung $\cancel{\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial^2 w}{\partial x\partial z}=0}$

Was folgt daraus?

$$(\sigma_1\nabla)\sigma_1=\tfrac{1}{2}\nabla\sigma_1^2+V\sigma_1\,\mathrm{curl}\,\sigma_1=\nabla^2\sigma_1-\frac{1}{\rho}\nabla P_1$$

$$\nabla(\sigma_1+\sigma)^2+V(\sigma_1+\sigma)\,\mathrm{curl}(\sigma_1+\sigma)=\nabla^2(\sigma_1+\sigma)-\frac{1}{\rho}\nabla P_{11}$$

$\cancel{\nabla\frac{P}{\sigma}}$

$$\cancel{\nabla\sigma_1^2} + \underbrace{\nabla(\sigma_1\sigma + \sigma\sigma_1)}_{2\nabla S\sigma\sigma_1} + \nabla\sigma^2 + \cancel{V\sigma_1 \operatorname{curl}\sigma_1} + V\sigma_1 \operatorname{curl}\sigma + V\sigma \operatorname{curl}\sigma_1 + V\sigma\operatorname{curl}\sigma$$

$$= \nabla^2\tau + \cancel{\nabla^2\tau_1} - \frac{1}{\rho}\nabla(P - P_1)$$

Wenn auch σ_1 eine Lös— ist:

$$2\nabla\sigma\sigma_1 + V(\sigma_1 \operatorname{curl}\sigma + \sigma\operatorname{curl}\sigma_1) = -\frac{1}{\rho}\nabla(P - P_1 - P')$$

P' kann daraus genügend bestimmt werden falls überhaupt:

$$\operatorname{curl} V(\sigma_1 \underbrace{\operatorname{curl}\sigma + \sigma\operatorname{curl}\sigma_1}) = 0$$

Also wenn $\quad = \tau\operatorname{curl}\tau$ gesetzt wird so muss τ eine mögliche (reibungslose) Flüssigkeitsbewegung sein

$$\operatorname{curl} Vab = a \operatorname{div} b - b\operatorname{div} a + (b\nabla)a - (a\nabla)b$$

$$(\operatorname{curl}\sigma\,\nabla)\sigma_1 - (\sigma_1\nabla)\operatorname{curl}\sigma + (\operatorname{curl}\sigma_1\,\nabla)\sigma - (\sigma\nabla)\operatorname{curl}\sigma_1 = 0$$

$x = \xi^{\frac{1}{2}} \quad y = \eta^{\frac{1}{2}} \quad z = \zeta^{\frac{1}{2}} \qquad P = \Pi^2$

$x^2 = \xi$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \xi}\frac{\partial \xi}{\partial x} \qquad \frac{\partial \xi}{\partial x} = 2x = 2\sqrt{\xi}$$

$$= 2\sqrt{\xi}\,\frac{\partial u}{\partial \xi}$$

$$\frac{\partial^2 u}{\partial x^2} = \left[2\sqrt{\xi}\frac{\partial^2 u}{\partial \xi^2} + \frac{1}{\sqrt{\xi}}\frac{\partial u}{\partial \xi}\right]2\sqrt{\xi} = 4\xi\frac{\partial^2 u}{\partial \xi^2} + 2\frac{\partial u}{\partial \xi}$$

$$\frac{2n-1}{W} = \frac{n-2}{W} \qquad u = U^{-1} = \frac{1}{U} \qquad P = \Pi^{-2}$$

$$n = -1 \qquad \frac{\partial u}{\partial x} = -\frac{1}{U^2}\frac{\partial U}{\partial x} \qquad \frac{\partial^2 u}{\partial x^2} = +\frac{2}{U^3}\left(\frac{\partial U}{\partial x}\right)^2 - \frac{1}{U^2}\frac{\partial^2 U}{\partial x^2}$$

Wenn man also eine Lösung der [illegible] hat so erhält man eine andere wenn man x, y, z m mal grösser macht aber u, v, w m mal kleiner → Druck [illegible] wird $(m)^2$ mal kleiner (aber dasselbe u).

I xyz	uvw	μ	P
II $m\,xyz$	uvw	$m\mu$	P
III xyz	$m\,uvw$	$m\mu$	m^2P
IV $m\,xyz$	$\frac{1}{m}\,uvw$	μ	$\frac{1}{m^2}P$

Daraus folgt
$xyz \quad m\,uvw \quad m\mu \quad m^2P$
sind also nur zwei davon unabhängige Transformation

III ist zur Anwendung geeignet

$$(\sigma\nabla)\sigma = F - \frac{1}{\rho}\nabla P + \frac{\mu}{3\rho}\nabla\,\mathrm{div}\,\sigma + \frac{\mu}{\rho}\nabla^2\sigma$$

$$\mathrm{div}(\rho\sigma) = 0 \qquad = \rho\,\mathrm{div}\,\sigma + S.\sigma\nabla\rho = 0$$

$$P = f(\rho) \quad \Big\| \quad \mathrm{div}\,\sigma\,\nabla\rho + \rho\nabla\,\mathrm{div}\,\sigma + \nabla(S.\sigma\nabla\rho) = 0$$

$\nabla S a\, b = a\,\mathrm{div}\,b + b\,\mathrm{div}$

$\left[i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y}\right][a_1 b_1 \ldots]$

$a_1\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + \ldots\right)$

$+$

$\left(\frac{\partial b_1}{\partial x} + \ldots\right)(i a_1 + j a_2 + k a_3)$

curl $A =$

$$\rho\left[\frac{1}{2}\nabla\sigma^2 + V\sigma\,\mathrm{curl}\,\sigma\right] = -\nabla P + \frac{\mu}{\rho}\nabla\sigma + \frac{\mu}{3\rho}\nabla\,\mathrm{div}\,\sigma$$

curl:

$$\frac{1}{2}\left[\cancel{\rho\,\mathrm{curl}\nabla\sigma^2} + V.\nabla\sigma^2\nabla\rho\right] + \rho\,\mathrm{curl}\,V\sigma\,\mathrm{curl}\,\sigma \pm V(\nabla\rho . V\sigma\,\mathrm{curl}\,\sigma) = \mu\,\mathrm{curl}\,\nabla^2\sigma$$

~~Wenn div σ~~

$$-V.\nabla\rho\,\underbrace{\left[\frac{1}{2}\nabla\sigma^2 + V\sigma\,\mathrm{curl}\,\sigma\right]}_{} + \rho\,\mathrm{curl}\,(V\sigma\,\mathrm{curl}\,\sigma) = \mu\,\mathrm{curl}\,\nabla^2\sigma$$

$$= -\frac{\nabla P}{\rho} + \frac{\mu}{3\rho}\nabla\,\mathrm{div}\,\sigma + \frac{\mu}{\rho}\nabla^2\sigma$$

$$= -V.\nabla\,\lg\rho\left[-\nabla P + \frac{\mu}{3}\nabla\,\mathrm{div}\,\sigma + \frac{\mu}{\rho}\nabla^2\sigma\right]$$

$$\mathrm{curl}.V\sigma\,\mathrm{curl}\,\sigma = \mathrm{curl}\left[-\frac{\nabla P}{\rho} + \frac{\mu}{3\rho}\nabla\,\mathrm{div}\,\sigma + \frac{\mu}{\rho}\nabla^2\sigma\right]$$

$$= \cancel{\frac{\mu}{3\rho}\,\mathrm{curl}\,\nabla\,\mathrm{div}\,\sigma} + \frac{\mu}{3\rho^2}V\nabla\rho\nabla\,\mathrm{div}\,\sigma$$

$$+ \frac{\mu}{\rho}\ldots$$

$$= \frac{\mu}{\rho}\,\mathrm{curl}\,\nabla^2\sigma + V.\nabla\rho\left[\frac{\mu}{3\rho^2}\nabla\,\mathrm{div}\,\sigma + \frac{\mu}{\rho^2}\nabla^2\sigma\right]$$

$$\nabla S ab = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)(a_1 b_1 + a_2 b_2 + a_3 b_3)$$

$$\operatorname{curl} m\bar{b} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ mb_1 & mb_2 & mb_3 \end{vmatrix} = i\, m \left(\frac{\partial b_3}{\partial y} - \frac{\partial b_2}{\partial z}\right) + \ldots$$

$$+ i\left(b_3 \frac{\partial m}{\partial y} - b_2 \frac{\partial m}{\partial z}\right) + \ldots$$

$$= m \operatorname{curl} b + V.\, b \nabla m$$

$$\operatorname{curl} \frac{\nabla P}{\rho} = \operatorname{curl} \frac{\nabla P}{\rho} - \frac{1}{\rho^2} V \nabla \rho \nabla P$$

∇P und $\nabla \rho$ sind gleichgerichtet

also falls $V \nabla \rho$ ...

und dispersion ...

$\operatorname{div} \ldots = - \int \sigma \nabla \ldots$

also falls $V \nabla \rho$ ...

$\nabla^2 \log \rho$ ... $= \frac{1}{4\pi}\int \nabla^2 \log \rho \, dv$

$\nabla \log \rho = \ldots \frac{\ldots}{\sigma_0}$

$\rho = \psi$

$\nabla \rho = \ldots$ $\quad \nabla \rho = \ldots \nabla V \cdot \nabla P$

[illegible]

$\frac{1}{2}\nabla \sigma^2 + V \times \mathrm{curl}\, V = -\nabla P + \ldots \nabla V$

[illegible] $\sigma = \rho \psi$

$\operatorname{div} \rho \psi = 0$ $\quad \mu = f(\rho)$

$\nabla \sigma^2 = \nabla \rho^2 \psi^2 = \rho^2 \nabla \psi^2 + \psi^2 \nabla \rho^2$

$\mathrm{curl}\, V = \rho\, \mathrm{curl}\, \psi + \nabla \rho \times \nabla \psi$

$V \times \mathrm{curl}\, V = \rho V \psi\, \mathrm{curl}\, \psi + \rho \psi \ldots$

$= -\nabla \rho \ldots + \psi \ldots \nabla \psi$

$= \rho^2 V \psi\, \mathrm{curl}\, \psi - \nabla \rho \cdot \psi^2 + \rho \psi \ldots \nabla \rho$

$\nabla^2(\rho\varphi) = $ [illegible] $\nabla^2\rho$ [illegible]

[illegible]

$\frac{1}{2}\rho\nabla\varphi^2$ [illegible] $+\rho\nabla\varphi$ [illegible] $= $ [illegible]

$$\left(\frac{\partial^2 \xi}{\partial x^2}\right)+\left(\frac{\partial^2 \xi}{\partial y^2}\right)=0$$

26. $\xi = x\left(y-\sqrt{a^2-x^2}\right)$

$$\frac{\partial\xi}{\partial x} = y-\sqrt{a^2-x^2} + \frac{x^2}{\sqrt{a^2-x^2}} \qquad \frac{\partial \xi}{\partial y}=x$$

$$\frac{\partial^2\xi}{\partial x^2} = \frac{x}{\sqrt{a^2-x^2}} + \frac{2x}{\sqrt{a^2-x^2}} + \frac{x^3}{\sqrt{(a^2-x^2)^3}} \qquad \frac{\partial^2\xi}{\partial y^2}=0$$

$$\xi = \left(y-\sqrt{a^2-x^2}\right)^n$$

$$\frac{\partial \xi}{\partial x} = \frac{x\,n}{\sqrt{\quad}}\left(y-\sqrt{\quad}\right)^{n-1}$$

$$\frac{\partial^2\xi}{\partial x^2} = \frac{n}{\sqrt{\quad}}\left(y-\sqrt{\quad}\right)^{n-1} + \frac{x^2 n}{\sqrt{(\quad)^3}}\left(y-\sqrt{\quad}\right)^{n-1} + \frac{n(n-1)\,x^2}{(a^2-x^2)}\left(y-\sqrt{\quad}\right)^{n-2}$$

$$\frac{\partial}{\partial x}\left(u\xi-\mu\frac{\partial\xi}{\partial x}\right)+\frac{\partial}{\partial y}\left(v\xi-\mu\frac{\partial\xi}{\partial y}\right)+\frac{\partial}{\partial z}\left(w\xi-\mu\frac{\partial\xi}{\partial z}\right)=0 \qquad k$$

$$\frac{\partial}{\partial x}\left(u\zeta-\mu\frac{\partial\zeta}{\partial x}\right)+\frac{\partial}{\partial y}\left(v\zeta-\mu\frac{\partial\zeta}{\partial y}\right)+\frac{\partial}{\partial z}\left(w\zeta-\mu\frac{\partial\zeta}{\partial z}\right)=0 \qquad i$$

$$\frac{\partial}{\partial x}\left(u\eta-\mu\frac{\partial\eta}{\partial x}\right)\ldots \qquad j$$

$$\frac{\partial\zeta}{\partial y}=n\left(y-\sqrt{\ }\right)^{n-1}$$

$$\frac{\partial^2\zeta}{\partial y^2}=n(n-1)\left(y-\sqrt{\ }\right)^{n-2}$$

$$\frac{\partial^2\zeta}{\partial x^2}+\frac{\partial^2\zeta}{\partial y^2}=n\frac{a^2-x^2+x^2}{\sqrt{(\ )^3}}\left(y-\sqrt{\ }\right)^{n-1}+\frac{n(n-1)x^2\left(y-\sqrt{\ }\right)^{n-2}}{a^2-x^2}+n(n-1)\left(y-\sqrt{\ }\right)^{n-2}$$

$$=n\left(y-\sqrt{\ }\right)^{n-2}\left[\frac{a^2y}{\sqrt{(\ )^3}}-\cancel{\frac{a}{a^2-x^2}}+\frac{nx^2}{a^2-x^2}-\cancel{\frac{x^2}{a^2-x^2}}+n-2\right]$$

$n>2$

10. $\varphi=x^2(y-a)^3$

$$\xi=2(y-a)^3+6x^2(y-a)$$

$$\frac{\partial\xi}{\partial x}=12x(y-a) \qquad \frac{\partial\xi}{\partial y}=6(y-a)^2+6x^2$$

$$\frac{\partial^2\xi}{\partial x^2}+\frac{\partial^2\xi}{\partial y^2}=12(y-a)+12(y-a)=24(y-a)$$

$$\frac{\partial \varphi}{\partial x}\ \cancel{\,}2x(y-a) + \frac{\partial \varphi}{\partial y}\left[\cancel{\,}(y-a)^2 + \cancel{\,}x^2\right] = \cancel{\,}4(y-a) -$$

$$- 3x^2(y-a)^2\ \cancel{\,}2x(y-a) + 2x(y-a)\left[\cancel{\,}(y-a)^2 + \cancel{\,}x^2\right]$$

$$\frac{\partial \varphi}{\partial x}\ 2x(y-a) + \frac{\partial \varphi}{\partial y}\left[(y-a)^2 + x^2\right] = 4(y-a) - 6x^3(y-a)^3 + 2x(y-a)^3 + 2x^3(y-a)$$

$$= 2(y-a)\left[2 + \left[(y-a)^2 + x^2 - 3x^2(y-a)^2\right]x\right]$$

$$P\frac{\partial \varphi}{\partial x} + Q\frac{\partial \varphi}{\partial y} = R$$

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{d\varphi}{R}$$

$$\frac{dx}{2x(y-a)} = \frac{dy}{(y-a)^2 + x^2} \qquad \left[\frac{y-a}{2x} + \frac{x}{2(y-a)}\right]dx = dy$$

$$\frac{y-a}{x} = z \qquad y-a = zx \qquad dy = z\,dx + x\,dz$$

$$\left(z + \frac{1}{z}\right)dx = 2(z\,dx + x\,dz)$$

$$dx = z^2 dx + 2x\,z\,dz$$

$$(1-z^2)\,dx = 2x\,z\,dz$$

$$\frac{dx}{x} = \frac{2z\,dz}{1-z^2} \qquad \lg x = \lg(1-z^2)$$

$$\frac{A}{x} = \left[1 - \left(\frac{y-a}{x}\right)^2\right]$$

$$A x = \left[x^2 - y^2 + 2ay - a^2\right]$$

$$x^2 - (y-a)^2 = Ax$$

$$\cancel{x\,dx = (y-a)\,dy}$$

$$(2x - A)\,dx = 2(y-a)\,dy \qquad \frac{dx}{dy} = \frac{(y-a)2x}{x^2 + \cancel{x^2 + y^2 - 2ay + a^2}}$$

$$(y-a)^2 = x^2 - Ax \qquad A = f_c(\varphi)$$

$$R = 2\sqrt{x^2-Ax}\left[2 + x\left[-Ax - 3x^2(x^2-Ax)\right]\right]$$

$$= 2\sqrt{x^2-Ax}\left[2 - Ax^2 - 3x^5 + 3Ax^4\right]$$

$$\frac{dx}{2x\sqrt{x^2-Ax}} = \frac{d\varphi}{2\sqrt{}\left[2 - \right]}$$

$$d\varphi = dx\left[\frac{2}{x} - Ax - 3x^4 + 3Ax^3\right] \text{ jixd. } A = \text{const}$$

$$\varphi = \log x - \frac{Ax^2}{2} - \frac{3}{5}x^5 + \frac{3}{4}Ax^4 + B = f_c(y)$$

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = 0!$$

$$u = \frac{1}{x} + 3x^2(y-a)^2$$

$$\text{fruit : } -\frac{1}{x^2} + \frac{\partial^2 B}{\partial y^2} = 0$$

$$v = \frac{y}{x^2} - 2x(y-a)^3$$

$$B = \frac{y^2}{2x^2}$$

$$\frac{v}{u} = \frac{\frac{y}{x^2} - 2x(y-a)^3}{\frac{1}{x} + 3x^2(y-a)^2} = \frac{y - 2x^3(y-a)^3}{x + 3x^4(y-a)^2}$$

$$\sqrt{u^2+v^2} =$$

Angenommen u und v werden an der Kreisoberfläche $=0$ von ~~..~~ algebraischer Ordnung

$u = (a-r)^n f(x,y)$ gerade in Bezug auf y, ~~ungerade~~ in x.

$v = (a-r)^m F(x,y)$ F ungerade in y, (ungerade) — x!

Für $x, y = \infty$: $u = c$ somit $f(x,y) = \frac{c}{r^n}$ ($x, y = \infty$)

$v = 0$ $F(x,y) = \frac{1}{r^{m+k}}$

$$\xi = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}$$

$$\xi = -n(a-r)^{n-1}\frac{y}{r}f(x,y) + (a-r)^n\frac{\partial f}{\partial y} - $$
$$+ m(a-r)^{m-1}\frac{x}{r}F(x,y) - (a-r)^m\frac{\partial F}{\partial x}$$

z.B. Probe mit: $n = m = 1$

$$\left.\begin{aligned}\xi = &-\frac{y}{r}f + (a-r)\frac{\partial f}{\partial y}\\ &+\frac{x}{r}F - (a-r)\frac{\partial F}{\partial x}\end{aligned}\right\} = 0 \text{ für } y = 0$$

Denn $\xi \gtrless 0$ an der Oberfläche

$$\frac{\partial \xi}{\partial x} = \frac{xy}{r^3}f - \frac{y}{r}\frac{\partial f}{\partial x} - \frac{x}{r}\frac{\partial f}{\partial y} + (a-r)\frac{\partial^2 f}{\partial x \partial y} +$$
$$+ \underbrace{\frac{1}{r}F - \frac{x^2}{r^3}F}_{\frac{y^2}{r^3}F} + 2\frac{x}{r}\frac{\partial F}{\partial x} - (a-r)\frac{\partial^2 F}{\partial x^2}$$

$$\frac{\partial^2 \xi}{\partial x^2} = \frac{y}{r^3}f - \frac{3x^2y}{r^5}f + \frac{xy}{r^3}\frac{\partial f}{\partial x} + \frac{xy}{r^3}\frac{\partial f}{\partial x} - \frac{y}{r}\frac{\partial^2 f}{\partial x^2} - \frac{1}{r}\frac{\partial f}{\partial y} + \frac{x^2}{r^3}\frac{\partial f}{\partial y} -$$
$$-\frac{x}{r}\frac{\partial^2 f}{\partial x \partial y} - \frac{x}{r}\frac{\partial^2 f}{\partial x \partial y} + (a-r)\frac{\partial^3 f}{\partial x^2 \partial y} - \frac{3y^2x}{r^5}F + \frac{y^2}{r^3}\frac{\partial F}{\partial x} + \frac{2}{r}\frac{\partial F}{\partial x} -$$
$$-\frac{2x^2}{r^3}\frac{\partial F}{\partial x} + \frac{2x}{r}\frac{\partial^2 F}{\partial x^2} + \frac{x}{r}\frac{\partial^2 F}{\partial x^2} - (a-r)\frac{\partial^3 F}{\partial x^3}$$

$$= \frac{(y^2-2x^y)}{r^5} f + 2\frac{xy}{r^3}\frac{\partial f}{\partial x} - \frac{y}{r}\frac{\partial^2 f}{\partial x^2} - \frac{y^2}{r^3}\frac{\partial f}{\partial y} - 2\frac{x}{r}\frac{\partial^2 f}{\partial x\partial y} + (a-r)\frac{\partial^3 f}{\partial x^2\partial y} -$$

$$- 3\frac{y^2 x}{r^5} F + \frac{(\cancel{x}x^2+2y^2)}{r^3}\frac{\partial F}{\partial x} + \frac{3x}{r}\frac{\partial^2 F}{\partial x^2} - (a-r)\frac{\partial^3 F}{\partial x^3}$$

$$\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} = -\frac{f}{r^3} + \frac{2xy}{r^3}\left(\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\right) - \frac{y}{r}\frac{\partial^2 f}{\partial x^2} - \frac{x}{r}\frac{\partial^2 f}{\partial y^2} - \frac{y^2}{r^3}\frac{\partial f}{\partial y} - \frac{x^2}{r^3}\frac{\partial f}{\partial x} -$$

$$- \frac{2(x+y)}{r}\frac{\partial^2 f}{\partial x\partial y} + (a-r)\left(\frac{\partial^3 f}{\partial x^2\partial y} + \frac{\partial^3 f}{\partial x\partial y^2}\right) - \frac{3(y+x)yx}{r^5} F +$$

$$+ \frac{2y^2-x^2}{r^3}\frac{\partial F}{\partial x} + \frac{2x^2-y^2}{r^3}\frac{\partial F}{\partial y} + \frac{3x}{r}\frac{\partial^2 F}{\partial x^2} + \frac{3y}{r}\frac{\partial^2 F}{\partial y^2} - (a-r)\left(\frac{\partial^3 F}{\partial x^3} + \frac{\partial^3 F}{\partial y^3}\right)$$

$$u = (a-r) f \qquad v = (a-r) F$$

$$= \frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y} \qquad = \frac{\partial \varphi}{\partial y} - \frac{\partial \psi}{\partial x}$$

$$\left.(a-r)\left(\frac{\partial f}{\partial x} + \frac{\partial F}{\partial y}\right) = \frac{xf + yF}{r}\right\}$$ Incompressibilitis

$\varphi =$ real part of $\Phi(x+iy)$

General problem in two dimensions

$$\frac{\partial\varphi}{\partial x} = \underbrace{\frac{\mu\left(\frac{\partial^4\psi}{\partial x^4} + 2\frac{\partial^4\psi}{\partial x^2\partial y^2} + \frac{\partial^4\psi}{\partial y^4}\right) - \frac{\partial\psi}{\partial y}\left(\frac{\partial^3\psi}{\partial x^3} + \frac{\partial^3\psi}{\partial x\partial y^2}\right) + \frac{\partial\psi}{\partial x}\left(\frac{\partial^3\psi}{\partial x^2\partial y} + \frac{\partial^3\psi}{\partial y^3}\right)}{\frac{\partial^3\psi}{\partial x^3} + \frac{\partial^3\psi}{\partial x\partial y^2}}}_{\Phi_1}$$

$$- \frac{\partial\psi}{\partial y}\underbrace{\frac{\frac{\partial^3\psi}{\partial x^2\partial y} + \frac{\partial^3\psi}{\partial y^3}}{\frac{\partial^3\psi}{\partial x^3} + \frac{\partial^3\psi}{\partial x\partial y^2}}}_{\Phi_2}$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{\partial\Phi_1}{\partial x} - \frac{\partial^2\psi}{\partial x\partial y}\Phi_2 - \frac{\partial\psi}{\partial y}\frac{\partial\Phi_2}{\partial x}$$

$$u = \frac{\mu\left(\frac{\partial^2\zeta}{\partial x^2} + \frac{\partial^2\zeta}{\partial y^2}\right)}{\frac{\partial\zeta}{\partial x}} - v\frac{\frac{\partial\zeta}{\partial y}}{\frac{\partial\zeta}{\partial x}}$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\Phi_1 - v\frac{\partial\Phi_2}{\partial x} - \Phi_2\overbrace{\frac{\partial v}{\partial x}}^{=-\zeta+\frac{\partial u}{\partial y}}$$

$$\frac{\partial u}{\partial y} = \frac{\partial\Phi_1}{\partial y} - v\frac{\partial\Phi_2}{\partial y} - \Phi_2\frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial x} = \frac{\partial\Phi_1}{\partial x} - v\frac{\partial\Phi_2}{\partial x} + \Phi_2\zeta - \Phi_2\frac{\partial\Phi_1}{\partial y} + v\Phi_2\frac{\partial\Phi_2}{\partial y} - \Phi_2^2\underbrace{\frac{\partial v}{\partial y}}_{=-\frac{\partial u}{\partial x}}$$

$$\frac{\partial u}{\partial x}\left[1 - \Phi_2^2\right] = \frac{\partial\Phi_1}{\partial x} + \Phi_2\zeta - \Phi_2\frac{\partial\Phi_1}{\partial y} + v\left[\Phi_2\frac{\partial\Phi_2}{\partial y} - \frac{\partial\Phi_2}{\partial x}\right]$$

If $v=0$ $\quad \zeta = \frac{\partial u}{\partial y} \quad \frac{\partial u}{\partial x} = 0$

$$\cancel{u \frac{\partial^2 u}{\partial x \partial y}} \quad \cancel{\frac{\partial^3 u}{\partial x^2 \partial y}} + \frac{\partial^3 u}{\partial y^3} = 0$$

$$u = Ay^2 + By + C$$

With neglection ~~second~~ of inertia-terms:

$$\nabla p - \mu \nabla^2 \mathfrak{v} = 0$$

$$\left.\begin{aligned} \frac{\partial p}{\partial x} &= \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \\ \frac{\partial p}{\partial y} &= \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \end{aligned}\right\} \quad \underbrace{\frac{\partial^3 u}{\partial x^2 \partial y} + \frac{\partial^3 u}{\partial y^3} - \frac{\partial^3 v}{\partial x^3} - \frac{\partial^3 v}{\partial y^2 \partial x}}_{= \frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial^2 \zeta}{\partial y^2}} = 0$$

$$\zeta = \text{Re } \text{P. } \Phi(x+iy) = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}$$

willkürlich $\qquad$ ebenso:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

$$\cancel{u = \frac{\partial \psi}{\partial y} \quad v = -\frac{\partial \psi}{\partial x}}$$

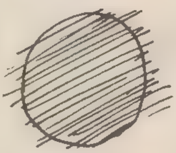
$$\frac{\partial \psi}{\partial x} = \int \frac{f\, x\, dx\, dy}{r^2}$$

$$(a-r) f = \frac{\partial F}{\partial y} \quad \cancel{\frac{\partial^2 \psi}{\partial x \partial y}} = (a-r)\frac{\partial f}{\partial x} + \frac{x}{r} f = (a-r)\frac{\partial F}{\partial y} - \frac{y}{r}F$$

$$F = \frac{\frac{\partial \psi}{\partial y}}{a-r}$$

$$\int \log r \cdot f(x,y)\, dx\, dy$$

$$\cancel{\frac{\partial u}{\partial x}} + \frac{\partial u}{\partial y}$$



$$V = \frac{1}{2}\int \log(x^2+y^2) \cdot f(x,y)\, dx\, dy$$

$$\frac{\partial V}{\partial x} = \frac{1}{2}\int \frac{x}{r^2} f + \log r^2 \frac{\partial f}{\partial x}$$

$$\frac{\partial^2 V}{\partial x^2} = \frac{1}{2}\int \frac{f}{r^2} - \frac{2x^2 f}{r^3} + \frac{2x}{r^2}\frac{\partial f}{\partial x} + \log r^2 \frac{\partial^2 f}{\partial x^2}$$

$$\int \frac{dx}{\sqrt{x^2+y^2+z^2}} = \int \frac{dz}{\sqrt{1+\left(\frac{z}{\sqrt{\ }}\right)^2}} = \log\left(z \pm \sqrt{x^2+y^2}\right)$$

$$= \log(z+r)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(p r) = \frac{\partial p}{\partial r} + \frac{p}{r} = \frac{\rho g a^2}{r^2}$$

$$p v^k = c$$

$$p = c\rho^k$$

$$c k \rho^{k-1}\frac{\partial \rho}{\partial r} + \frac{c\rho^k}{r} = \rho g \frac{a^2}{r^2}$$

$$c k \rho^{k-2}\frac{d\rho}{dr} + \frac{c\rho^{k-1}}{r} = g\frac{a^2}{r^2}$$

$$\cancel{c k \rho^{k-2} d\rho = \left(g\frac{a^2}{r^2} - \frac{c\rho^{k-1}}{r}\right)dr = \frac{dr}{r^2}\left(g a^2 - c r \rho^{k-1}\right)}$$

$$\rho^{k-1} = z$$

$$\frac{k}{k-1}\frac{dz}{dr} + \frac{z}{r} = \frac{g a^2}{c r^2}$$

$$\cancel{z = \frac{A}{r} + \log r\,(Br + C) + Cr^2 \log r} + D$$

$$-\frac{k}{k-1}A + A = \frac{g a^2}{c}$$

$$A = \frac{g a^2}{c}\,\frac{1}{1-\frac{k}{k-1}} = 1-k$$

$$\rho^{k-1} = \frac{g a^2 (1-k)}{c}\,\frac{1}{r}$$

$$p =$$

$$+2Cr\log r + \frac{C}{r}$$

$$\frac{k}{k-1}\left(-\frac{A}{r^2} + \frac{B}{r}\log r + B + \frac{C}{r}\right) + \frac{A}{r^2} + B\log r + Cr\log r + \frac{D}{r} = \frac{g a^2}{c r^2}$$

$$z = u v \qquad \frac{du}{dr}v + u\frac{dv}{dr}$$

$$\underbrace{\left(\frac{k}{k-1}\frac{dv}{dr}+\frac{v}{r}\right)}_{=0}u+\frac{k}{k-1}v\frac{du}{dr}=\frac{ga^2}{cr^2}$$

$$\frac{k}{k-1}\frac{dv}{v}+\frac{dr}{r}=0 \qquad du=\frac{k-1}{k}\frac{ga^2}{c}b^{-\frac{1}{k}}\underbrace{\frac{r^{\frac{k-1}{k}}}{r^2}}_{=r^{-\frac{1}{k}-1}}dr$$

$$v^k r^{k-1}=b$$

$$v=b^{\frac{1}{k}}r^{\frac{1-k}{k}} \qquad u=\frac{\frac{k-1}{k}\frac{ga^2}{c}b^{-\frac{1}{k}}r^{-\frac{1}{k}}}{-\frac{1}{k}}+C$$

$$z=(1-k)\frac{ga^2}{c}\frac{1}{r}+Cr^{\frac{1-k}{k}}=\rho^{k-1}$$

$$\cancel{\left(-\frac{k-1}{k}\frac{ga^2}{c}\frac{1}{r^2}+C\frac{1-k}{k}r^{\frac{1}{k}-2}\right)\frac{k}{k-1}+\frac{k-1}{k}\frac{ga^2}{c}\frac{1}{r}+Cr^{\frac{1}{k}-2}=\frac{ga^2}{cr^2}}$$

$$\cancel{\frac{k}{k-1}\left(-(1-k)\frac{ga^2}{c}\frac{1}{r^2}+\frac{1-k}{k}Cr^{\frac{1}{k}-2}\right)+(1-k)\frac{ga^2}{c}\frac{1}{r^2}+Cr^{\frac{1}{k}-2}=\frac{ga^2}{cr^2}}$$

$$\cancel{r=a \quad \rho=\rho_0 \quad \rho_0^{k-1}=(1-k)\frac{ga}{c}+Ca^{\frac{1-k}{k}}} \qquad C=\left[\rho_0^{k-1}+(k-1)\frac{ga}{c}\right]a^{\frac{k-1}{k}}$$

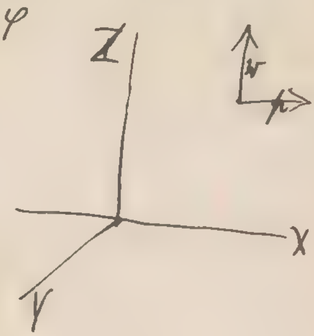
$$\rho^{k-1}=(1-k)\frac{ga^2}{c}\frac{1}{r}+\left[\rho_0^{k-1}+(k-1)\frac{ga}{c}\right]\left(\frac{r}{a}\right)^{\frac{1-k}{k}}$$

$$=\rho_0^{k-1}\left(\frac{r}{a}\right)^{\frac{1}{k}-1}+(1-k)\frac{ga}{c}\left[\frac{a}{r}-\left(\frac{r}{a}\right)^{\frac{1}{k}-1}\right]$$

$$=\rho_0^{k-1}\left(\frac{r}{a}\right)^{\frac{1-k}{k}}+(1-k)\frac{ga^2}{cr}\left[1-\left(\frac{r}{a}\right)^{\frac{1}{k}}\right]$$

$$RT=pv=\frac{p}{\rho}=c\rho^{k-1}=\uparrow \qquad \mu\sim\sqrt{T}\sim\rho^{\frac{k-1}{2}}$$

Transformation of ~~general~~ special equations (for rotational symmetry) in polar coordinates $r\,\cancel{\varphi}\,\theta\,\varphi$



z = axis of symmetry $= r\cos\theta$

$x = r\sin\theta\cos\varphi$

$y = r\sin\theta\sin\varphi$

~~Note:~~ $\cancel{\frac{\partial u}{\partial\varphi} = \frac{\partial v}{\partial\varphi} = \frac{\partial w}{\partial\varphi} = 0}$

Equation of Continuity: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

$$\frac{\partial u}{\partial r}\sin\theta\cos\varphi + \frac{\partial u}{\partial\theta}\frac{\cos\theta\cos\varphi}{r} - \frac{\partial u}{\partial\varphi}\frac{\sin\varphi}{r\sin\theta} = \frac{\partial u}{\partial x} \qquad u = 6\sin\theta\cos\varphi$$

$$\frac{\partial u}{\partial r}\sin\theta\sin\varphi + \frac{\partial u}{\partial\theta}\frac{\cos\theta\sin\varphi}{r} + \frac{\partial u}{\partial\varphi}\frac{\cos\varphi}{r\sin\theta} = \frac{\partial u}{\partial y} \qquad v = 6\sin\theta\sin\varphi$$

$$\frac{\partial u}{\partial r}\cos\theta - \frac{\partial u}{\partial\theta}\frac{\sin\theta}{r} = \frac{\partial u}{\partial z} \qquad w = 6\cos\theta$$

New variable: $p = \sqrt{u^2+v^2}$ $\qquad \frac{\partial p}{\partial\varphi} = 0$

$$\frac{\partial w}{\partial\varphi} = 0$$

$u = p\cos\varphi$

$v = p\sin\varphi$

$$\frac{\partial u}{\partial x} = \frac{\partial p}{\partial r}\sin\theta\cos^2\varphi + \frac{\partial p}{\partial\theta}\frac{\cos\theta\cos^2\varphi}{r} + p\frac{\sin^2\varphi}{r\sin\theta}$$

$$\frac{\partial u}{\partial y} = \frac{\partial p}{\partial r}\sin\theta\sin\varphi\cos\varphi + \frac{\partial p}{\partial\theta}\frac{\cos\theta\sin\varphi\cos\varphi}{r} - p\frac{\sin\varphi\cos\varphi}{r\sin\theta}$$

$$\frac{\partial u}{\partial z} = \frac{\partial p}{\partial r}\cos\theta\cos\varphi - \frac{\partial p}{\partial\theta}\frac{\sin\theta\cos\varphi}{r}$$

$$\frac{\partial v}{\partial x} = \frac{\partial p}{\partial r}\sin^2\theta\cos\varphi\sin\varphi + \frac{\partial p}{\partial \theta}\frac{\cos\theta\sin\varphi\cos\varphi}{r} - p\,\frac{\sin\varphi\cos\varphi}{r\sin\theta}$$

$$\frac{\partial v}{\partial y} = \frac{\partial p}{\partial r}\sin\theta\sin^2\varphi + \frac{\partial p}{\partial \theta}\frac{\cos\theta\sin^2\varphi}{r} + p\,\frac{\cos^2\varphi}{r\sin\theta}$$

$$\frac{\partial v}{\partial z} = \frac{\partial p}{\partial r}\cos\theta\sin\varphi - \frac{\partial p}{\partial \theta}\frac{\sin\theta\sin\varphi}{r}$$

$$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial r}\sin\theta\cos\varphi + \frac{\partial w}{\partial \theta}\frac{\cos\theta\cos\varphi}{r}$$

$$\frac{\partial w}{\partial y} = \frac{\partial w}{\partial r}\sin\theta\sin\varphi + \frac{\partial w}{\partial \theta}\frac{\cos\theta\sin\varphi}{r}$$

$$\frac{\partial w}{\partial z} = \frac{\partial w}{\partial r}\cos\theta - \frac{\partial w}{\partial \theta}\frac{\sin\theta}{r}$$

$$\frac{\partial u}{\partial x} = \frac{\partial \sigma}{\partial r}\sin^2\theta\cos^2\varphi + \frac{\sigma}{r}\cos^2\theta\cos^2\varphi + \frac{\sigma}{r}\sin^2\varphi$$

$$\frac{\partial w}{\partial y} = \frac{\partial \sigma}{\partial r}\sin^2\theta\sin^2\varphi + \frac{\sigma}{r}\cos^2\theta\sin^2\varphi + \frac{\sigma}{r}\cos^2\varphi$$

$$\frac{\partial w}{\partial z} = \frac{\partial \sigma}{\partial r}\cos^2\theta + \frac{\sigma}{r}\sin^2\theta$$

$$\frac{\partial u}{\partial y} = \frac{\partial \sigma}{\partial r}\sin^2\theta\sin\varphi\cos\varphi + \frac{\sigma}{r}(\cos^2\theta\sin\varphi\cos\varphi - \sin\varphi\cos\varphi)$$

$$= \sin^2\theta\sin\varphi\cos\varphi\left(\frac{\partial \sigma}{\partial r} - \frac{\sigma}{r}\right)$$

$$\frac{\partial u}{\partial x} = \left[\frac{\partial \sigma}{\partial r} - \frac{\sigma}{r}\right]\sin^2\theta\cos^2\varphi + \frac{\sigma}{r} \qquad \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} = \left[\frac{\partial \sigma}{\partial r} - \frac{\sigma}{r}\right]\sin^2\theta\sin\varphi\cos\varphi$$

$$\frac{\partial v}{\partial y} = \left[\frac{\partial \sigma}{\partial r} - \frac{\sigma}{r}\right]\sin^2\theta\sin^2\varphi + \frac{\sigma}{r} \qquad \frac{\partial u}{\partial z} = \frac{\partial w}{\partial x} = \left[\frac{\partial \sigma}{\partial r} - \frac{\sigma}{r}\right]\sin\theta\cos\theta\cos\varphi$$

$$\frac{\partial w}{\partial z} = \left[\frac{\partial \sigma}{\partial r} - \frac{\sigma}{r}\right]\cos^2\theta + \frac{\sigma}{r} \qquad \frac{\partial v}{\partial z} = \frac{\partial w}{\partial y} = \left[\frac{\partial \sigma}{\partial r} - \frac{\sigma}{r}\right]\sin\theta\cos\theta\sin\varphi$$

Reduction of general equations to one differential equation

$$6 = \nabla U + \text{curl}\,\varphi$$

$$\tfrac{1}{2}\nabla 6^2 + V 6\,\text{curl}\,6 = \nabla P - \mu\,\text{curl}^2 6$$

$$\text{curl}\, V 6\,\text{curl}\,6 = -\mu\,\text{curl}^3 6$$

$$= (\text{curl}\,6\,\nabla)6 - (6\,\nabla)\,\text{curl}\,6$$

$$(b\nabla)a = V(\text{curl}\,a.\,b) - \nabla_a\, ab$$

$$\text{curl}\, V 6\,\text{curl}\,6 = V(\text{curl}^2 6.\,6) - \cancel{V(\text{curl}\,6)^2} - \nabla_{\text{curl}\,6}\, 6\,\text{curl}\,6 + \nabla_6\, 6\,\text{curl}\,6$$

$$\cancel{\nabla(6\nabla) = \nabla\left(i\tfrac{\partial}{\partial x} + j\tfrac{\partial}{\partial y} + k\tfrac{\partial}{\partial z}\right)(\ldots)} = 6^2 \nabla \frac{\text{curl}\,6}{6} =$$

$$\nabla \frac{6\,\text{curl}\,6}{6^2} = 6\,\text{curl}\,6\;\nabla\frac{1}{6^2} + \frac{1}{6^2}\nabla 6\,\text{curl}\,6$$

$$= \cancel{2\,6\,\text{curl}\,6\,\nabla 6}$$

?

$$\cancel{\nabla\tfrac{1}{6^2}} \quad = \frac{6\,\text{curl}\,6}{6^2}\,\delta\,\frac{\nabla 6}{6}$$

$$\frac{\partial p}{\partial r} = -\rho \frac{g a^2}{r^2} \qquad \Big\| \; p v^k = c \qquad p = c \rho^k \qquad c = p_0 v_0^k = p_0 \rho_0^{-k}$$

$$k c \rho^{k-1} \frac{\partial \rho}{\partial r} = -\frac{\rho}{r^2} g a^2$$

$$k c \rho^{k-2} d\rho = -\frac{dr}{r^2} g a^2 \qquad\qquad RT = p v = c \rho^{k-1}$$

$$k c \frac{\rho^{k-1}}{k-1} = \frac{g a^2}{r} + \text{const}$$

$$k c \frac{\rho_0^{k-1}}{k-1} = g a \; \text{const}$$

$$\frac{k c}{k-1}\left(\rho^{k-1} - \rho_0^{k-1}\right) = g a \left(\frac{a}{r} - 1\right) \qquad\qquad R \frac{dT}{dr} = -\frac{k-1}{k} \frac{g a^2}{r^2}$$

$$R(T - T_0) = \frac{k-1}{k} g a \left(\frac{a}{r} - 1\right)$$

---

für $r = \infty$: $$\rho^{k-1} = \rho_0^{k-1} - \frac{k-1}{kc} g a$$

$$= \rho_0^{k-1}\Big[1 - \underbrace{\frac{k-1}{k p_0} \rho_0 g a}_{<0}\Big] \qquad \text{nicht möglich!}$$

$\rho_0 = 0{\cdot}0013$

$p_0 = 9{\cdot}8 \times 13{\cdot}6 \times 0{\cdot}76$

$g = 9{\cdot}8$

$a = 6366200$

$$\frac{\cancel{0{\cdot}0013} \,.\, \cancel{9{\cdot}8} \,.\, 636620\cancel{0}}{\cancel{9{\cdot}8} \,.\, 13{\cdot}6 \,.\, \cancel{0{\cdot}76}}$$

$r = a(1+\delta)$

Für $p = 0$: $$\frac{k c \rho_0^{k-1}}{k-1} = g a \left(1 - \frac{a}{r}\right)$$

$$\frac{1}{ga} \frac{k}{k-1} \frac{p_0}{\rho_0} = \frac{\cancel{9{\cdot}8} \,.\, 13{\cdot}6 \,.\, 0{\cdot}76}{0{\cdot}0013 \,.\, \cancel{9{\cdot}8} \,.\, 63662\cancel{00}} \qquad \frac{1{\cdot}4}{0{\cdot}4}$$

$$\delta = \frac{76 \,.\, 7}{\cancel{126000}\; 18000} = \frac{1}{210}$$

Falls dagegen die Abnahme der Schwere wegen Centrifugalkraft dieselbe wäre (mit wachsender Höhe) wie am Äquator

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = -\frac{ga^2}{r^2} + \omega(r-a) = kc\rho^{k-2}\frac{\partial \rho}{\partial r}$$

$$\frac{kc}{k-1}\left(\rho^{k-1}-\rho_0^{k-1}\right) = \cancel{\ldots}^2 + \frac{\omega}{2}(r-a)^2$$

$$= ga\left(\frac{a}{r}-1\right) \qquad = (a-r)\left[\frac{ga}{r} - \frac{\omega}{2}(a-r)\right]$$

$$\omega = \frac{\pi}{2}\,\frac{1}{365.\,24.\,60.\,60} \qquad \omega a = \frac{\pi}{2}\,\frac{\cancel{6366\,000}}{\cancel{365}.\,24.\,\cancel{36}}\;\frac{\cancel{10610}}{\cancel{6.73}.\,3}\;53$$

$$= \frac{\cancel{3\ldots}.\,\cancel{53}}{\cancel{22}.\,\cancel{24}} = \frac{27}{88} = 0{\cdot}40$$

Macht sehr wenig aus.

Obiges nicht ganz genau; eigentlich:

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = -\frac{ga^2}{r^2} + \omega r = kc\rho^{k-2}\frac{\partial\rho}{\partial r}$$

$$\frac{kc}{k-1}\left(\rho^{k-1}-\rho_0^{k-1}\right) = ga\left(\frac{a}{r}-1\right) + \frac{\omega(r^2-a^2)}{2} \quad - - \quad -$$

Damit ρ für $x=\infty$ $\rho=0$ wird müsste sein:

$$\frac{k-1}{k}\, a g \frac{\rho_0}{p_0} = 1 \qquad \frac{\rho_0}{p_0} = R T_0 = \frac{0.0013}{100} = 0.000013 = 1.3 \cdot 10^{-5}$$

Eigentemp

$$\frac{k-1}{k}\, a g = \frac{0.4}{\cancel{1.4}}\; 6{,}366{,}000 \cdot \cancel{981}^{7} = 17{,}800{,}000$$

4456

1782

$$= 18 \cdot 10^{6}$$

$$\frac{1}{18}\, 10^{-6} = 5.5 \cdot 10^{-8}$$

Es müsste also die Temperatur $\frac{1}{\sqrt{200}}$ der jetzigen betragen

---

Dissipation of energy pro unit of time:

$$\Phi = -\frac{2}{3}\mu\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2 + \mu\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \dots\right\}$$

$$\frac{\partial \rho u}{\partial x} + \dots = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \frac{1}{\rho}\left[u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z}\right]$$

$$\operatorname{div}(\rho\sigma) = \frac{\partial(\rho\sigma)}{\partial r} + 2\frac{\rho\sigma}{r} = 0 = \rho\underbrace{\left[\frac{d\sigma}{dr} + \frac{2\sigma}{r}\right]}_{\operatorname{div}\sigma} + \sigma\frac{d\rho}{dr} = 0$$

$$\frac{d\log\sigma}{dr} + \frac{2}{r} + \frac{d\log\rho}{dr} = 0 \qquad \operatorname{div}\sigma = -\frac{\sigma}{\rho}\frac{d\rho}{dr}$$

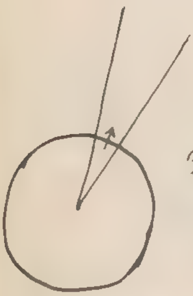
$$\sigma\rho\, r^2 = \text{const}$$

Falls man annimmt, dass der Unterschied zwischen aerostatischem und aerodynamischem Druck zu vernachlässigen ist; dagegen der Temperatureinfluss der Bewegung zu berücksichtigen:

$$\left.\begin{aligned}\frac{1}{\rho}\frac{\partial p}{\partial r} &= g\frac{a^2}{r^2}\\ \frac{p}{\rho} &= R\theta\end{aligned}\right\}$$

Betreffs θ Annahme:
Es bewege sich von der Erdoberfläche aus ein Luftstrom mit der Geschw. c aufwärts; welche Temp.verteilung entsteht?
σ = Radialgeschw. $\sigma_\alpha = c$

Arbeit = Ausdehnungsarbeit ~~und~~ Reibungsarbeit und Schwere



Falls keine Reibung wäre, so wäre die andere gleich

$$\rho\frac{g a^2 dr}{r^2} = p\frac{\partial v}{\partial r} + c\frac{\partial\theta}{\partial r} + \mu\frac{\partial}{\partial r}$$

$$\frac{d\rho}{dt} = \frac{\partial\rho}{\partial r}\sigma$$

$$p = R\rho\theta$$

$$\frac{R}{\rho}\left[\frac{\partial\theta}{\partial r}\rho + \theta\frac{\partial\rho}{\partial r}\right] = g\frac{a^2}{r^2} \qquad \mathrm{I}).$$

$$\frac{\partial\theta}{\partial r} = \frac{AR\theta}{c\rho}\frac{\partial\rho}{\partial r} - \frac{\frac{4}{3}A\mu\left(\frac{b}{\rho^2 r^2}\right)^2\left[\frac{\partial\rho}{\partial r}+\frac{3\rho}{r}\right]^2 r^2}{c\, b}$$

$$= \frac{AR}{c}\frac{\theta}{\rho}\frac{\partial\rho}{\partial r} - \frac{4}{3}\frac{A\mu}{c}\frac{b}{r^2\rho^4}\left[\frac{\partial\rho}{\partial r} + \frac{3\rho}{r}\right]^2 \qquad \mathrm{II}).$$

Work done by internal friction:

$$\Phi = -\frac{2}{3}\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)^2 + \mu\left[2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right] + \left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)^2 + \cdots\right]$$

$$2\left[\frac{\partial \sigma}{\partial r}-\frac{\sigma}{r}\right]^2\left[\sin^4\theta(\cos^4\varphi+\sin^4\varphi)+\cos^4\theta\right] + 6\frac{\sigma^2}{r^2} + 4\frac{\sigma}{r}\left[\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right]$$

$$+4\left[\frac{\partial \sigma}{\partial r}-\frac{\sigma}{r}\right]^2\left[\sin^2\theta\cos^2\theta + \sin^4\theta\sin^2\varphi\cos^2\varphi\right] - \frac{2}{3}\left[\frac{\partial\sigma}{\partial r}+2\frac{\sigma}{r}\right]^2$$

$$= 2\left[\frac{\partial \sigma}{\partial r}-\frac{\sigma}{r}\right]^2\left[\underbrace{\sin^4\theta\left[\cos^4\varphi+2\sin^2\varphi\cos^2\varphi+\sin^4\varphi\right]+2\sin^2\theta\cos^2\theta+\cos^4\theta}_{=1}\right] + \cdots$$

$$= 2\left[\frac{\partial \sigma}{\partial r}-\frac{\sigma}{r}\right]^2 + 6\frac{\sigma^2}{r^2} + 4\frac{\sigma}{r}\left[\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right] - \frac{2}{3}\left[\frac{\partial\sigma}{\partial r}+2\frac{\sigma}{r}\right]^2$$

$$= \left(\frac{\partial\sigma}{\partial r}\right)^2\left[2-\frac{2}{3}\right] + \frac{\partial\sigma}{\partial r}\cdot\frac{\sigma}{r}\left[-4+4-\frac{8}{3}\right] + \frac{\sigma^2}{r^2}\left[2+6-4-\frac{8}{3}\right]$$

$$= \left(\frac{\partial\sigma}{\partial r}\right)^2\frac{4}{3} - \frac{\partial\sigma}{\partial r}\cdot\frac{\sigma}{r}\,\frac{8}{3} + \frac{\sigma^2}{r^2}\,\frac{4}{3} = \frac{4}{3}\left[\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right]^2$$

Equation of continuity: $\rho\sigma\cdot r^2 = \text{const} = \rho_0\sigma_0 a^2 = b$

$$\sigma = \frac{b}{\rho r^2} \qquad \frac{\partial\sigma}{\partial r} = -\frac{b}{\rho^2 r^2}\frac{\partial\rho}{\partial r} - 2\frac{b}{\rho r^3}$$

$$\Phi = -\frac{4}{3}\mu\left[\frac{b}{\rho^2 r^2}\frac{\partial\rho}{\partial r} + \frac{2b}{\rho r^3} + \frac{b}{\rho r^3}\right]^2 = -\frac{4}{3}\left(\frac{b}{\rho^2 r^2}\right)^2\mu\left[\frac{\partial\rho}{\partial r}+3\frac{\rho}{r}\right]^2$$

$$\frac{\partial\rho}{\partial r} + \frac{3\rho}{r} = \frac{1}{r^3}\frac{\partial}{\partial r}\left(\rho r^3\right)$$

Transformation of Equations of Motion for spherical symmetry:

Continuity: $\rho\sigma r^2 = \text{const.}$

$$\underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}} = X - \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{3\rho}\frac{\partial}{\partial x}\operatorname{div}\sigma + \frac{\mu}{\rho}\nabla^2 u$$

$$\sigma\left(\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right)\sin^3\theta\cos^3\varphi + \sin^3\theta\cos\varphi\sin^2\varphi + \sin\theta\cos^2\theta\cos\varphi$$

$$+\frac{\sigma^2}{r}\sin\theta\cos\varphi = \sigma\left(\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right)\sin\theta\cos\varphi + \frac{\sigma^2}{r} \sim \text{-}$$

$$= \sigma\frac{\partial\sigma}{\partial r}\sin\theta\cos\varphi$$

$$\operatorname{div}\sigma = -\frac{\sigma}{\rho}\frac{\partial\rho}{\partial r} \qquad \frac{\partial\operatorname{div}\sigma}{\partial x} = \frac{\partial\operatorname{div}\sigma}{\partial r}\sin\theta\cos\varphi$$

$$= -\frac{\partial}{\partial r}\left(\frac{\sigma}{\rho}\frac{\partial\rho}{\partial r}\right)$$

$$\frac{\partial^2 u}{\partial x^2} = \sin\theta\cos\varphi\left[\left[\frac{\partial^2\sigma}{\partial r^2} - \frac{1}{r}\frac{\partial\sigma}{\partial r} + \frac{\sigma}{r^2}\right]\sin^2\theta\cos^2\varphi + \frac{\partial\sigma}{\partial r}\frac{1}{r} - \frac{\sigma}{r^2}\right]$$

$$\frac{\partial^2 u}{\partial y^2} = \sin\theta\sin\varphi\left[\frac{\partial^2\sigma}{\partial r^2} - \frac{1}{r}\frac{\partial\sigma}{\partial r} + \frac{\sigma}{r^2}\right]\sin\theta\sin\varphi\cos\varphi$$

$$\frac{\partial^2 u}{\partial z^2} = \cos\theta\left[\qquad\qquad\right]\sin\theta\cos\theta\cos\varphi$$

$$\nabla^2 u = \left[\frac{\partial^2\sigma}{\partial r^2} - \frac{1}{r}\frac{\partial\sigma}{\partial r} + \frac{\sigma}{r^2}\right]^{+\frac{1}{r}\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r^2}}\sin\theta\cos\varphi + \cancel{\frac{\sin\theta\cos\varphi}{r}2\sin\theta\cos\theta\cos^2\varphi\left[\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right]}$$

$$\cancel{+\frac{\sin\varphi}{r\sin\theta}2\cos^2\theta\sin\varphi\cos\varphi\Big]}$$

$$= \frac{\partial^2\sigma}{\partial r^2}\sin\theta\cos\varphi$$

$$+\frac{[\;]}{r}\cos\theta\sin\varphi\;\cancel{2\sin\theta\cos\theta\sin\varphi\cos\varphi + \frac{\cos\varphi}{\sin\theta}\sin^2\theta(\cos^2\varphi - \sin^2\varphi)} - \sin\theta(\cos^2\theta\cos\varphi - \sin^2\theta\cos\varphi)$$

Equation of Motion:

$$\sigma \frac{\partial \sigma}{\partial r} = R - \frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{\mu}{3\rho}\frac{\partial}{\partial r}\left(\frac{\sigma}{\rho}\frac{\partial \rho}{\partial r}\right) + \frac{\mu}{\rho}\left[\frac{\partial^2 \sigma}{\partial r^2} + \frac{2}{r}\frac{\partial \sigma}{\partial r}\right]$$

$$\rho \sigma r^2 = c$$

$$\sigma = \frac{c}{\rho r^2}$$

$$\frac{\partial \sigma}{\partial r} = -\frac{2c}{\rho r^3} - \frac{c}{\rho r^2}\frac{\partial \rho}{\partial r}$$

$$\frac{\partial}{\partial r}\left(\frac{\sigma}{\rho}\frac{\partial \rho}{\partial r}\right) = \frac{\partial \sigma}{\partial r}\frac{1}{\rho}\frac{\partial \rho}{\partial r} - \frac{\sigma}{\rho^2}\left(\frac{\partial \rho}{\partial r}\right)^2 + \frac{\sigma}{\rho}\frac{\partial^2 \rho}{\partial r^2} \ldots$$

$$\frac{\partial^2 \sigma}{\partial r^2} = +\frac{6c}{\rho r^4}$$

$$-\frac{2c^2}{\rho^2 r^5} - \frac{c^2}{\rho^2 r^4}\frac{\partial \rho}{\partial r} = R - \frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{\mu}{3\rho}\left(-\frac{2c}{\rho^2 r^3}\frac{\partial \rho}{\partial r} - \frac{c}{\rho^3 r^2}\left(\frac{\partial \rho}{\partial r}\right)^2 + \frac{c}{\rho^2 r^2}\frac{\partial^2 \rho}{\partial r^2}\right) + \frac{\mu}{\rho}\frac{6c}{\rho r^4} + \ldots$$

When $\mu = 0$: $\quad p = a \rho^k \qquad \frac{\partial p}{\partial r} = a k \rho^{k-1}\frac{\partial \rho}{\partial r}$

$$\frac{1}{2}\frac{\partial(\sigma^2)}{\partial r} = -ak\rho^{k-2}\frac{\partial \rho}{\partial r} = -\frac{ak}{k-1}\frac{\partial}{\partial r}(\rho^{k-1})$$

$$\frac{1}{2}\frac{c^2}{\rho^2 r^4} = \frac{-ak}{k-1}\rho^{k-1} + \text{const} \qquad \frac{c^2}{2}\left(\frac{1}{\rho^2 r^4} - \frac{1}{A^2 a^2}\right) = \frac{-ak}{k-1}\left(\rho^{k-1} - \rho_0^{k-1}\right)$$

$$= \left[\frac{\partial \sigma}{\partial r} - \frac{\sigma}{r}\right]\frac{1}{r}\Big[2(\underbrace{\sin\theta\cos^2\theta\cos^3\varphi + \sin\theta\cos^2\theta\sin^2\varphi\cos\varphi}_{\sin\theta\cos^2\theta\cos\varphi}) + \cos\varphi(\sin^3\theta - \sin\theta\cos^2\theta) + \frac{1}{\sin\theta}\left(\cos^2\theta\sin^2\varphi\cos\varphi + \sin^2\theta(\cos^3\varphi - \cos\varphi\sin^2\varphi)\right)\Big] =$$

$$\nabla^2 u = \frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 u}{\partial \varphi^2} + \frac{1}{r^2}\frac{\partial^2 \theta}{\partial \theta^2} + \frac{\cos\theta}{r^2 \sin\theta}\frac{\partial u}{\partial \theta}$$

$$u = \sigma \cos\varphi \sin\theta$$

$$\nabla^2 u = \cos\varphi \sin\theta \left[\frac{\partial^2 \sigma}{\partial r^2} + \frac{2}{r}\frac{\partial \sigma}{\partial r}\right] \underbrace{- \frac{\cos\varphi}{r^2 \sin^2\theta} \sin\theta\, \sigma - \frac{1}{r^2}\cos\varphi \sin\theta + \frac{\cos^2\theta}{r^2 \sin\theta}\cos\varphi}$$

$$\frac{\sigma}{r^2}\,\frac{\cos\varphi}{\sin\theta}\left[\cos^2\theta - 1 - \sin^2\theta\right] = -2\frac{\sigma}{r^2}\sin\theta\cos\varphi$$

$$\nabla^2 u = \cos\varphi \sin\theta \left[\frac{\partial^2 \sigma}{\partial r^2} + \frac{2}{r}\frac{\partial \sigma}{\partial r} - \frac{2\sigma}{r^2}\right]$$

$$\frac{\partial^2 \sigma}{\partial r^2} + 2\frac{\partial}{\partial r}\left(\frac{\sigma}{r}\right) = \frac{\partial}{\partial r}\underbrace{\left(\frac{\partial \sigma}{\partial r} + \frac{2\sigma}{r}\right)}_{= \operatorname{div}\sigma = -\frac{\sigma}{\rho}\frac{\partial \rho}{\partial r}}$$

Equation of Motion for spherical symmetry:

$$\sigma \frac{\partial \sigma}{\partial r} = R - \frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{\mu}{3\rho}\frac{\partial}{\partial r}\left(\frac{\sigma}{\rho}\frac{\partial \rho}{\partial r}\right) + \frac{\mu}{\rho}\underbrace{\left[\frac{\partial^2 \sigma}{\partial r^2} + \frac{2}{r}\frac{\partial \sigma}{\partial r} - \frac{2\sigma}{r^2}\right]}_{= -\frac{\partial}{\partial r}\left(\frac{\sigma}{\rho}\frac{\partial \rho}{\partial r}\right)}$$

$\rho \sigma r^2 = \text{const}$

$$\sigma \frac{\partial \sigma}{\partial r} = R - \frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{4\mu}{3\rho}\frac{\partial}{\partial r}\left(\frac{\sigma}{\rho}\frac{\partial \rho}{\partial r}\right)$$

$$\frac{\partial \theta}{\partial r} + \frac{\theta}{\rho}\frac{\partial \rho}{\partial r} = \frac{g a^2}{R r^2}$$

$$\frac{\partial \theta}{\partial r} \qquad = \frac{AR}{c}\frac{\theta}{\rho}\frac{\partial \rho}{\partial r} - \frac{4}{3}\frac{A\mu}{c}\frac{b}{r^2\rho^4}\left[\frac{\partial \rho}{\partial r} + \frac{3\rho}{r}\right]^2$$

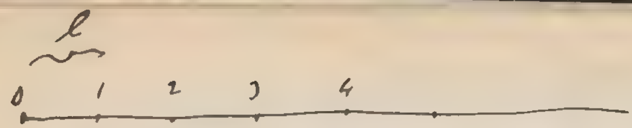
$$\frac{\theta}{\rho}\frac{\partial \rho}{\partial r}\underbrace{\left[1 + \frac{AR}{c}\right]}_{=C} = \frac{g a^2}{R r^2} + \frac{4}{3}\frac{A\mu}{c}\frac{b}{r^2\rho^4}\left[\frac{\partial \rho}{\partial r} + \frac{3\rho}{r}\right]^2$$

$$\theta = \frac{\rho}{C}\,\frac{[\;\ldots\;]}{\frac{\partial \rho}{\partial r}}$$

$$\frac{\partial \theta}{\partial r} = \frac{1}{C}\left[\qquad\right] + \frac{\rho}{C}\frac{\left[\qquad\right]\frac{\partial^2 \rho}{\partial r^2}}{\left(\frac{\partial \rho}{\partial r}\right)^2} + \frac{\rho}{C}\frac{\frac{\partial [\quad]}{\partial r}}{\frac{\partial \rho}{\partial r}}$$

$$= \frac{g a^2}{R r^2} - \frac{1}{C}\left[\qquad\right]$$

$$\left[\frac{g a^2}{R r^2} + \frac{4}{3}\frac{A\mu}{c}\frac{b}{r^2\rho^4}\left[\frac{\partial \rho}{\partial r} + \frac{3\rho}{\partial r}\right]^2\right]\left[2 + \rho\frac{\frac{\partial^2 \rho}{\partial r^2}}{\left(\frac{\partial \rho}{\partial r}\right)^2}\right] + \frac{\rho}{\frac{\partial \rho}{\partial r}}\frac{\partial [\qquad]}{\partial r} - g\frac{a^2 C}{R r^2} = 0$$



A ~~st~~ cord stretching out from 0 to ∞; equal masses 1, 2, .. in equal distances; point 0 begins to move: $\sigma = a \sin \alpha t$; how will the motion of agitation ~~repeat itself~~ spread out on the chord?

$\frac{d^2\sigma_0}{dt^2} = f(t)$ ~~the law of force. At~~ attracting forces between the said points

$$\frac{d^2\sigma_1}{dt^2} = -f(x_1 - x_0) + f(x_2 - x_1)$$

$$= -k(\sigma_0 - \sigma_1) + k(\sigma_1 - \sigma_2) = k(2\sigma_1 - \sigma_0 - \sigma_2)$$

$$\frac{d^2\sigma_2}{dt^2} = k(2\sigma_2 - \sigma_1 - \sigma_3) = -k(\sigma_1 - \sigma_2) + k(\sigma_2 - \sigma_3)$$

$$\frac{d^2\sigma_3}{dt^2} = k(2\sigma_3 - \sigma_2 - \sigma_4) = -k(\sigma_2 - \sigma_3) + k(\sigma_3 - \sigma_4)$$

. . . .

$$\sum \frac{d^2\sigma_n}{dt^2} = k(\sigma_1 - \sigma_0)$$

$$\frac{d^2(\sigma_0 - \sigma_1)}{dt^2} = f + k(\sigma_0 - \sigma_1) - k(\sigma_1 - \sigma_2)$$

$$\frac{d^2}{dt^2}(\sigma_1 - \sigma_2) = k(-\sigma_0 + 3\sigma_1 - 3\sigma_2 + \sigma_3) = k[-(\sigma_0 - \sigma_1) + 2(\sigma_1 - \sigma_2) - (\sigma_2 - \sigma_3)]$$

. . . . .

$$\frac{d^4}{dt^4}(\sigma_0 - \sigma_1) = \frac{d^2 f}{dt^2} + k\frac{d^2}{dt^2}(\sigma_0 - \sigma_1) + k(\sigma_0 - \sigma_1) + 2\frac{d^2}{dt^2}(\sigma_0 - \sigma_1) \pm 2f - 2k(\sigma_0 - \sigma_1) - k(\sigma_2 - \sigma_3)$$

$$2\frac{d^2\sigma_2}{dt^2} - \frac{d^2\sigma_1}{dt^2} - \frac{d^2\sigma_3}{dt^2} = k(4\sigma_2 - 2\sigma_1 - 2\sigma_3 - 2\sigma_1 + \sigma_0 + \sigma_2 - 2\sigma_3 + \sigma_2 + \sigma_4)$$

$$= k(\sigma_0 - 4\sigma_1 + 6\sigma_2 - 4\sigma_3 + \sigma_4) = \frac{1}{k}\frac{d^4\sigma_2}{dt^4}$$

$$\int \left(\frac{d\sigma_n}{dt}\right)^2 dt = ? \quad = t\left(\frac{d\sigma_n}{dt}\right)^2 - 2\int t \frac{d\sigma}{dt}\frac{d^2\sigma}{dt^2}\,dt$$

$$= t\left(\frac{d\sigma_n}{dt}\right)^2 - 2t\sigma\frac{d^2\sigma}{dt^2} + 2\int \sigma \frac{\partial}{\partial t}\left(t\frac{d^2\sigma}{dt^2}\right)dt$$

$$\sigma_2 \frac{d^2\sigma_2}{dt^2} = k\left(2\sigma_2^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3\right) \qquad \sum \sigma \frac{d^2\sigma_n}{dt^2} = k\left(2\sum\sigma_n^2 - 2\sum \sigma_n\sigma_{n-1}\right)$$

$$\sigma_3 \frac{d^2\sigma_3}{dt^2} = k\left(2\sigma_3^2 - \sigma_2\sigma_3 - \sigma_3\sigma_4\right)$$

$$\sigma_3 \frac{d^2\sigma_2}{dt^2} = k\left(2\sigma_2\sigma_3 - \sigma_1\sigma_3 - \sigma_3^2\right)$$

$$\sigma_2 \frac{d^2\sigma_3}{dt^2} = k\left(2\sigma_2\sigma_3 - \sigma_2^2 - \sigma_2\sigma_4\right)$$

$$\int \sigma_3 \frac{d^2\sigma_2}{dt^2}dt = \sigma_3\frac{d\sigma_2}{dt}\Big| - \int \frac{d\sigma_2}{dt}\frac{d\sigma_3}{dt}dt = \sigma_3\frac{d\sigma_2}{dt}\Big| - \sigma_2\frac{d\sigma_3}{dt}\Big| + \int \sigma_2 \frac{d^2\sigma_3}{dt^2}dt$$

$$\sum \int \left(\frac{d\sigma}{dt}\right)^2 dt$$

$$\frac{d^2\sigma_n}{dt^2} = k\left(2\sigma_n - \sigma_{n-1} - \sigma_{n+1}\right)$$

$$\frac{d\sigma_n}{dt} = k\int\left(2\sigma_n - \sigma_{n-1} - \sigma_{n+1}\right)dt = k\,t\left(2\sigma_n - \sigma_{n-1} - \sigma_{n+1}\right) - k\int t\left(2\frac{d\sigma_n}{dt} - \dots\right)$$

$$= k\,t\left(2\sigma_n - \sigma_{n-1} - \sigma_{n+1}\right) - k\frac{t^2}{2}\left(2\frac{d\sigma_n}{dt} - \frac{d\sigma_{n+1}}{dt} - \frac{d\sigma_{n-1}}{dt}\right) +$$

$$+ \frac{k}{2}\int t^2 \underbrace{\left(2\frac{d^2\sigma_n}{dt^2} - \dots\right)}_{k\left[6\sigma_n - 4(\sigma_{n-1}+\sigma_{n+1}) + \sigma_{n-2} + \sigma_{n+2}\right]} dt$$

$$= k\,t\left(2\sigma_n - \sigma_{n-1} - \sigma_{n+1}\right) \dots\dots + k$$

$$\frac{d\sigma_n}{dt}\frac{d^2\sigma_n}{dt^2} = k\left[2\sigma_n\frac{d\sigma_n}{dt} - \sigma_{n-1}\frac{d\sigma_n}{dt} - \sigma_{n+1}\frac{d\sigma_n}{dt}\right]$$

$$\frac{1}{2}\frac{d}{dt}\left(\frac{d\sigma_n}{dt}\right)^2 = k\left[\frac{d}{dt}\left(\sigma_n^2\right) - \frac{d\sigma_n}{dt}\left(\sigma_{n-1}+\sigma_{n+1}\right)\right]$$

$$\frac{1}{2}\left(\frac{d\sigma_n}{dt}\right)^2 = k\left[\sigma_n^2 - \sigma_n\left(\sigma_{n-1}+\sigma_{n+1}\right) + \int\sigma_n\left(\frac{d\sigma_{n-1}}{dt} + \frac{d\sigma_{n+1}}{dt}\right)dt\right]$$

$$\sum\frac{1}{2}\left(\frac{d\sigma_n}{dt}\right)^2 = k\sum\sigma_n^2 - k\int\underbrace{\sum\left\{\begin{matrix}\frac{d\sigma_{n-1}}{dt}\sigma_{n-2} & + & \frac{d\sigma_{n-1}}{dt}\sigma_n \\ \frac{d\sigma_n}{dt}\sigma_{n-1} & + & \frac{d\sigma_n}{dt}\sigma_{n+1} \\ \frac{d\sigma_{n+1}}{dt}\sigma_n & + & \frac{d\sigma_{n+1}}{dt}\sigma_{n+2} \\ \ldots & & \end{matrix}\right\}}_{=\ \sum\sigma_n\sigma_{n-1}}dt$$

$$\sum\frac{1}{2}\left(\frac{d\sigma_n}{dt}\right)^2 = k\left[\sum\sigma_n^2 - \sum\sigma_n\sigma_{n-1}\right] = L$$

$$\sum\sigma_n\frac{d^2\sigma_n}{dt^2} = 2k\left[\sum\sigma_n^2 - \sum\sigma_n\sigma_{n-1}\right]$$

$$\boxed{\begin{aligned}&\sum\left(\frac{d\sigma_n}{dt}\right)^2 - \sigma_n\frac{d^2\sigma_n}{dt^2} = 0\\ &\cancel{\frac{d\sigma}{dt}\frac{d^2\sigma}{dt^2}} - \cancel{\frac{d\sigma}{dt}\frac{d^2\sigma}{dt^2}} - \sum\sigma\frac{d^3\sigma}{dt^3} = 0\end{aligned}}$$

$$\frac{\partial L}{\partial t} = \sum\frac{d\sigma}{dt}\frac{d^2\sigma}{dt^2}$$

$$= k\left[\sum 2\sigma_n\frac{d\sigma_n}{dt} - \ldots\right]$$

$$\frac{\partial L}{\partial x} = 2k\left[\sum\sigma_{n+1}^2 - \sum\sigma_{n+1}\sigma_n - \sum\sigma_n^2 + \sum\sigma_n\sigma_{n-1}\right]$$

$$= 2k\left(\sigma_n^2 - \sigma_n\sigma_{n+1}\right)$$

$$\frac{\partial^2 L}{\partial t^2} = k \sum \underbrace{2\left(\frac{d\sigma_n}{dt}\right)^2 + 2\sigma_n\left(\frac{d^2\sigma_n}{dt^2}\right)} - \frac{d\sigma_{n-1}}{dt}\frac{d\sigma_n}{dt} - \sigma_{n-1}\frac{d^2\sigma_n}{dt^2}$$

$$- \frac{d\sigma_{n+1}}{dt}\frac{d\sigma_n}{dt} - \sigma_{n+1}\frac{d^2\sigma_n}{dt^2}$$

$$= 8kL - \sum \frac{d\sigma_n}{dt}\left(\frac{d\sigma_{n-1}}{dt} + \frac{d\sigma_{n+1}}{dt}\right) + \sigma_{n-1}(2\sigma_{n-2} - \sigma_{n-3} - \sigma_{n-1})$$

$$+ \sigma_{n-2}(2\sigma_{n-1} - \sigma_{n-2} - \sigma_n) + \sigma_n(2\sigma_{n-1} - \sigma_{n-2} - \sigma_n)$$

$$- \sum \sigma_{n-1}(2\sigma_n - \sigma_{n-1} - \sigma_{n+1}) + \sigma_{n+1}(2\sigma_n - \sigma_{n-1} - \sigma_{n+1})$$

$$+ \sigma_n(2\sigma_{n+1} - \sigma_n - \sigma_{n+2}) + \sigma_{n+2}(2\sigma_{n+1} - \sigma_n - \sigma_{n+2})$$

$$+ \sigma_{n+1}(2\sigma_{n+2} - \sigma_{n+1} - \sigma_{n+3}) + \sigma_{n+3}(2\sigma_{n+2} - \sigma_{n+1} - \sigma_{n+3})$$

$$+ \sigma_{n+2}(2\sigma_{n+3} - \sigma_{n+2} - \sigma_{n+4}) + \sigma_{n+4}(- \quad - \quad)$$

$$= 2\sum \sigma_n\sigma_{n+1} - \sigma_n^2 - \sigma_n\sigma_{n+2} + 2\sigma_n\sigma_{n+1} - \sigma_n\sigma_{n+2} - \sigma_n^2$$

$$\sigma_1 = f_1(t) = f_1(0) + t f_1'(0) + \frac{t^2}{2} f_1''(0) + \dots$$

$$= \cancel{\sigma_1(0)} + t \cancel{\frac{d\sigma_1(0)}{dt}} + \frac{t^2}{2}\frac{d^2\sigma_1(0)}{dt^2} + \frac{t^3}{3!}\frac{d^3\sigma_1(0)}{dt^3} + \frac{t^4}{4!}\frac{d^4\sigma_1(0)}{dt^4} + \dots$$

$$\frac{d^2\sigma_1}{dt^2}(0) = -k\,\sigma_0(0)$$

$$\frac{d^3\sigma_1(0)}{dt^3} = -k\left(2\cancel{\frac{d\sigma_1}{dt}(0)} - \frac{d\sigma_0}{dt}(0) - \cancel{\frac{d\sigma_2(0)}{dt}}\right) = -k\frac{d\sigma_0}{dt}(0)$$

$$\frac{d^4\sigma_1(0)}{dt^4} = -k\left(2\frac{d^2\sigma_1}{dt^2}(0) - \frac{d^2\sigma_0}{dt^2}(0) - \frac{d^2\sigma_2(0)}{dt^2}\right) =$$

$$= +2k^2\sigma_0(0) + k\frac{d^2\sigma_0}{dt^2}(0)$$

$$d^5 = 2k^2\frac{d\sigma_0}{dt} + k\frac{d^3\sigma_0}{dt^3}$$

$$d^6 = k\left[2\frac{d^4\sigma_1}{dt^2} - \frac{d^4\sigma_0}{dt^2} - \frac{d^4\sigma_2}{dt^2}\right] = 4k^3\sigma_0 + 2k^2\frac{d^2\sigma_0}{dt^2} - k\frac{d^4\sigma_0}{dt^4} - \cancel{k^3\sigma_0}$$

$$\frac{d^2\sigma_2(0)}{dt^2} = 0 = \frac{d^2\sigma_3}{dt^2}(0) \quad \text{etc.}$$

$$= 3k^3\sigma_0 + 2k^2\frac{d^2\sigma_0}{dt^2} - k\frac{d^4\sigma_0}{dt^4}$$

$$d^7 = k\left[2d^5\sigma_1 - d^5\sigma_0 - d^5\sigma_2\right] = 4k^3\frac{d\sigma_0}{dt} + 2k^2\frac{d^3\sigma_0}{dt^3} - k\frac{d^5\sigma_0}{dt^5} - k^3\frac{d\sigma_0}{dt}$$

$$= 3k^3\frac{d\sigma_0}{dt} + 2k^2\frac{d^3\sigma_0}{dt^3} - k\frac{d^5\sigma_0}{dt^5}$$

$$d^8 = k\left[2d^6\sigma_1 - \sigma_0^6 - \sigma_2^6\right] = 6k^4\sigma_0 + 4k^3\frac{d^2\sigma_0}{dt^2} - 2k^2\frac{d^4\sigma_0}{dt^4} + k^3\frac{d^2\sigma_0}{dt^2} - k\frac{d^6\sigma_0}{dt^6}$$

$$= 6k^4\sigma_0 + 5k^3\frac{d^2\sigma_0}{dt^2} - 2k^2\frac{d^4\sigma_0}{dt^4} - k\frac{d^6\sigma_0}{dt^6}$$

$$\frac{d^2\sigma_2}{dt^2}(0)=0$$

$$\frac{d^3\sigma_2}{dt^3}(0) = k\left(2\frac{d\sigma_2}{dt} - \frac{d\sigma_1}{dt} - \frac{d\sigma_3}{dt}\right) = 0$$

$$\frac{d^4\sigma_2}{dt^4}(0) = k\left(\cancel{2\frac{d^2\sigma_2}{dt^2}} - \frac{d^2\sigma_1}{dt^2} - \cancel{\frac{d^2\sigma_3}{dt^2}}\right) = +k^2\sigma_0(0)$$

$$\frac{d^5\sigma_2}{dt^5}(0) = k\left(\cancel{2\frac{d^3\sigma_2}{dt^3}} - \frac{d^3\sigma_1}{dt^3} - \cancel{\frac{d^3\sigma_3}{dt^3}}\right) = +k^2\frac{d\sigma_0}{dt}(0)$$

$$\frac{d^6\sigma_2}{dt^6}(0) = k\left(2\frac{d^4\sigma_2}{dt^4} - \frac{d^4\sigma_1}{dt^4} - \cancel{\frac{d^4\sigma_3}{dt^4}}\right) = \cancel{2k^3\sigma_0(0)} - \cancel{2k^3\sigma_0(0)} - k^2\frac{d^2\sigma_0}{dt^2} \;*$$

$$d7 \quad = k\left(2\,d^5\sigma_2 - \sigma_1 - \sigma_3\right) = \cancel{2k^3\frac{d\sigma_0}{dt}} - \cancel{2k^3\frac{d\sigma_0}{dt}} - k^2\frac{d^3\sigma_0}{dt^3}$$

σ_3 $\qquad \dfrac{d^5 \sigma_3}{dt^5} = 0$

$$d^6 = k\, d^4 (2\cancel{\sigma_3} - \sigma_2 - \cancel{\sigma_4}) = -k^3 \sigma_0$$

$$d^7 = k\, d^5 (2\cancel{\sigma_3} - \sigma_2 - \cancel{\sigma_4}) = -k^4 \frac{d\sigma_0}{dt}$$

$$d^8 = k\, d^6 (2\sigma_3 - \sigma_2 - \sigma_4) = -2k^4 \sigma_0$$

[illegible]

$6 \theta \theta_1 = \Gamma - \frac{1}{2}\Gamma_1 - \frac{1}{2g}$ [illegible]

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$= \ldots \rho$ [illegible] $\qquad p = c\rho^{k}$

[illegible] $= -\ldots$ [illegible]

[illegible] $= -2\,\frac{\ldots}{\ldots} = -\frac{9a^2 c}{\ldots}$ [illegible] $\qquad$ ~~[illegible]~~ $+ \ldots = -\ldots$ ~~[illegible]~~

[illegible] $= -\frac{9a^2}{c}\ldots$

[illegible] $= -\frac{9a^2}{c}\ldots\,dr$

$\frac{\ldots}{\ldots} = -\frac{9a^2}{c}\, r^{-\frac{2}{k}}\,dr$ $\qquad$ ~~[illegible]~~ [illegible]

[illegible] $= -\frac{9a^2}{c}\ldots$

[illegible]

[illegible] $= -\frac{9a^2}{c}\ldots$

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$$\frac{\gamma k}{k-1}\frac{\alpha}{R}\left(\theta_0-\theta\right)=-g\frac{a^2}{r}+\text{const}$$

$$-\frac{d\theta}{dx}=\frac{g}{\alpha}\frac{k-1}{k}\frac{a^2}{r^2}$$

$$\frac{p}{\rho^{k}} = C\theta$$

$$\rho^{k-1} = C\theta$$

$$\sigma = \theta^{\frac{1}{k-1}}$$

$$\sigma = \frac{1}{\theta^{1-k}}$$

$$\frac{d\sigma}{dx} = \frac{1}{1-k}\,\theta^{\frac{1}{1-k}-1}$$

$$= \frac{1}{1-k}\,\theta^{\frac{k}{1-k}}$$

$$\theta = \sigma^{1-k}$$

$$\frac{d\theta}{dx} = (1-k)\,\sigma^{-k}$$

$$\frac{1}{\rho} = \frac{1}{\rho_0}\left(\frac{p_0}{p}\right)^{\frac{1}{k}}$$

$$\frac{1}{\frac{1}{k}+1}\,\frac{p_0^{\frac{1}{k}}}{\rho_0}\left(\frac{1}{p}\right)^{\frac{1}{k}+1}$$

$$\frac{k}{k+1}\,\frac{p_0^{\frac{1}{k}}}{\rho_0}\,p^{-\frac{1+k}{k}} = \frac{k}{k+1}\left(\frac{p}{\rho}\right)^{k}$$

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Dla przewodzenia ciepła i zaniedbując bezwładności ale uwzględniając tarcie

1). $\rho\,\sigma r^2 = \text{const} = \rho_0\sigma_0 r^2 = b$

$$\alpha = \frac{076 \cdot 98 \cdot 136}{0{,}0013 \cdot 280} = \frac{98}{0{,}36} = 270$$

2). $p = \overbrace{\frac{p_0}{\rho_0\theta_0}}^{\alpha}\,\rho\theta$

3). $0 = -\frac{g a^2}{r^2} - \frac{1}{\rho}\frac{dp}{dr} - \frac{4\mu}{3\rho}\frac{d}{dr}\left(\underbrace{\frac{\sigma}{\rho}\frac{d\rho}{dr}}_{= \frac{b}{\rho^2 r^2}\frac{d\rho}{dr}}\right)$

4). $\rho\,\sigma r^2\left[c\frac{d\theta}{dr} + A\,p\,\frac{d\left(\frac{1}{\rho}\right)}{dr}\right] = \frac{4}{3}\left(\frac{\sigma}{\rho^2 r^2}\right)^2\mu\left[\frac{d\rho}{dr} + \frac{3\rho}{r}\right]^2$

---

3). $\frac{g a^2}{r^2} + \alpha\frac{1}{\rho}\frac{d}{dr}[\rho\theta] - \frac{4\mu}{3\rho}\frac{d}{dr}\left[\frac{b}{\rho^2 r^2}\frac{d\rho}{dr}\right] = 0$

4). $b\,c\frac{d\theta}{dr} - A b\alpha\frac{\theta}{\rho}\frac{d\rho}{dr} = \frac{4\mu}{3}\left(\frac{b}{\rho^2 r^2}\right)^2\left(\frac{d\rho}{dr} + \frac{3\rho}{r}\right)^2$

$$\frac{1}{\rho^2 r^2}\left(\frac{d\rho}{dr} + \frac{3\rho}{r}\right) = \frac{\frac{d}{dr}(\rho r^3)}{\rho^2 r^5} = \left|\frac{1}{\rho r^2}\frac{d}{dr}\log(\rho r^3)\right| = -\,r\frac{d}{dr}\left(\frac{1}{\rho r^3}\right)$$

$\mu = \gamma\theta$

3). $\frac{g a^2}{r^2} + \frac{\alpha}{\rho}\frac{d}{dr}(\rho\theta) - \frac{4\gamma\theta}{3\rho}\frac{d}{dr}\left[\frac{b}{\rho^2 r^2}\frac{d\rho}{dr}\right] = 0$

4). $c\frac{d\theta}{dr} - A\alpha\frac{\theta}{\rho}\frac{d\rho}{dr} - \frac{4\gamma}{3}\frac{b\,\theta}{\rho^4 r^4}\left[\frac{d\rho}{dr} + \frac{3\rho}{r}\right]^2 = 0$

4). $c\frac{d\theta}{dr}\frac{1}{\theta} = A\alpha\frac{d\rho}{dr}\frac{1}{\rho} + \frac{4\gamma b}{3\rho^4 r^4}\left[\frac{d\rho}{dr} + \frac{3\rho}{r}\right]^2$

$c\log\theta + \text{const} = \int \quad dr$

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$$c \log\theta + \text{konst} = A\alpha \log\rho + \frac{4\gamma b}{3} \int \left[\frac{1}{\rho^2 r^2}\left(\frac{d\rho}{dr} + \frac{3\rho}{r}\right)\right]^2 dr$$

Zmodyfikowane ¹równanie (3):

$$\frac{\rho a^2}{r^2} + \frac{\alpha}{\rho}\frac{d}{dr}(\rho\theta) = 0 = \frac{g a^2}{r^2} + \alpha\frac{d\theta}{dr} + \alpha\frac{\theta}{\rho}\frac{d\rho}{dr}$$

$$\text{konst}\times\theta^c = \rho^{\alpha A} \cdot e^{\frac{4\gamma b}{3}\int}$$

$$\left(\frac{\theta}{\theta_0}\right)^c = \left(\frac{\rho}{\rho_0}\right)^{\alpha A}$$

$$\text{konst}\ \rho^c \rho^{-c} = \rho^{\alpha A} \qquad \text{konst}\times\rho^c = \rho^{\alpha A + c} \qquad \text{konst}\ \rho = \rho^{\underbrace{1+\frac{\alpha A}{c}}_{=1{,}5}}$$

$$\frac{\alpha A}{c} = 0{,}5 \qquad \frac{\alpha A}{c} - 1 = -\frac{1}{2}$$

$$\text{konst}\ \theta = \rho^{\frac{\alpha A}{c}} e^{\frac{4\gamma b}{3c}\int}$$

$$0 = \frac{g a^2}{r^2} + \left\{\frac{A\alpha^2}{c}\frac{1}{\rho}\frac{d\rho}{dr} + \frac{4\gamma b\alpha}{3\rho^4 r^4 c}\left[\frac{d\rho}{dr} + \frac{3\rho}{r}\right]^2 + \frac{\alpha}{\rho}\frac{d\rho}{dr}\right\}\rho^{\frac{\alpha A}{c}} e^{\frac{4\gamma b}{3c}\int\left(\frac{\frac{d\rho}{dr}+\frac{3\rho}{r}}{\rho^2 r^2}\right)^2}$$

$$= \frac{g a^2}{r^2} + \rho^{\frac{\alpha A}{c}-1}\frac{d\rho}{dr}\left(\frac{A\alpha}{c}+1\right)\alpha\, e^{\frac{4\gamma b}{3c}\int\dots} + \frac{4\gamma b\alpha}{3c}\left(\frac{\frac{d\rho}{dr}+\frac{3\rho}{r}}{\rho^2 r^2}\right)^2$$

$$g a^2 e^{-\frac{4\gamma b}{3c}\int} + r^2 \rho^{\frac{\alpha A}{c}-1}\frac{d\rho}{dr}\left(\frac{A\alpha^2}{c}+\alpha\right) + \frac{4\gamma b\alpha}{3c} r^2\left(\frac{\frac{d\rho}{dr}+\frac{3\rho}{r}}{\rho^2 r^2}\right)^2 = 0$$

$$\int\left[r\frac{d}{dr}\left(\frac{1}{r^3\rho}\right)\right]^2 dr = f(r)\log r$$

$$r\frac{d}{dr}\left(\frac{1}{r^3\rho}\right) = \left[f'\log r + \frac{f}{r}\right]^{\frac{1}{2}} = \frac{\frac{d\rho}{dr} + \frac{3\rho}{r}}{\rho^2 r} = \frac{k}{\sqrt{r}}$$

$$= \varphi$$

$$\frac{1}{r^3\rho} = \int\frac{1}{r}\left[\quad\right]^2 dr$$

$$\frac{d}{dr}\left(\frac{1}{r^3\rho}\right) = k'\int r^{-\frac{3}{2}} dr$$

$$\frac{1}{r^3\rho} = kr^{\frac{5}{2}} + k'$$

$$\rho = \frac{1}{k}r^{-\frac{11}{2}}$$

$$\int \qquad = f(\log r)$$

$$\frac{1}{r}\frac{d}{dr}\left(\frac{1}{r^3\rho}\right) = \left[\frac{f'(\log r)}{r}\right]^{\frac{1}{2}}$$

$$\int \qquad dr = k^2\log r$$

$$\frac{1}{r^3\rho} = \int\left[r f'(\log r)\right]^{\frac{1}{2}} dr$$

$$e^{-m\log r} = r^{-m}$$

$$M\cancel{e^{-m\log r}}$$

$$M r^{-m} + r^2 N r^{-\frac{9}{4}} r^{-\frac{11}{2}} + P r^2 r^{-1} = 0$$

$$\frac{d}{dr}\left(\frac{1}{r^3\rho}\right) = f$$

$$\frac{d}{dr}\left(\frac{1}{r^3\rho}\right) = f$$

$$\frac{1}{r^3\rho} = \int f\,dr$$

$$\rho = \frac{1}{r^3\int f\,dr}$$

$$M = e^{-\int f^2 r^2 dr}$$

$$\rho r^3 = z \qquad \rho = r^3 z \qquad \frac{d\rho}{dr} = 3r^2 z + r^3\frac{dz}{dr}$$

$$\frac{d\rho}{dr} = \frac{3}{r^4\int f\,dr} +$$

$$e^{-\int\left[r\frac{d}{dr}\left(\frac{1}{\rho r^3}\right)\right]^2 dr} = F \qquad \frac{d}{dr}\left(\frac{1}{\rho r^3}\right) = \frac{1}{r}\sqrt{\frac{F'}{F}}$$

$$-\int\left[r\frac{d}{dr}\left(\frac{1}{\rho r^3}\right)\right]^2 dr = \log F \qquad \frac{1}{\rho r^3} = \int\frac{dr}{r}\sqrt{\frac{F'}{F}}$$

$$\left[r\frac{d}{dr}\left(\frac{1}{\rho r^3}\right)\right]^2 = \frac{F'}{F}.$$

$$e^{-\int\ldots} = 1$$

Jak dtego r bliskie = a

$$g a^2 + r^2 \rho^{\frac{\alpha A}{c}-1}\frac{d\rho}{dr}\left(\frac{A\alpha}{c}+1\right)\alpha + \frac{4\mu b\alpha}{3c\,a^2}\underbrace{\left(\frac{1}{\rho^2}\frac{d\rho}{dr}+\frac{3}{\rho a}\right)^2}_{\frac{1}{a^2}\left[\frac{d}{dr}\left(\frac{1}{\rho}\right)\right]^2} = 0$$

$$\frac{4}{3}\frac{\mu\, b\,6\,a^2\, m\, \rho}{c\, a^2\, \rho^4\,\theta}\left(\frac{d\rho}{dr}\right)^2$$

Zaniedbujemy wpływ tarcia w 4 a uwzględniając w 3:

$$\left(\frac{\theta}{\theta_0}\right) = \left(\frac{\rho}{\rho_0}\right)^{\frac{\alpha A}{c}}$$

$$\theta = \theta_0\left(\frac{\rho}{\rho_0}\right)^{\frac{\alpha A}{c}} = \beta\,\rho^{\frac{\alpha A}{c}}$$

3). $$\frac{g a^2}{r^2} + \frac{\alpha}{m}\beta\left(\frac{\alpha A}{c}+1\right)\rho^{\frac{\alpha A}{c}-1}\frac{d\rho}{dr} - \frac{4\mu}{3}\beta\,\rho^{\frac{\alpha A}{c}-1}\frac{d}{dr}\left[\frac{b}{\rho^2 r^2}\frac{d\rho}{dr}\right] = 0$$

$$\frac{\alpha A}{c} = A$$

1). $p = \alpha \theta \rho$

$+ \frac{1}{3}\frac{\mu}{\rho}\frac{d\,\mathrm{div}}{dx}$ fehlt

2). $\sigma \frac{d\sigma}{dx} = -\frac{g a^2}{x^2} - \frac{1}{\rho}\frac{dp}{dx} + \frac{4}{3}\frac{\mu}{\rho}\left(\frac{d^2\sigma}{dx^2} + \frac{d^2\sigma}{dy^2} + \frac{d^2\sigma}{dz^2}\right)$

3). $\left[c\frac{d\theta}{dx} + Ap\frac{d(\frac{1}{\rho})}{dx}\right] = \cancel{\mu\left(\frac{du}{dz}\right)^2 + \left(\frac{du}{dy}\right)^2}$

4). $\rho\sigma = \text{const} = b$ $\quad = A\mu\left[\left(\frac{du}{dx}\right)^2 + \left(\frac{du}{dy}\right)^2 + \left(\frac{du}{dz}\right)^2\right] + \frac{1}{3}\mu\left(\frac{du}{dx}\right)^2$

$\sigma = f(x)$

1). $p = \alpha\theta\rho$

2). $0 = -\frac{g a^2}{x^2} - \frac{1}{\rho}\frac{dp}{dx} + \frac{4}{3}\frac{\mu}{\rho}\frac{d^2\sigma}{dx^2}$

3). $b\left[c\frac{d\theta}{dx} + Ap\frac{d(\frac{1}{\rho})}{dx}\right] = \frac{4}{3}\mu\left(\frac{d\sigma}{dx}\right)^2$

$\mu = \mu\theta$

4). $\rho\sigma = b$

$\frac{1}{\rho} = \frac{\alpha\theta}{p}$

$\frac{d}{dx}\left(-\frac{1}{\rho^2}\frac{d\rho}{dx}\right) =$

$\frac{2}{\rho^3}\left(\frac{d\rho}{dx}\right)^2 - \frac{1}{\rho^2}\frac{d^2\rho}{dx^2}$

---

3). $b\left[\frac{c}{A}\frac{d\theta}{dx} - \frac{\alpha\theta}{\rho}\frac{d\rho}{dx}\right] = \frac{4}{3}\mu\theta\frac{b^2}{\rho^4}\left(\frac{d\rho}{dx}\right)^2$

2). $0 = -\frac{g a^2}{x^2} - \frac{\alpha}{\rho}\left(\theta\frac{d\rho}{dx} + \rho\frac{d\theta}{dx}\right) + \frac{4}{3}\frac{\mu\theta b}{\rho}\frac{d^2}{dx^2}\left(\frac{1}{\rho}\right)$

$\cancel{-\frac{\alpha}{\rho^2}\frac{d\rho}{dx}\frac{d\rho}{dx} = \frac{\alpha}{\rho}\frac{d\rho}{dx}}$

$$0 = -g\frac{a^2}{x^2} - \frac{c}{A}\frac{d\theta}{dx} - \alpha\frac{d\theta}{dx} + \mu\theta\left[\frac{b}{\rho}\frac{4}{3}\frac{d^2}{dx^2}\left(\frac{1}{\rho}\right) + \frac{4}{3}\frac{b^4}{\rho^4}\left(\frac{d\rho}{dx}\right)\right]$$

bei vollständiger Vernachlässigung von $\int^x$

$$\text{const} = \frac{a^2}{x} - \left(\frac{c}{A} + \alpha\right)\theta \quad + \int dx\ \mu\theta\left[\quad\right]$$

$$\frac{1}{\rho} = \frac{\sigma}{b} \qquad p = \frac{\alpha b\theta}{\sigma}$$

2). $0 = -g\frac{a^2}{x^2} - \frac{\sigma}{b}\frac{d}{dx}\left(\frac{\alpha\theta \cancel{b}}{\sigma}\right) - \mu\frac{\theta\sigma}{b}\frac{d^2\sigma}{dx^2}$

3). $\frac{c}{A}\frac{d\theta}{dx} + \frac{\alpha\theta}{\sigma \cancel{b}}\frac{d\sigma}{dx} = \frac{4}{3}\mu\frac{\theta}{b}\left(\frac{d\sigma}{dx}\right)^2$

3). $\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dx} + \alpha\frac{1}{\sigma}\frac{d\sigma}{dx} = \frac{4}{3}\frac{\mu}{b}\left(\frac{d\sigma}{dx}\right)^2$

2). $0 = -g\frac{a^2}{x^2} - \alpha\frac{d\theta}{dx} + \frac{\alpha}{\sigma}\theta\frac{d\sigma}{dx} + \frac{4}{3}\mu\frac{\theta\sigma}{b}\frac{d^2\sigma}{dx^2}$ ergänzt $\frac{4}{3}\frac{\mu}{b}\sigma\frac{d}{dx}\left(\theta\frac{d\sigma}{dx}\right)$

$$0 = -g\frac{a^2}{x^2} - \left(\alpha + \frac{c}{A}\right)\frac{d\theta}{dx} + \frac{4}{3}\frac{\mu\theta}{b}\left[+\sigma\frac{d^2\sigma}{dx^2} + \left(\frac{d\sigma}{dx}\right)^2\right]$$

$\frac{\alpha A}{c} = k-1$

$$\frac{c}{A}\log\theta + \alpha\log\sigma = \frac{4}{3}\frac{\mu}{b}\int\left(\frac{d\sigma}{dx}\right)^2 dx \;-$$

$$\text{const}\times\theta^{\frac{c}{A}}\cdot\sigma^{\alpha} = e^{\frac{4}{3}\frac{\mu}{b}\int\cdots} = f_c(x) > 1 \text{ wachsend mit } x$$

$\theta = \text{const}\cdot\rho^{\frac{\alpha A}{c}}$

$$\left(\frac{\theta}{\theta_0}\right)^{\frac{c}{A}}\left(\frac{\sigma}{\sigma_0}\right)^{\alpha} = \qquad = \left(\frac{\theta}{\theta_0}\right)^{\frac{c}{A}}\left(\frac{\rho_0}{\rho}\right)^{\alpha}$$

$$g\frac{a^2}{x^2} = \left(\alpha + \frac{c}{A}\right)\frac{d\theta}{dx} + \gamma\theta\left[\ \sim\ \right] \quad \Big| \quad \theta'$$

$$0 = \left(\alpha + \frac{c}{A}\right)\frac{d\theta'}{dx} + \gamma\theta'\left[\quad\right] \quad \Big| \quad \theta$$

$$\frac{g\frac{a^2}{x^2}}{\theta'} = \left(\alpha + \frac{c}{A}\right)\underbrace{\left(\frac{\frac{d\theta}{dx}}{\theta'} - \frac{\theta}{\theta'^2}\frac{d\theta'}{dx}\right)}_{= \frac{d}{dx}\left(\frac{\theta}{\theta'}\right)}$$

$$g\frac{a^2}{x^2} = \left(\alpha + \frac{c}{A}\right)\frac{d\theta'}{dx}$$

σ' und θ' [illegible]

$$-\frac{g a^2}{x} = \left(\alpha + \frac{c}{A}\right)\theta' + \text{const}$$

$$\theta' = M - \frac{N}{x} \qquad \frac{d\theta'}{dx} = \frac{N}{x^2}$$

$$\frac{c}{A}\frac{N}{x^2} + \frac{\alpha}{6}\left(M - \frac{N}{x}\right)\frac{d\sigma}{dx} = \frac{4}{3}\gamma\left(M - \frac{N}{x}\right)\left(\frac{d\sigma}{dx}\right)^2$$

$$\frac{4}{3}\gamma\left(\frac{d\sigma}{dx}\right)^2 - \frac{\alpha}{6}\frac{d\sigma}{dx} = \frac{cN}{A}\frac{1}{x^2\left(M - \frac{N}{x}\right)}$$

$$\left(\alpha + \frac{c}{A}\right)\theta = \left(\alpha + \frac{c}{A}\right)\frac{c}{\alpha}\rho \quad = c\left(1 + \frac{c}{\alpha A}\right) = \frac{k}{k-1}$$

$$\cancel{0 = \frac{g a^2}{\sigma^2} - \left(\alpha + \frac{c}{A_1}\frac{d\sigma}{dx} + y\,\theta\right[}$$

!

$$\cancel{\sigma'\frac{d^2\sigma'}{dx^2} = \frac{4}{3}\left(\frac{d\sigma}{dx}\right)^2}$$

$$\cancel{\frac{\frac{d^2\sigma'}{dx^2}}{\frac{d\sigma'}{dx}} = \frac{4}{3}\frac{1}{\sigma'}\frac{d\sigma'}{dx}}$$

$$\cancel{\log\frac{d\sigma'}{dx} = \frac{4}{3}\log\sigma' + \text{const}}$$

$$\cancel{\frac{d\sigma'}{dx} = A\,\sigma'^{\frac{4}{3}}}$$

$$\cancel{\sigma'^{1-\frac{4}{3}}\,d\sigma' = A\,dx}$$

$$\cancel{-3\sigma^{-\frac{1}{3}} = -B + Ax}$$

$$\cancel{\frac{1}{\sigma} = \left(\frac{B - Ax}{3}\right)^3}$$

$$\cancel{\sigma = \frac{27}{(B - Ax)^3}}$$

$$\sigma\frac{d^2\sigma}{dx^2} + \left(\frac{d\sigma}{dx}\right)^2 = 0 = \frac{d}{dx}\left(\sigma\frac{d\sigma}{dx}\right)$$

$$\sigma\frac{d\sigma}{dx} = \text{const}$$

$$\sigma^2 = \text{const}\;x + \text{const}'$$

$$\sigma = \sqrt{a^2x + m}$$

$$\frac{d\sigma}{dx} = \frac{1}{2}\frac{a^2}{\sqrt{\;}}$$

$$\frac{d\sigma'}{dx} = A\frac{81}{(B - Ax)^4}$$

$$\sigma = c\,\theta^{-\frac{c}{\alpha A}}$$

$$\frac{d\sigma}{dx} = -\frac{c}{\alpha A}\,\theta^{-\frac{c}{\alpha A}-1}$$

in der abgekürzten Gleichung ist

$$\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dx} = -\alpha\frac{1}{\sigma}\frac{d\sigma}{dx}$$

$$\theta = 0 \qquad \sigma = (\infty)^{\frac{1}{k-1}}$$

$$\text{aber}\quad \frac{d\sigma}{dx} = (\infty)^{\frac{k}{k-1}}$$

wenn $\theta = 0$ wird $\frac{d\theta}{dx}$ aber endlich ~~...~~ ist, so muss $\frac{d\theta}{dx} < 0$ sein also

$\frac{\alpha}{\sigma} > -\frac{d\sigma}{dx}$ und zwar $\sigma = [0]^1$ und $\frac{d\sigma}{dx}$ endlich positiv

Was aber nicht möglich ist wenn σ für kleinere x positiv sein soll.

Wenn dagegen $\theta = 0$ $\frac{d\theta}{dx}$ aber von höherer Ordnung 0 so kann z.B. $\sigma = \infty$ und $\left(\frac{d\sigma}{dx}\right)$ von höherer Ordnung 0 werden

Wenn $\frac{d\theta}{dx}$ von derselben Ordnung $= 0$, sodass $\frac{1}{\theta}\frac{d\theta}{dx}$ = endlich : $\sigma = 0$ $\frac{d\sigma}{dx} = 0$ nicht möglich ebenso wie oben

und σ und $\frac{d\sigma}{dx}$ endlich so würde Gleichg. 2. verlangen dass $\frac{d^2\sigma}{dx^2}$ von 1ter Ordnung unendlich wird (am Ende $x = \infty$) was aber x = ... verlangen

$$\theta = \theta_0 \left[\left(\frac{\sigma_0}{\sigma}\right)^{\alpha} f(x)\right]^{\frac{A}{c}}$$

$$\frac{d\theta}{dx} = \theta_0 \frac{A}{c}\left[\left(\frac{\sigma_0}{\sigma}\right)^{\alpha} f(x)\right]^{\frac{A}{c}-1}\left[\left(\frac{\sigma}{\sigma_0}\right)^{-\alpha}\frac{df}{dx} - \alpha f\left(\frac{\sigma}{\sigma_0}\right)^{-\alpha-1}\frac{d\sigma}{dx}\right]$$

$$= \theta_0 \frac{A}{c}\left[\left(\frac{\sigma}{\sigma_0}\right)^{-\alpha} f\right]^{\frac{A}{c}}\left[\frac{1}{f}\frac{df}{dx} - \alpha\left(\frac{\sigma}{\sigma_0}\right)^{-1}\frac{d\sigma}{dx}\right]$$

$$g\frac{a^2}{x^2} + \left(\frac{c}{A}+\alpha\right) \qquad = \frac{4}{3}\frac{\gamma}{b}\left[+\sigma\frac{d^2\sigma}{dx^2} + \left(\frac{d\sigma}{dx}\right)^2\right]\theta_0\left[\left(\frac{\sigma_0}{\sigma}\right)^{\alpha} f\right]^{\frac{A}{c}}$$

$$f(x) = e^{\frac{4}{3}\frac{\gamma}{b}\int_0^x \left(\frac{d\sigma}{dx}\right)^2 dx}$$

$$\frac{1}{f}\frac{df}{dx} = \frac{d}{dx}\lg f(x) = \frac{4}{3}\frac{\gamma}{b}\left(\frac{d\sigma}{dx}\right)^2$$

$$g\frac{a^2}{x^2} = \frac{4}{3}\frac{\gamma}{b}\left[\left(\frac{d\sigma}{dx}\right)^2 + \sigma\frac{d^2\sigma}{dx^2}\right]\theta_0\left[\left(\frac{\sigma_0}{\sigma}\right)^{\alpha} f\right]^{\frac{A}{c}} - \left(\frac{c}{A}+\alpha\right)\theta_0\frac{A}{c}\left[\left(\frac{\sigma_0}{\sigma}\right)^{\alpha} f\right]^{\frac{A}{c}}\left[\frac{4}{3}\frac{\gamma}{b}\left(\frac{d\sigma}{dx}\right)^2 - \alpha\frac{\sigma_0}{\sigma}\frac{d\sigma}{dx}\right]$$

$$= \left[\left(\frac{\sigma_0}{\sigma}\right)^{\alpha} f\right]^{\frac{A}{c}}\theta_0\left\{\frac{4}{3}\frac{\gamma}{b}\left(\frac{d\sigma}{dx}\right)^2 + \frac{4}{3}\frac{\gamma}{b}\sigma\frac{d^2\sigma}{dx^2} - \left(1+\frac{\alpha A}{c}\right)\left[\frac{4}{3}\frac{\gamma}{b}\left(\frac{d\sigma}{dx}\right)^2 - \alpha\frac{\sigma_0}{\sigma}\frac{d\sigma}{dx}\right]\right\}$$

$$= \left[\left(\frac{\sigma_0}{\sigma}\right)^{\alpha} f\right]^{\frac{A}{c}}\theta_0\left\{-\frac{4}{3}\frac{\gamma}{b}\frac{\alpha A}{c}\left(\frac{d\sigma}{dx}\right)^2 + \frac{4}{3}\frac{\gamma}{b}\sigma\frac{d^2\sigma}{dx^2} + \alpha\frac{\sigma_0}{\sigma}\left(1+\frac{\alpha A}{c}\right)\frac{d\sigma}{dx}\right\}$$

$$g\frac{a^2}{x^2} = \underbrace{\left(\frac{\sigma_0}{\sigma}\right)^{\frac{\alpha A}{c}}\theta_0\left[\frac{4\gamma}{b3}\left[-\frac{\alpha A}{c}\left(\frac{d\sigma}{dx}\right)^2 + \sigma\frac{d^2\sigma}{dx^2}\right] + \alpha\frac{\sigma_0}{\sigma}\left(1+\frac{\alpha A}{c}\right)\frac{d\sigma}{dx}\right]}_{\Phi} e^{\frac{4}{3}\frac{\gamma}{b}\frac{A}{c}\int_0^x\left(\frac{d\sigma}{dx}\right)^2 dx}$$

$$-2g\frac{a^2}{x^3} = \frac{d\Phi}{dx}e^{\cdots} + \Phi\frac{4}{3}\frac{\gamma}{b}\frac{A}{c}\left(\frac{d\sigma}{dx}\right)^2 e^{\cdots}$$

$$-\frac{2}{\alpha} = \frac{d\Phi}{dx} + \frac{4}{3}\frac{\gamma}{b}\frac{A}{c}\left(\frac{d\sigma}{dx}\right)^2\Phi$$

Versuche:

$\sigma = x^n$

$$\frac{c}{A}\frac{1}{\theta}\frac{\partial\theta}{\partial x} + \frac{\alpha n}{x} = \frac{4}{3}\frac{\gamma}{b}n^2 x^{2n-2}$$

$$\frac{c}{A}\lg\theta + \alpha n \lg x = \frac{4}{3}\frac{\gamma}{b}n^2\frac{x^{2n-1}}{2n-1}$$

$$\theta = \ldots x^{-n\frac{\alpha A}{c}} e^{\ldots x^{2n-1}}$$

$$\frac{d\theta}{dx} = e^{x^{2n-1}}\left[\cdot x^{-n\frac{\alpha A}{c}-1} + x^{-n\frac{\alpha A}{c}} x^{2n-2}\right]$$

$$\frac{1}{x^2} = e^{x^{2n-1}}$$

$\sigma = e^x$

$$\frac{1}{\sigma}\frac{d\theta}{dx} = e^{2x}$$

$$\lg\theta = e^{2x}.$$

$$\theta = \cdot$$

$\sigma = \lg x \qquad \frac{d\sigma}{dx} = \frac{1}{x} \qquad \int\cdots = \frac{1}{x}$

$$\sigma\frac{d^2\sigma}{dx^2} = x^{-1} \qquad \left(\frac{d\sigma}{dx}\right)^2 = x^{-1}$$

$$\int\frac{d\sigma}{dx}\,dx = \lg x$$

$$\left(\frac{d\sigma}{dx}\right)^2 = \frac{1}{x}$$

$$\frac{d\sigma}{dx} = x^{-\frac{1}{2}}$$

$$\sigma = x^{\frac{1}{2}}$$

$$\frac{d^2\sigma}{dx^2} = x^{-\frac{3}{2}}$$

$$\theta = \sigma^{-\frac{\alpha A}{c}} e^{\int\cdots\frac{4}{3}\frac{\gamma}{b}\cdots}$$

$$= x^{-\frac{\alpha A}{2c}}\cdot x$$

$$0 = g\frac{a^2}{x} + m - \left(\alpha + \frac{c}{A}\right)\theta + \frac{\mu}{b}\,\theta\,\frac{d\theta}{dx}$$

$\theta = 0$

$\cancel{\theta = u v}$

$$\cancel{0 = g\frac{a^2}{x} + m - uv\left(\alpha + \frac{c}{A}\right) + \frac{\mu}{b}\left(u^2 v \frac{dv}{dx} + u v^2 \frac{du}{dx}\right)}$$

$$u\left[\cancel{-v\left(\alpha + \frac{c}{A}\right) + \frac{\mu}{b}\left(u v \frac{dv}{dx} + v^2 \frac{du}{dx}\right)}\right]$$

$$\theta = u + v$$

$$0 = g\frac{a^2}{x} + m - \left(\alpha + \frac{c}{A}\right)(u+v) + \frac{\mu}{b}(u+v)\left(\frac{du}{dx} + \frac{dv}{dx}\right)$$

$$0 = g\frac{a^2}{x} + m - \left(\alpha + \frac{c}{A}\right)u + \frac{\mu}{b}\,u\left(\frac{du}{dx} + \frac{dv}{dx}\right)$$

$$+ v\left[\left(\alpha + \frac{c}{A}\right) + \frac{\mu}{b}\left(\frac{du}{dx} + \frac{dv}{dx}\right)\right]$$

$$\frac{\mu}{b}\frac{d\theta}{dx} - \left(\alpha + \frac{c}{A}\right) + \frac{m}{\theta} + \frac{g a^2}{\theta x} = 0$$

$$\frac{\mu}{b}\frac{d\theta_1}{dx} - \qquad + \frac{m}{\theta_1} + \frac{g a^2}{\theta_1 x} = 0$$

$$\frac{\mu}{b}\left(\theta\frac{d\theta}{dx} - \theta_1\frac{d\theta_1}{dx}\right) - \left(\alpha + \frac{c}{A}\right)\theta \qquad = 0$$

$$\theta\frac{d\theta}{dx} \qquad - \frac{b}{\mu}\left(\alpha + \frac{c}{A}\right)\theta + \theta_1\frac{d\theta_1}{dx} = 0$$

$$\cancel{\frac{\mu}{b}\frac{d}{dx}(\theta - \theta_1)}$$

$$\frac{\mu}{b}\frac{d^2\theta}{dx^2} - \frac{m}{\theta^2}\frac{d\theta}{dx} - \frac{ga^2}{\theta^2 x}\frac{d\theta}{dx} - \frac{ga^2}{\theta x^2} = 0$$

$$u = (\log f(x))^2 \qquad \frac{du}{dx} = 2\log f' \cdot \frac{f'}{f}$$

$$\frac{ga^2}{x} - \left(\alpha + \frac{c}{A}\right)\log f + \underbrace{\frac{\mu}{b}\frac{f'}{f}\log f}_{\log f\left(\frac{\mu}{b}\frac{f'}{f} - \alpha + \frac{c}{A}\right)} + \text{const}$$

$$f = e^{\frac{1}{x}}$$

$$\frac{\mu}{2b}\frac{du}{dx} - \left(\alpha + \frac{c}{A}\right) u^{\frac{1}{2}} + \frac{ga^2}{x} + m = 0$$

$$\left(\alpha + \frac{c}{A}\right)^2 u = \left[\frac{\mu}{2b}\frac{du}{dx} + \frac{ga^2}{x} + m\right]^2$$

$$\frac{du}{dx} = p \qquad \left(\alpha + \frac{c}{A}\right)^2 p = 2\left[\frac{\mu}{2b}p + \frac{ga^2}{x} + m\right]\left[\frac{\mu}{2b}\frac{dp}{dx} - \frac{ga^2}{x^2}\right]$$

$$\frac{\frac{1}{2}\left(\alpha + \frac{c}{A}\right)^2 p}{\frac{\mu}{2b}p + \frac{ga^2}{x} + m} + \frac{ga^2}{x^2} = \frac{\mu}{2b}\frac{dp}{dx}$$

$$\frac{ga^2}{x} = \left(\alpha + \frac{c}{A}\right)u^{\frac{1}{2}} - m - \frac{\mu}{2b}\frac{du}{dx}$$

$$x = \frac{ga^2 \frac{dx}{du}}{\left[\left(\alpha + \frac{c}{A}\right)u^{\frac{1}{2}} - m\right]\frac{dx}{du} - \frac{\mu}{2b}} \qquad 1 =$$

$$\left[\left(\alpha + \frac{c}{A}\right)\sqrt{u} - m - \frac{\mu}{2b}\frac{du}{dx}\right]^2 = ga^2\left[\left(\alpha + \frac{c}{A}\right)\frac{1}{2}\frac{du}{dx}\frac{1}{\sqrt{u}} - \frac{\mu}{2b}\frac{d^2u}{dx^2}\right]$$

$$g\cancel{\mu}x + m - \left(\alpha + \frac{c}{A}\right)\sqrt{u} + \sqrt{\frac{\mu}{2b}}\,\frac{du}{dx} = 0 \qquad \frac{dx}{du} = p$$

$$g\cancel{\mu}x + m - \left(\alpha + \frac{c}{A}\right)\sqrt{u} + \sqrt{\frac{\mu}{2b}}\,\frac{1}{p} = 0$$

$$g\cancel{\mu}p - \left(\alpha + \frac{c}{A}\right)\frac{1}{2}\frac{1}{\sqrt{u}} - \sqrt{\frac{\mu}{2b}}\,\frac{1}{p^2}\frac{dp}{du} = 0$$

p Dimens . . . $\frac{1}{\sqrt{u}}$

$$p = \frac{z}{\sqrt{u}} \qquad \frac{dp}{du} = \frac{dz}{du}\frac{1}{\sqrt{u}} - \frac{z}{2u\sqrt{u}}$$

$$g\cancel{\mu}\frac{z}{\cancel{\sqrt{u}}} - \left(\alpha + \frac{c}{A}\right)\frac{1}{2}\cancel{\frac{1}{\sqrt{u}}} - \sqrt{\frac{\mu}{2b}}\,\frac{u}{z^2}\left(\frac{dz}{du}\cancel{\frac{1}{\sqrt{u}}} - \frac{z}{2u\cancel{\sqrt{u}}}\right)$$

$$g\cancel{\mu}z - \frac{1}{2}\left(\alpha + \frac{c}{A}\right) + \sqrt{\frac{\mu}{4b}}\,\frac{1}{z} = \sqrt{\frac{\mu}{2b}}\,\frac{u}{z^2}\frac{dz}{du}$$

$$\frac{du}{u} = \sqrt{\frac{\mu}{2b}}\,\frac{dz}{\left[g\cancel{\mu}z^3 - \frac{1}{2}\left(\alpha + \frac{c}{A}\right)z^2 + \sqrt{\frac{\mu}{4b}}\,z\right]}$$

$$\log u = f(z)$$

$$\log u = f(p\sqrt{u})$$

$$= f\left(\frac{dx}{du}\sqrt{u}\right)$$

$$2\log\theta = f\left[\frac{dx}{d(\theta^2)}\,\theta\right]$$

$$\log u = \log\frac{z^2}{z^2 - \frac{1}{2g}(\quad) + \sqrt{\frac{\mu}{4bg}}}\left(\frac{z - \quad + \sqrt{\quad}}{z \quad - \sqrt{\quad}}\right)^{-\frac{1}{4g}(\alpha \ldots)}$$

$$\theta = \vartheta + n$$

$$0 = g\frac{a^2}{x} + m - \left(\alpha + \frac{c}{A}\right)\vartheta - \left(\alpha + \frac{c}{A}\right)n + \frac{\mu}{b}(\vartheta + n)\frac{d\vartheta}{dx}$$

$$m - \left(\alpha + \frac{c}{A}\right)n = 0 \qquad n = \frac{m}{\alpha + \frac{c}{A}}$$

$$0 = g\frac{a^2}{x} - \left(\alpha + \frac{c}{A}\right)\vartheta + \frac{\mu}{b}\, n\,\frac{d\vartheta}{dx} + \frac{\mu}{b}\,\vartheta\,\frac{d\vartheta}{dx}$$

$$\frac{du}{u} = 2\left[\frac{dz}{z} + \frac{-gz + \frac{1}{2}\left(\alpha + \frac{c}{A}\right)}{gz^2 - \frac{1}{2}\left(\alpha + \frac{c}{A}\right)z + \frac{\mu}{4b}}\,dz\right]$$

$$= 2\left[\frac{dz}{z} - \frac{\left[z - \frac{1}{2g}\left(\alpha + \frac{c}{A}\right)\right]dz}{z^2 - \frac{1}{2g}\left(\alpha + \frac{c}{A}\right)z + \frac{\mu}{4bg}}\right]$$

$$= 2\frac{dz}{z} - \frac{2z - \frac{1}{2g}\left(\alpha + \frac{c}{A}\right)}{z^2 - \frac{1}{2g}\left(\alpha + \frac{c}{A}\right)z + \frac{\mu}{4bg}} + \frac{1}{2g}\left(\alpha + \frac{c}{A}\right)\frac{dz}{z^2 - \frac{1}{2g}\left(\alpha + \frac{c}{A}\right)z + \frac{\mu}{4bg}}$$

$$= 2\log z - \log\left(z^2 - \frac{1}{2g}(\;\;)z + \frac{\mu}{4bg}\right) + \frac{1}{2g}\left(\alpha + \frac{c}{A}\right)\int\frac{dz}{z^2 - \cdots}$$

$$\int\frac{1}{\left[z - \frac{1}{4g}\left(\alpha + \frac{c}{A}\right)\right]^2 - \left(\left[\frac{1}{4g}\left(\alpha + \frac{c}{A}\right)\right]^2 - \frac{\mu}{4bg}\right)} = \int\frac{d\,\frac{z - \frac{1}{4g}\left(\alpha + \frac{c}{A}\right)}{\sqrt{\;}}}{\left[\frac{z - \frac{1}{4g}\left(\alpha + \frac{c}{A}\right)}{\sqrt{\left(\frac{1}{4g}\left(\alpha + \frac{c}{A}\right)\right)^2 - \frac{\mu}{4bg}}}\right]^2 - 1}\cdot\frac{1}{\sqrt{\;}}$$

$$= \frac{1}{2}\,\frac{1}{\sqrt{\left(\frac{\alpha + \frac{c}{A}}{4g}\right)^2 - \frac{\mu}{4bg}}}\,\log\left(\frac{z - \frac{1}{4g}\left(\alpha + \frac{c}{A}\right) + \sqrt{\;}}{z - \frac{1}{4g}\left(\alpha + \frac{c}{A}\right) - \sqrt{\;}}\right)$$

[illegible]

[illegible]

$$x = 2\theta \quad [illegible]$$

[illegible]

$$\alpha + \frac{c}{A} - \frac{dx}{d\theta} - [illegible] = 2$$

$$x = \frac{\theta}{g}\left[[illegible] \right] \qquad \theta = \frac{m - gx}{[illegible] - \frac{c}{A}}$$

$$\log\theta = \log([illegible]) - [illegible]$$

$$\frac{1}{\theta}\frac{d\theta}{dx} = \frac{g}{gx - m} - \frac{1}{[illegible] - \frac{c}{A}}\,[illegible] = [illegible]$$

$$\cancel{\frac{dx}{d\theta}} = \cancel{\alpha + \frac{c}{A} - \frac{m}{\theta}\ 2}$$

$$x = \frac{1}{g}\left(\alpha + \frac{c}{A}\right)\theta + \frac{m}{g} - \frac{2\theta}{g}$$

$$\frac{dx}{d\theta} = \frac{1}{g}\left(\alpha + \frac{c}{A}\right) - \frac{2}{g} - \frac{\theta\,\frac{dx}{d\theta}}{g}$$

$\cancel{2c}\,\alpha$

$$\frac{2c}{A}z = \frac{d}{dx}\left[\frac{\alpha+\frac{c}{A}}{g}\theta - \frac{z\theta}{g} - \frac{\theta^2\frac{dz}{d\theta}}{g}\right]\left[z(2-\alpha)\right]$$

$$\frac{2c}{A}z\frac{d\alpha}{d\theta} = \frac{d}{d\theta}\left[\;\ldots\right.$$

$$\frac{2c}{A}\frac{z}{\cancel{g}}\left[\frac{\alpha+\frac{c}{A}}{\cancel{g}} - \frac{z}{\cancel{g}} - \theta\frac{dz}{d\theta}\right] = z(2-\alpha)\left[\left(\alpha+\frac{c}{A}\right) - z - 3\theta\frac{dz}{d\theta} - \theta^2\frac{d^2z}{d\theta^2}\right]$$

$$+\left[2z - \alpha\right]\frac{dz}{d\theta}\left[\left(\alpha+\frac{c}{A}\right)\theta - z\theta - \theta^2\frac{dz}{d\theta}\right]$$

$$\frac{dx}{dz} = \frac{1}{g}\left[\left(\alpha+\frac{c}{A} - z\right)\frac{d\theta}{dz} - \theta\right]$$

$$\frac{2c}{A}z\frac{dx}{dz} = \cancel{\frac{d}{dz}\left[\frac{1}{g}\left(\alpha+\frac{c}{A}\right) - \frac{z}{g} - \frac{\theta\frac{dz}{d\theta}}{g}\right]}$$

$$= \frac{d}{dz}\left[\theta\frac{dz}{d\theta}\; z(2-\alpha)\frac{1}{g}\left[\left(\alpha+\frac{c}{A}-z\right)\frac{d\theta}{dz} - \theta\right]\right]$$

$$= \frac{d}{dz}\left[\theta\; z(2-\alpha)\left[\alpha+\frac{c}{A} - z - \frac{\theta}{\frac{d\theta}{dz}}\right]\right]$$

$$\frac{2c}{A}z\left[\left(\alpha+\frac{c}{A}-z\right)\frac{d\theta}{dz} - \theta\right] = z(2-\alpha)\left[\left(\alpha+\frac{c}{A}\right)\frac{d\theta}{dz} - z\frac{d\theta}{dz} - \theta\right] +$$

$$+\theta(2z-\alpha)\left[\alpha+\frac{c}{A} - z - \frac{\theta}{\frac{d\theta}{dz}}\right] +$$

Jistě možno $2(2-\alpha) = 2^2$ postaviti:

$$\frac{2cz}{A} = \frac{d}{dx}\left[\frac{z^2}{\frac{1}{\theta}\frac{d\theta}{dx}}\right]$$

$$\frac{2c}{A} z = \frac{2z\frac{dz}{dx}}{\frac{1}{\theta}\frac{d\theta}{dx}} - z^2 \frac{\frac{d}{dx}\left(\frac{1}{\theta}\frac{d\theta}{dx}\right)}{\left(\frac{1}{\theta}\frac{d\theta}{dx}\right)^2} .$$

$$\frac{2c}{A} \frac{1}{\theta}\frac{d\theta}{dx} = 2\frac{dz}{dx} - z\frac{d}{dx}\left(\lg \frac{1}{\theta}\frac{d\theta}{dx}\right)$$

$$\frac{2c}{A} \underbrace{\frac{1}{\theta}\frac{d\theta}{dz}}_{\frac{d(\lg\theta)}{dz}} = 2 - z\frac{d}{dz}\left(\lg \frac{1}{\theta}\frac{d\theta}{dx}\right)$$

$$\left[\frac{c^2}{A^2} + \frac{c\alpha}{A} + \frac{\alpha}{u} - \frac{1}{u^2}\right]$$

$$\left(u + \frac{du}{dz}\right)^2 \overbrace{\left[\cancel{\frac{2c\alpha}{A}} + \cancel{\frac{2c^2}{A^2}} - \cancel{\frac{2c}{Au}} - \cancel{\frac{\alpha c}{A}} + \frac{\alpha}{u} - \cancel{\frac{c^2}{A^2}} + \cancel{\frac{\alpha c}{A\alpha}} - \frac{1}{u^2}\right]}$$

Przyblizone: $\frac{g\alpha^2}{x^2} = g$

$$\frac{2c}{A}\left[\alpha+\frac{c}{A}+\frac{m}{\theta}-\frac{gx}{\theta}\right] = \frac{d}{dx}\left[\frac{\left[\alpha+\frac{c}{A}+\frac{m}{\theta}-\frac{gx}{\theta}\right]\left[\frac{c}{A}+\frac{m}{\theta}-\frac{gx}{\theta}\right]}{\frac{1}{\theta}\frac{d\theta}{dx}}\right]$$

$gx - m = y \qquad g\,dx = dy$

$$\frac{2c}{A}\left[\alpha+\frac{c}{A}-\frac{y}{\theta}\right] = \frac{d}{dy}\left[\frac{\left[\alpha+\frac{c}{A}-\frac{y}{\theta}\right]\left[\frac{c}{A}-\frac{y}{\theta}\right]}{\frac{1}{\theta}\frac{d\theta}{dy}}\right]$$

$$= \frac{-\left[\alpha+\frac{2c}{A}-\frac{2y}{\theta}\right]\frac{d}{dy}\left[\frac{y}{\theta}\right]}{\frac{1}{\theta}\frac{d\theta}{dy}} - \frac{\left[\alpha+\frac{c}{A}-\frac{y}{\theta}\right]\left[\frac{c}{A}-\frac{y}{\theta}\right]\frac{d}{dy}\left(\frac{1}{\theta}\frac{d\theta}{dy}\right)}{\left(\frac{1}{\theta}\frac{d\theta}{dy}\right)^2}$$

$$\frac{2c}{A}\left[\alpha+\frac{c}{A}-\frac{y}{\theta}\right]\left(\frac{1}{\theta}\frac{d\theta}{dy}\right)^2 = -\left[\alpha+\frac{2c}{A}-\frac{2y}{\theta}\right]\frac{1}{\theta}\frac{d\theta}{dy}\left[\frac{1}{\theta}-\frac{y}{\theta^2}\frac{d\theta}{dy}\right] - \left[\alpha+\frac{c}{A}-\frac{y}{\theta}\right]\left[\frac{c}{A}-\frac{y}{\theta}\right]\left[\frac{1}{\theta}\frac{d^2\theta}{dy^2}-\frac{1}{\theta^2}\left(\frac{d\theta}{dy}\right)^2\right]$$

$y = e^z \qquad \theta = u e^z$

$\frac{d\theta}{dy} = \left(u+\frac{du}{dz}\right)$

$\frac{d^2\theta}{dy^2} = \left(\frac{du}{dz}+\frac{d^2u}{dz^2}\right)e^{-z}$

$$\frac{2c}{A}\left[\alpha+\frac{c}{A}-\frac{y}{\theta}\right]\left(\frac{d\theta}{dy}\right)^2 = -\left[\alpha+\frac{2c}{A}-\frac{2y}{\theta}\right]\left[1-\frac{y}{\theta}\frac{d\theta}{dy}\right]\frac{d\theta}{dy} - \left[\alpha+\frac{c}{A}-\frac{y}{\theta}\right]\left[\frac{c}{A}-\frac{y}{\theta}\right]\left[\theta\frac{d^2\theta}{dy^2}-\left(\frac{d\theta}{dy}\right)^2\right]$$

$$\frac{2c}{A}\left[\alpha+\frac{c}{A}-\frac{1}{u}\right]\left(u+\frac{du}{dz}\right)^2 = -\left[\alpha+\frac{2c}{A}-\frac{2}{u}\right]\left[\underbrace{1-\frac{1}{u}\left(u+\frac{du}{dz}\right)}_{-\frac{1}{u}\frac{du}{dz}}\right]\left(u+\frac{du}{dz}\right) - \left[\alpha+\frac{c}{A}-\frac{1}{u}\right]\left[\frac{c}{A}-\frac{1}{u}\right]\left[u\left(\frac{du}{dz}+\frac{d^2u}{dz^2}\right)-\left(u+\frac{du}{dz}\right)^2\right]$$

$$\left[\frac{c^2}{A^2} + \frac{c\alpha}{A} + \frac{\alpha}{u} - \frac{1}{u^2}\right]\left[u + \frac{du}{dz}\right]^2 = \left[\alpha + \frac{2c}{A} - \frac{2}{u}\right]\left[u + \frac{du}{dz}\right]\frac{1}{u}\frac{du}{dz} -$$

$$- \left[\alpha + \frac{c}{A} - \frac{1}{u}\right]\left[\frac{c}{A} - \frac{1}{u}\right]\left[\frac{du}{dz} + \frac{d^2u}{dz^2}\right]u = 0$$

$$\cancel{\frac{c^2}{A^2} + \frac{2c\alpha}{A} + \alpha - \frac{1}{u}}$$

$$\left[\left(\frac{c^2}{A^2} + \frac{c\alpha}{A}\right)u^2 + \alpha u - 1\right] + \frac{du}{dz}\left[\left(\frac{2c^2}{A^2}u + \frac{2c\alpha}{A}u\right) + \cancel{2\alpha} - \cancel{\frac{2}{u}} - \cancel{\alpha} - \frac{2c}{A} + \cancel{\frac{2}{u}} + \right.$$

$$\left. + \left(\cancel{\frac{\alpha c}{A}u} + \frac{c^2}{A^2}u\right) - \frac{c}{A} - \cancel{\alpha} - \frac{c}{A} + \frac{1}{u}\right]$$

$$+ \left(\frac{du}{dz}\right)^2\left[\frac{c^2}{A^2} + \frac{c\alpha}{A} + \cancel{\frac{\alpha}{u}} - \cancel{\frac{1}{u^2}} - \cancel{\frac{\alpha}{u}} - \frac{2c}{Au} + \cancel{\frac{2}{u^2}}\right]$$

$$+ \frac{d^2u}{dz^2}\left[\frac{\alpha c}{A}u + \frac{c^2}{A^2}u - \frac{c}{A} - \alpha - \frac{c}{A} + \frac{1}{u}\right] = 0$$

$$\left[\left(\frac{c^2}{A^2} + \frac{c\alpha}{A}\right)u^2 + \alpha u - 1\right] + \frac{du}{dz}\left[3\left(\frac{c^2}{A^2} + \frac{c\alpha}{A}\right)u - \frac{4c}{A} + \frac{1}{u}\right] +$$

$$+ \left(\frac{du}{dz}\right)^2\left[\frac{c^2}{A^2} + \frac{c\alpha}{A} - \frac{2c}{A}\frac{1}{u} + \frac{1}{u^2}\right] + \frac{d^2u}{dz^2}\left[\left(\frac{c^2}{A^2} + \frac{\alpha c}{A}\right)u - \alpha - \frac{2c}{A} + \frac{1}{u}\right] =$$

$$\frac{du}{dz} = p \qquad \frac{d^2u}{dz^2} = p\frac{dp}{du}$$

$$U_0 + p\,U_1 + p^2\,U_2 + p\frac{dp}{du}\,U_3 = 0$$

$$\alpha + \frac{c}{A} = \cancel{\frac{pq}{\theta_0 \varrho_0}} = \alpha\left(1 + \frac{c}{\alpha A}\right) = \alpha\left(1 + \frac{1}{k-1}\right) = \frac{\alpha k}{k-1}$$

$$\frac{\alpha A}{c} = k - 1 \qquad \frac{c}{A} = \frac{\alpha}{k-1}$$

$$\frac{dq}{du}\underbrace{\left(\alpha + \frac{c}{A} - u\right)\left(\frac{c}{A} - u\right)}_{U_0} + q\overbrace{\left[\frac{c}{A}\left(\alpha + \frac{c}{A}\right) - 2u\left(\alpha + \frac{2c}{A}\right) + 3u^2\right]}^{U_1} = \underbrace{\left[\frac{2c}{A}\left(\alpha + \frac{c}{A}\right) - u\left(\alpha + \frac{4c}{A}\right) + 2u^2\right]}_{U_2}$$

$$q = e^{-\int \frac{U_1}{U_0} du}\left[C + \int \frac{U_2}{U_0} e^{\int \frac{U_1}{U_0} du} du\right]$$

$$\frac{U_1}{U_0} = \frac{\frac{c}{A}\left(\alpha + \frac{c}{A}\right) - 2u\left(\alpha + \frac{2c}{A}\right) + 3u^2}{\left(\alpha + \frac{c}{A} - u\right)\left(\frac{c}{A} - u\right)} = 3 - \frac{2\frac{c}{A}\left(\alpha + \frac{c}{A}\right) - u\left(\alpha + \frac{2c}{A}\right)}{\frac{c}{A}\left(\alpha + \frac{c}{A}\right) - \left(\alpha + \frac{2c}{A}\right)u + u^2}$$

$$= 3 - \left[\frac{\left(\alpha + \frac{c}{A}\right)}{\alpha + \frac{c}{A} - u} + \frac{\frac{c}{A}}{\frac{c}{A} - u}\right]$$

$$\int \frac{U_1}{U_0} du = 3u + \log\left[\left(\alpha + \frac{c}{A} - u\right)^{+\left(\alpha + \frac{c}{A}\right)}\left(\frac{c}{A} - u\right)^{+\frac{c}{A}}\right]$$

$$\frac{U_2}{U_0} = \frac{\frac{2c}{A}\left(\alpha + \frac{c}{A}\right) - u\left(\alpha + \frac{4c}{A}\right) + 2u^2}{\frac{c}{A}\left(\alpha + \frac{c}{A}\right) - \left(\alpha + \frac{2c}{A}\right)u + u^2} = 2 -$$

U_0

$$\frac{dq}{du}\left(\alpha+\frac{c}{A}-u\right)\left(\frac{c}{A}-u\right)+q\left[\cancel{\left(\alpha+\frac{c}{A}-u\right)\left(\frac{c}{A}-u\right)\left(\alpha-2u\right)}\right]=\cancel{2c}$$

U_1

$$q\left[\left(\alpha+\frac{c}{A}-u\right)\left(\frac{c}{A}-u\right)-u\left(\alpha+\frac{2c}{A}-2u\right)\right]$$

$$=\frac{2c}{A}\left(\alpha+\frac{c}{A}-u\right)-u\left(\alpha+\frac{2c}{A}-2u\right)$$

U_2

$$U_1=\left(\alpha+\frac{c}{A}-u\right)\left(\frac{c}{A}-u\right)-u\left(\alpha+\frac{c}{A}-u+\frac{c}{A}-u\right)$$

$$\frac{U_1}{U_0}=1-u\left[\frac{1}{\frac{c}{A}-u}+\frac{1}{\alpha+\frac{c}{A}-u}\right]$$

$$=1+\frac{u}{u-\frac{c}{A}}+\frac{u}{u-\alpha-\frac{c}{A}}=3-\frac{\frac{c}{A}}{\frac{c}{A}-u}-\frac{\alpha+\frac{c}{A}}{\left(\alpha+\frac{c}{A}\right)-u}$$

$$\frac{U_2}{\cancel{U_0}}=\frac{2c}{A}\left(\alpha+\frac{c}{A}-u\right)-u\left(\alpha+\frac{c}{A}-u+\frac{c}{A}-u\right)$$

$$\frac{U_2}{U_0}=\frac{\frac{2c}{A}-u}{\frac{c}{A}-u}-\frac{u}{\alpha+\frac{c}{A}-u}=1+\frac{\frac{c}{A}}{\frac{c}{A}-u}+1-\frac{\alpha+\frac{c}{A}}{\alpha+\frac{c}{A}-u}$$

$$=2+\frac{\frac{c}{A}}{\frac{c}{A}-u}-\frac{\frac{\alpha+c}{A}}{\left(\alpha+\frac{c}{A}\right)-u}$$

$$\int\frac{U_1}{U_0}\,du=3u+\log\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}}$$

$$e^{\int\frac{U_1}{U_0}du}=e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}}$$

$$\int -\frac{\frac{c}{A}\,du}{\frac{c}{A}-u} = \frac{c}{A}\lg\left(\frac{c}{A}-u\right) = \lg\left(\frac{c}{A}-u\right)^{\frac{c}{A}}$$

$$= \int \frac{\frac{c}{A}\,du}{u-\frac{c}{A}} = \frac{c}{A}\lg\left(u-\frac{c}{A}\right) = \lg\left(u-\frac{c}{A}\right)^{\frac{c}{A}} \quad \Bigg\} \; ?$$

$$\int \frac{u_2}{u_0} e^{\int \frac{u_1}{u_0}du} du = \int \left[2 \frac{\alpha+\frac{c}{A}}{\alpha+\frac{c}{A}-u} + \frac{\frac{c}{A}}{\frac{c}{A}-u}\right] e^{3u} \left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}} \left(\frac{c}{A}-u\right)^{\frac{c}{A}} du$$

$$= 2\underbrace{\int e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}} du}_{J_0} - \left(\alpha+\frac{c}{A}\right)\underbrace{\int e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}-1}\left(\frac{c}{A}-u\right)^{\frac{c}{A}} du}_{J_1}$$

$$+ \frac{c}{A}\underbrace{\int e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}-1} du}_{J_2}$$

$$\left(\alpha+\frac{c}{A}-u\right)\left(\frac{c}{A}-u\right) = z$$

$$z\left(\frac{dq}{du}+q\right) - (q-1)u\frac{dz}{du} = \frac{2c}{A}\left(\alpha+\frac{c}{A}-u\right)$$

$$\frac{q}{z} = t$$

$$z\frac{dq}{du} - \frac{q}{u}\frac{dz}{du} + zq + \cancel{\ldots} = f_{c}(u)$$

$$\frac{1}{z}\frac{dq}{du} - \frac{q}{z^2}\frac{dz}{du} + \frac{q}{z} = \frac{\frac{2c}{A}\left(\alpha+\frac{c}{A}-u\right) - \frac{dz}{du}}{z^2} = \frac{u_2}{z^2}$$

$$\left(\alpha+\frac{c}{A}\right) J_1 = e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}} - \underbrace{\int \left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}} \frac{d}{du}\left[e^{3u}\left(\frac{c}{A}-u\right)^{\frac{c}{A}}\right]}_{3J_0 - \frac{c}{A}J_2}$$

$$\left(\alpha+\frac{c}{A}\right)J_1 + \frac{c}{A}J_2 - 3J_0 + e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}} = 0$$

$$J_0 = \int e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}}du = u\,e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}} -$$

$$-3J_0 + \left(\alpha+\frac{c}{A}\right)J_1 + \frac{c}{A}J_2$$

$$4J_0 - \left(\alpha+\frac{c}{A}\right)J_1 - \frac{c}{A}J_2 = u\,e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}}$$

$$-3J_0 + \left(\alpha+\frac{c}{A}\right)J_1 + \frac{c}{A}J_2 = -e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}}$$

$$J_0 = e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}}(u-1) \;?$$

$$\frac{d}{du} = e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}-1}\left(\frac{c}{A}-u\right)^{\frac{c}{A}-1}\Big[(u-1)\Big[3\left(\alpha+\frac{c}{A}-u\right)\left(\frac{c}{A}-u\right) -$$

$$-\left(\alpha+\frac{c}{A}\right)\left(\frac{c}{A}-u\right) - \left(\alpha+\frac{c}{A}-u\right)\frac{c}{A}\Big] + \left(\alpha+\frac{c}{A}-u\right)\left(\frac{c}{A}-u\right)\Big]$$

$$\left[3\alpha\frac{c}{A} + 3\frac{c^2}{A^2} - 6\frac{c}{A}u - 3u\alpha + 3u^2 - \alpha\frac{c}{A} + \alpha u + \frac{c}{A}u - \frac{c^2}{A^2} - \alpha\frac{c}{A} - \frac{c^2}{A^2} + u\frac{c}{A}\right]$$

$$= \alpha\frac{c}{A} + \frac{c^2}{A^2} - 4\frac{c}{A}u - 2\alpha u + 3u^2$$

$$= \left(\alpha+\frac{c}{A}\right)\frac{c}{A} - 2\left(\alpha+\frac{c}{A}\right)u - 2\frac{c}{A}u + 3u^2 \quad \Big\| -\alpha\frac{c}{A} - \frac{c^2}{A^2} + \frac{2c}{A}u + \alpha u - u^2$$

$$= -\frac{2c}{A}u - \alpha u + 2u^2$$

$$3u^3 - 4\frac{c}{A}u^2 - 2\alpha u^2 + \frac{\alpha c}{A}u + \frac{c}{A}u + \frac{2c}{A}u + \alpha u - 2u^2$$

$$-3\int u\,e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}}du + \int u\,e^{3u}\left(\alpha+\frac{c}{A}\right)\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}-1}\left(\frac{c}{A}-u\right)^{\frac{c}{A}}du$$

$$+\int u\,e^{3u}\,\frac{c}{A}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}-1}du$$

$q = f_1(u) = \frac{1}{1-p} = \frac{1}{1-\frac{1}{u}\frac{du}{dz}}$ $\qquad u = \frac{y}{\theta} \qquad z = \log y$

$$\frac{du}{dz} = y\frac{du}{dy}$$

$$\frac{du}{dz} = u\left[1-\frac{1}{f(u)}\right]$$

$$\left.\int\frac{du}{u\left[1-\frac{1}{f(u)}\right]} = \int dz = \int\frac{dy}{y}\quad\right\}\; u = F(y)$$

$$0 = g x - m - \left(\alpha+\frac{c}{A}\right)\theta + \frac{4}{3}\frac{k}{b}\theta\,\sigma\frac{d\sigma}{dx}$$

$$0 = y - \left(\alpha+\frac{c}{A}\right)\theta + \frac{4}{3}\frac{k}{b}\theta\,\sigma\frac{d\sigma}{dy}$$

$$0 = u - \left(\alpha+\frac{c}{A}\right) + \frac{2}{3}\frac{k}{b}\frac{d}{dy}(\sigma^2)$$

$$\cancel{0 = u - \left(\alpha+\frac{c}{A}\right) + \frac{2}{3}\frac{k}{b}\frac{d(\sigma^2)}{dx}\,u\left[1-\frac{1}{f(u)}\right]}$$

$$\frac{2}{3}\frac{k}{b}\sigma^2 + \text{const} = \left(\alpha+\frac{c}{A}\right)y - \int u\,dy = \left(\alpha+\frac{c}{A}\right)y - \int F(y)\,dy$$

$$\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dx} + \frac{1}{2}\frac{\alpha}{\sigma^2}\frac{d(\sigma^2)}{dx} = \frac{2}{3}\frac{k}{b}\frac{1}{\sigma^2}\left[\frac{d(\sigma^2)}{dx}\right]^2$$

jak mogą być oznaczone 3 stałe przez b, θ_0 ?

Systematyczne Eliminacye:

$$\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dx} + \alpha\frac{1}{\sigma}\frac{d\sigma}{dx} = \frac{4}{3}\frac{\nu}{b}\left(\frac{d\sigma}{dx}\right)^2$$

$$\frac{d}{dx}\left[\frac{4}{3}\frac{\nu}{b}\sigma\theta\frac{d\sigma}{dx}\right]$$

$$g + \left(\alpha+\frac{c}{A}\right)\frac{d\theta}{dx} = \frac{4}{3}\frac{\nu}{b}\left[\sigma\frac{d}{dx}\left(\theta\frac{d\sigma}{dx}\right) + \theta\left(\frac{d\sigma}{dx}\right)^2\right]$$

$$g + \alpha\frac{d\theta}{dx} - \alpha\frac{\theta}{\sigma}\frac{d\sigma}{dx} = \frac{4}{3}\frac{\nu}{b}\sigma\frac{d}{dx}\left(\theta\frac{d\sigma}{dx}\right)$$

$$= \frac{4}{3}\frac{\nu}{b}\sigma\frac{d\sigma}{dx}\frac{d\theta}{dx} + \frac{4}{3}\frac{\nu}{b}\theta\sigma\frac{d^2\sigma}{dx^2}$$

$$\frac{d\sigma}{dx} = v$$

$$\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dx} = \frac{4}{3}\frac{\nu}{b}v^2 - \alpha\frac{v}{\sigma}$$

$$g + \alpha\frac{d\theta}{dx} - \alpha\frac{\theta}{\sigma}v = \frac{4}{3}\frac{\nu}{b}\sigma v\frac{d\theta}{dx} + \frac{4}{3}\frac{\nu}{b}\theta\sigma\frac{dv}{dx}$$

I). $\frac{d\sigma}{dx} = v$

II). $\frac{d\theta}{dx} = \frac{\theta}{\frac{c}{A}}\frac{4}{3}\frac{\nu}{b}v^2 - \frac{\alpha}{\frac{c}{A}}\frac{\theta v}{\sigma}$

$$\frac{4}{3}\frac{\nu}{b}\theta\sigma\frac{dv}{dx} = g - \alpha\frac{\theta v}{\sigma} + \left[\alpha - \frac{4}{3}\frac{\nu}{b}\sigma v\right]\left[\frac{\frac{4}{3}\frac{\nu}{b}}{\frac{c}{A}}\theta v^2 - \frac{\alpha}{\frac{c}{A}}\frac{\theta v}{\sigma}\right]$$

$$= g - \left(\alpha + \frac{\alpha^2}{\frac{c}{A}}\right)\frac{\theta v}{\sigma} + \frac{\frac{4}{3}\frac{\nu}{b}}{\frac{c}{A}}\alpha\,\theta v^2 - \frac{\left(\frac{4}{3}\frac{\nu}{b}\right)^2}{\frac{c}{A}}\theta\sigma v^3$$

III). $\frac{dv}{dx} = \frac{g}{\frac{4}{3}\frac{\nu}{b}}\frac{1}{\theta\sigma} - \frac{\alpha\left(1+\frac{\alpha}{\frac{c}{A}}\right)}{\frac{4}{3}\frac{\nu}{b}}\frac{v}{\sigma^2} + \frac{2\alpha}{\frac{c}{A}}\frac{v^2}{\sigma} - \frac{\left(\frac{4}{3}\frac{\nu}{b}\right)}{\frac{c}{A}}v^3$

$$g x + \underbrace{\left(\alpha - \frac{4}{3}\frac{\eta}{b}\sigma\frac{d\sigma}{dx}\right)}_{=-\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dx}\cdot\frac{\sigma}{\frac{d\sigma}{dx}}}\frac{d\theta}{dx} = \alpha\frac{\theta}{\sigma}\frac{d\sigma}{dx} + \frac{4}{3}\frac{\eta}{b}\theta\sigma\frac{d^2\sigma}{dx^2}$$

$$\cancel{\frac{c}{A}\frac{1}{\theta}\frac{d^2\theta}{dx^2} - \frac{c}{A}\frac{1}{\theta^2}\left(\frac{d\theta}{dx}\right)^2 + \alpha\frac{1}{\sigma}\frac{d^2\sigma}{dx^2} - \frac{\alpha}{\sigma^2}\left(\frac{d\sigma}{dx}\right)^2 - \frac{8}{3}\frac{\eta}{b}\frac{d\sigma}{dx}\frac{d^2\sigma}{dx^2}}$$

$$\frac{c}{A}\theta\frac{d\theta}{dx} + \alpha\frac{\theta}{\sigma}\cdot\theta\frac{d\sigma}{dx} = \frac{4}{3}\frac{\eta}{b}\left(\theta\frac{d\sigma}{dx}\right)^2$$

$$\left(\theta\frac{d\sigma}{dx}\right)^2 - \frac{\alpha}{\frac{4}{3}\frac{\eta}{b}}\frac{\theta}{\sigma}\theta\frac{d\sigma}{dx} = \frac{\frac{c}{A}}{\frac{4}{3}\frac{\eta}{b}}\theta\frac{d\theta}{dx}$$

$$\theta\frac{d\sigma}{dx} = \frac{\alpha}{\frac{8}{3}\frac{\eta}{b}}\frac{\theta}{\sigma} \pm \sqrt{\left(\frac{\alpha}{\frac{8}{3}\frac{\eta}{b}}\right)^2\left(\frac{\theta}{\sigma}\right)^2 + \frac{\frac{c}{A}}{\frac{4}{3}\frac{\eta}{b}}\theta\frac{d\theta}{dx}}$$

$$g + \left(\alpha + \frac{c}{A}\right)\frac{d\theta}{dx} = \frac{d}{dx}\left[\frac{\alpha}{2}\frac{\theta}{\sigma} \pm \sqrt{\frac{\alpha^2}{4}\left(\frac{\theta}{\sigma}\right)^2 + \frac{4}{3}\frac{\eta}{b}\frac{c}{A}\theta\frac{d\theta}{dx}}\right]$$

$$= \frac{\alpha}{2}\frac{1}{\sigma}\frac{d\theta}{dx} - \frac{\alpha}{2}\frac{\theta}{\sigma^2}\frac{d\sigma}{dx} \pm \frac{1}{2}\frac{2\frac{\alpha^2}{4}\left(\frac{\theta}{\sigma^2}\frac{d\theta}{dx} - \frac{\theta^2}{\sigma^3}\frac{d\sigma}{dx}\right)}{\sqrt{\quad}}$$

$$\pm\frac{1}{2}\frac{\frac{4}{3}\frac{\eta}{b}\frac{c}{A}\frac{d}{dx}\left(\theta\frac{d\theta}{dx}\right)}{\sqrt{\quad}}$$

$$\frac{c}{A}\frac{1}{\theta}\frac{d^2\theta}{dx^2} - \frac{c}{A}\frac{1}{\theta^2}\left(\frac{d\theta}{dx}\right)^2 + \cancel{} - \frac{\alpha}{\sigma^2}\left(\frac{d\sigma}{dx}\right)^2 = \left[\frac{8}{3}\frac{\eta}{b}\frac{d\sigma}{dx} - \frac{\alpha}{\sigma}\right]\left[\frac{g + \alpha\frac{d\theta}{dx} - \alpha\frac{\theta}{\sigma}\frac{d\sigma}{dx}}{\frac{4}{3}\frac{\eta}{b}\theta\sigma} - \frac{\frac{c}{A}\frac{d\theta}{dx}}{\theta}\right.$$

$$\left[\frac{8}{3}\frac{\gamma}{b}\frac{d\sigma}{dx}-\frac{\alpha}{\sigma}\right]\frac{d^2\sigma}{dx^2}+\frac{\alpha}{\sigma^2}\left(\frac{d\sigma}{dx}\right)^2=\frac{c}{A}\frac{d}{dx}\left(\frac{1}{\theta}\frac{d\theta}{dx}\right) \quad \Big| \quad \frac{4}{3}\frac{\gamma}{b}\theta\sigma$$

$$\frac{4}{3}\frac{\gamma}{b}\theta\sigma\frac{d^2\sigma}{dx^2}+\sigma\frac{d\sigma}{dx}\frac{d\theta}{dx}\Big]\frac{4}{3}\frac{\gamma}{b}+\alpha\frac{\theta}{\sigma}\frac{d\sigma}{dx}=g+\alpha\frac{d\theta}{dx} \quad \Big| \quad \frac{8}{3}\frac{\gamma}{b}\frac{d\sigma}{dx}-\frac{\alpha}{\sigma}$$

$$-\frac{4}{3}\frac{\gamma}{b}\alpha\frac{\theta}{\sigma}\left(\frac{d\sigma}{dx}\right)^2+2\left(\frac{4}{3}\frac{\gamma}{b}\right)^2\frac{d\theta}{dx}\left(\frac{d\sigma}{dx}\right)^2\sigma-\frac{4}{3}\frac{\gamma}{b}\alpha\frac{d\sigma}{dx}\frac{d\theta}{dx}+\frac{8}{3}\frac{\gamma}{b}\alpha\frac{\theta}{\sigma}\left(\frac{d\sigma}{dx}\right)^2-$$

$$-\alpha^2\frac{\theta}{\sigma^2}\frac{d\sigma}{dx}=\frac{8}{3}\frac{\gamma}{b}g\frac{d\sigma}{dx}-\frac{\alpha g}{\sigma}+\frac{8}{3}\frac{\alpha\gamma}{b}\frac{d\sigma}{dx}\frac{d\theta}{dx}-\frac{\alpha^2}{\sigma}\frac{d\theta}{dx}-\frac{4}{3}\frac{\gamma}{b}\frac{c}{A}\theta\sigma\frac{d}{dx}\left(\frac{1}{\theta}\frac{d\theta}{dx}\right)$$

$$\frac{4}{3}\frac{\gamma\alpha}{b}\frac{\theta}{\sigma}\left(\frac{d\sigma}{dx}\right)^2+2\left(\frac{4}{3}\frac{\gamma}{b}\right)^2\frac{d\theta}{dx}\sigma\left(\frac{d\sigma}{dx}\right)^2-4\frac{\gamma\alpha}{b}\frac{d\sigma}{dx}\frac{d\theta}{dx}-\alpha^2\frac{\theta}{\sigma^2}\frac{d\sigma}{dx}$$

$$=\frac{8}{3}\frac{\gamma g}{b}\frac{d\sigma}{dx}-\frac{\alpha}{\sigma}g-\frac{\alpha^2}{\sigma}\frac{d\theta}{dx}-\frac{4}{3}\frac{\gamma}{b}\frac{c}{A}\theta\sigma\frac{d}{dx}\left(\frac{1}{\theta}\frac{d\theta}{dx}\right)$$

$$\frac{4}{3}\frac{\gamma}{b}\left(\frac{d\sigma}{dx}\right)^2=\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dx}+\alpha\frac{1}{\sigma}\frac{d\sigma}{dx}$$

$$\frac{\alpha c}{A}\frac{1}{\sigma}\frac{d\theta}{dx}+\cancel{\frac{\alpha^2}{\sigma^2}\theta\frac{d\sigma}{dx}}+\frac{4}{3}\frac{\gamma}{b}\frac{d\theta}{dx}\left[\frac{c}{A}\frac{\sigma}{\theta}\frac{d\theta}{dx}+\alpha\frac{d\sigma}{dx}\right]\cancel{-4\frac{\gamma\alpha}{b}\frac{d\sigma}{dx}\frac{d\theta}{dx}-\frac{\alpha^2\theta}{\sigma^2}\frac{d\sigma}{dx}}$$

$$=\frac{8}{3}\frac{\gamma g}{b}\frac{d\sigma}{dx}-\frac{\alpha}{\sigma}g-\frac{\alpha^2}{\sigma}\frac{d\theta}{dx}-\frac{4}{3}\frac{\gamma}{b}\frac{c}{A}\theta\sigma\underbrace{\Big[\quad\Big]}_{\frac{1}{\theta}\frac{d^2\theta}{dx^2}-\cancel{\frac{1}{\theta^2}\left(\frac{d\theta}{dx}\right)^2}}$$

$$\frac{4}{3}\frac{\gamma}{b}\frac{d\sigma}{dx}\left[\alpha\frac{d\theta}{dx}-g\right]+\frac{\alpha g}{\sigma}+\left[\alpha^2+\frac{\alpha c}{A}\right]\frac{1}{\sigma}\frac{d\theta}{dx}+$$

$$+\frac{4}{3}\frac{\gamma}{b}\frac{c}{A}\frac{\sigma}{\theta}\left(\frac{d\theta}{dx}\right)^2+\frac{4}{3}\frac{\gamma}{b}\frac{c}{A}\sigma\frac{d^2\theta}{dx^2}=0$$

$$\frac{2}{3}\frac{\gamma}{b}\frac{d(\sigma^2)}{dx}\left[\alpha\frac{d\theta}{dx}-g\right]+\alpha g+\alpha\left(\alpha+\frac{c}{A}\right)\frac{d\theta}{dx}+\frac{4}{3}\frac{\gamma}{b}\frac{c}{A}\frac{\sigma^2}{\theta}\frac{d}{dx}\left(\theta\frac{d\theta}{dx}\right)=0$$

$$\frac{1}{3}\frac{\gamma}{b}\left[\frac{d(\sigma^2)}{dx}\right]^2-\frac{\alpha}{2}\frac{d(\sigma^2)}{dx}=\frac{c}{A}\frac{\sigma^2}{\theta}\frac{d\theta}{dx} \quad \Big| \quad \frac{4}{3}\frac{\gamma}{b}$$

$$\left[\alpha g + \alpha\left(\alpha + \frac{c}{A}\right)\frac{d\theta}{dx} + \frac{4}{3}\vartheta_b \frac{c}{A}\,\frac{6^2}{\theta}\frac{d}{dx}\left(\theta\frac{d\theta}{dx}\right)\right]^2 - \alpha\left[\alpha\frac{d\theta}{dx} - g\right]\left[\alpha g + \alpha\left(\alpha + \frac{c}{A}\right)\frac{d\theta}{dx} + \frac{4}{3}\ldots\right]$$

$$= \frac{4}{3}\vartheta_b \frac{c}{A}\left[\alpha\frac{d\theta}{dx} - g\right]^2 \frac{6^2}{\theta}\frac{d\theta}{dx}$$

$$\cancel{\left[\quad\right]^2 - \alpha^2\left[\alpha\frac{d\theta}{dx} - g\right]\left[\alpha g + \left(\alpha + \frac{c}{A}\right)\frac{d\theta}{dx}\right] - \alpha\left[\alpha\frac{d\theta}{dx} - g\right]\frac{4}{3}\vartheta_b}$$

$$6^2 = \ldots\ldots$$

$$\left[\frac{6^2}{\theta}\frac{d}{dx}\left(\theta\frac{d\theta}{dx}\right)\frac{4}{3}\vartheta_b\frac{c}{A}\right]^2 + \left[\frac{6^2}{\theta}\frac{d}{dx}\left(\theta\frac{d\theta}{dx}\right)\frac{4}{3}\vartheta_b\frac{c}{A}\right]\left[2\alpha\left[g + \left(\alpha + \frac{c}{A}\right)\frac{d\theta}{dx}\right] - \alpha^2\frac{d\theta}{dx} + \alpha g\right]$$

$$- \frac{4}{3}\vartheta_b\frac{c}{A}\,\frac{6^2}{\theta}\frac{d\theta}{dx}\left[\alpha\frac{d\theta}{dx} - g\right]^2 = \alpha^2\left[\alpha\frac{d\theta}{dx} - g\right]\left[g + \left(\alpha + \frac{c}{A}\right)\frac{d\theta}{dx}\right] -$$

$$- \alpha^2\left[g + \left(\alpha + \frac{c}{A}\right)\frac{d\theta}{dx}\right]^2$$

$$= \alpha^2\left[g + \left(\alpha + \frac{c}{A}\right)\frac{d\theta}{dx}\right]\left[\alpha\frac{d\theta}{dx} - g - g - \left(\alpha + \frac{c}{A}\right)\frac{d\theta}{dx}\right]$$

Wstawiając to 6^2 w równanie otrzymalibyśmy równanie 3 stopnia dla ϑ (nie zawierające już ϑ_b)

To równanie $6^2 = \ldots\ldots$ będzie służyć do oznaczenia stałych

Ówczesny przykład:

Rozkład izotermiczny:

$$\rho = \frac{p}{\alpha\theta}$$

$$\frac{d^2\sigma}{dx^2} = -\frac{\partial}{dx}\left(\frac{\sigma}{\rho}\frac{d\rho}{dx}\right)$$

$$\rho\frac{\partial\sigma}{\partial x} + \sigma\frac{\partial\rho}{\partial x} = 0$$

$$\left\{\begin{aligned} & p = \alpha\theta\rho \\ & 0 = -g - \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{4}{3}\frac{\mu}{\rho}\frac{d^2\sigma}{dx^2} \\ & \rho\sigma = b = \rho_0\sigma_0 = \frac{p_0\sigma_0}{\alpha\theta_0} \qquad \rho = \frac{b}{\sigma} \end{aligned}\right.$$

$$0 = -g - \alpha\theta\frac{1}{\rho}\frac{\partial\rho}{\partial x} + \frac{4}{3}\frac{\mu}{b}\sigma\frac{d^2\sigma}{dx^2}$$

$$0 = -g + \alpha\theta\frac{1}{\sigma}\frac{d\sigma}{dx} + \frac{4}{3}\frac{\mu}{b}\sigma\frac{d^2\sigma}{dx^2}$$

$$\frac{d\sigma}{dx} = u \qquad \frac{d^2\sigma}{dx^2} = \frac{du}{dx} = u\frac{du}{d\sigma} \qquad 2\sigma\frac{d\sigma}{dx} = u \qquad \frac{du}{dx} = 2\left(\frac{d\sigma}{dx}\right)^2 + 2\sigma\frac{d^2\sigma}{dx^2}$$

$$0 = -\frac{g}{\sigma} + \alpha\theta\frac{u}{\sigma^2} + \frac{4}{3}\frac{\mu}{b}u\frac{du}{d\sigma}$$

$$2\sigma\frac{d^2\sigma}{dx^2} = \frac{du}{dx} - \frac{1}{2}\left(\frac{u}{\sigma}\right)^2$$

$$\frac{1}{\sigma}\frac{d\sigma}{dx} = \frac{u}{2\sigma^2} \qquad = \frac{du}{d(\sigma^2)}u - \frac{1}{2}\frac{u^2}{\sigma^2}$$

$$\frac{d\sigma}{du}\left(g - \alpha\theta\frac{u}{\sigma}\right) = \frac{4}{3}\frac{\mu}{b}u\sigma$$

$$\cancel{0 = -g + \alpha\theta\frac{u}{\sigma^2} + \frac{4}{3}\frac{\mu}{b}\left[\frac{du}{dx} - \left(\frac{u}{\sigma}\right)^2\right]}$$

$$0 = -g + \alpha\theta\frac{u}{2} + \frac{4}{3}\frac{\mu}{b}\left[\frac{1}{2}\frac{du}{d(\sigma^2)}u - \frac{1}{4}\frac{u^2}{\sigma^2}\right]$$

$$\frac{2}{3}\frac{\mu}{b}\frac{du}{d\sigma^2} - \frac{1}{3}\frac{\mu}{b}\frac{u}{\sigma^2} + \frac{\alpha\theta}{2} - \frac{g}{u} = 0$$

$$u\sigma = x \qquad \frac{u}{\sigma} = y$$

$$u^2 = xy \qquad \frac{x}{y} = \sigma^2$$

$$u\,d\sigma + \sigma\,du = dx$$

$$-\frac{u}{\sigma^2}d\sigma + \frac{du}{\sigma} = dy \quad \Big| \cdot\sigma^2$$

$$2u\,d\sigma = dx - \sigma^2 dy$$

$$2\sigma\,du = dx + \sigma^2 dy$$

$$\frac{d\sigma}{du} = \frac{1 - \frac{dy}{dx}\frac{x}{y}}{1 + \frac{dy}{dx}\frac{x}{y}}\,\frac{1}{y}$$

$$0 = -\frac{g}{\sigma} + \frac{\alpha\theta}{\sigma^2}u'v + \frac{4}{3}\frac{\mu}{b}u'v\left[u'\frac{dv}{d\sigma} + v\frac{du'}{d\sigma}\right]$$

$$u'v\left[\frac{\alpha\theta}{\sigma^2} + \frac{4}{3}\frac{\mu}{b}u'\frac{dv}{d\sigma}\right.$$

2.

$\sigma^2 = y$

$2\sigma \frac{d\sigma}{dx} = \frac{dy}{dx}$

$2\left(\frac{d\sigma}{dx}\right)^2 + 2\sigma \frac{d^2\sigma}{dx^2} = \frac{d^2y}{dx^2}$

$\sigma \frac{d^2\sigma}{dx^2} = \frac{1}{2}\frac{d^2y}{dx^2} - \frac{1}{4}\frac{1}{\sigma^2}\left(\frac{dy}{dx}\right)^2$

$$0 = g + \frac{\alpha\theta}{2y}\frac{dy}{dx} + \frac{4}{3}\sqrt{\frac{\mu}{b}}\left[\frac{1}{2}\frac{d^2y}{dx^2} - \frac{1}{4y}\left(\frac{dy}{dx}\right)^2\right]$$

$= \cancel{g + \frac{1}{y}\frac{\alpha}{dx}\left[\frac{\theta}{2}y + \frac{4}{3}\right]}$ $\frac{\frac{d}{dx}\left[\frac{1}{y}\left(\frac{dy}{dx}\right)^2\right]}{\frac{dy}{dx}} = y$

$\cancel{= \frac{1}{y}\frac{d^2y}{dx^2} - \frac{1}{y^2}\left(\frac{dy}{dx}\right)^2}$

$\frac{2}{y}\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2$

$$0 = -g + \frac{\alpha\theta}{2}\frac{1}{y}\frac{dy}{dx} + \frac{1}{3}\sqrt{\frac{\mu}{b}}\frac{\frac{d}{dx}\left[\frac{1}{y}\left(\frac{dy}{dx}\right)^2\right]}{\frac{1}{y}\frac{dy}{dx}}$$

$\frac{1}{y}\frac{dy}{dx} = z$

dy

$$0 = -g + \frac{\alpha\theta}{2}z + \frac{1}{3}\sqrt{\frac{\mu}{b}}\frac{\frac{d}{dy}(z^2y)}{z}\,\cancel{z}\,y$$

$$\theta = -\frac{g}{y} + \frac{\alpha\theta}{2}\frac{z}{y} + \frac{1}{3}\sqrt{\frac{\mu}{b}}z^2 + \frac{2}{3}\sqrt{\frac{\mu}{b}}\,y\,z\frac{dz}{dy}$$

$\frac{du}{d\sigma}\frac{4}{3}\sqrt{\frac{\mu}{b}} = -\frac{\theta\alpha}{\sigma^2}$

$\alpha\theta\frac{u'}{\sigma^2} + \frac{4}{3}\sqrt{\frac{\mu}{b}}u'\frac{du'}{d\sigma} = 0$ $\qquad u = u' + v$

$$0 = -\frac{g}{\sigma} + \alpha\theta\left[\frac{u'}{\sigma^2} + \frac{v}{\sigma^2}\right] + \frac{4}{3}\sqrt{\frac{\mu}{b}}\left[u' + v\right]\left(\frac{du'}{d\sigma} + \frac{dv}{d\sigma}\right) = 0$$

$$0 = -\frac{g}{\sigma} + \cancel{\alpha\theta\frac{v}{\sigma^2}} + \frac{4}{3}\sqrt{\frac{\mu}{b}}u'\frac{dv}{d\sigma} + \cancel{\frac{4}{3}\sqrt{\frac{\mu}{b}}v\frac{du'}{d\sigma}} + \frac{4}{3}\sqrt{\frac{\mu}{b}}v\frac{dv}{d\sigma} = 0$$

$uz = t$

$u + z\frac{du}{dz} = \frac{dt}{dz}$

$$0 = -\frac{g}{\sigma} + \frac{4}{3}\sqrt{\frac{\mu}{b}}\frac{dv}{d\sigma}(u' + v)$$

$uz = t$

$u = \frac{1}{tz}$

$\frac{du}{dt} = \frac{1}{t^2 z} - \frac{1}{tz^2}\frac{dz}{dt}$

$\cancel{= -\frac{g}{\sigma} + \frac{4}{3}\sqrt{\frac{\mu}{b}}v\frac{dv}{d\sigma} + \frac{4}{3}\sqrt{\frac{\mu}{b}}\frac{d(u'v)}{d\sigma} + v\frac{\theta\alpha}{\sigma^2}}$

$\frac{1}{\sigma} = z \qquad \frac{dz}{d\sigma} = -\frac{1}{\sigma^2}$

$$0 = -gz + \alpha\theta u z^2 - \frac{4}{3}\sqrt{\frac{\mu}{b}}u\frac{du}{dz}z^2$$

$$\frac{4}{3}\sqrt{\frac{\mu}{b}}\frac{du}{dz} = \alpha\theta - \frac{g}{uz}$$

$u\frac{d}{dz}\left(\frac{4}{3}\sqrt{\frac{\mu}{b}}u - \alpha\theta z\right) + \frac{g}{z}dz = 0$

$$u = \frac{\frac{g}{\sigma}}{\frac{\alpha\theta}{\sigma^2} + \frac{4}{3}\frac{\mu}{\sigma}\, p}$$

$$p = \frac{-\frac{g}{\sigma^2}}{\frac{\alpha\theta}{\sigma^2} + \frac{4}{3}\frac{\mu}{\sigma}\, p} - \frac{\frac{g}{\sigma}\left[\frac{2\alpha\theta}{\sigma^3} + \frac{4}{3}\frac{\mu}{\sigma^2}\frac{dp}{dx}\right]}{\left(\qquad\right)^2}$$

$h = 0 \qquad x = a \qquad y = \left(\alpha + \frac{\varepsilon}{A}\right)\theta - \varepsilon \qquad u = \left(1 + \frac{\varepsilon}{A\alpha}\right) - \delta$

$$= \frac{k}{k-1} - \delta < \frac{7}{2}$$

$$> 0$$

$$\frac{2c}{A\alpha}\left[1+\frac{c}{A\alpha}+\frac{m-gx}{\alpha\theta}\right]=\frac{d}{dx}\left[\frac{\left[1+\frac{c}{A\alpha}+\frac{m-gx}{\alpha\theta}\right]\left[\frac{c}{A\alpha}+\frac{m-gx}{\alpha\theta}\right]}{(\alpha\frac{1}{\theta})\frac{d(\alpha\theta)}{dx}}\right]$$

m przybliżeniu $= ga-(\alpha+\frac{c}{A})\theta_0+z$

$$\left[\;y>(\alpha+\frac{c}{A})\theta_0-z\right.$$

$$\frac{\alpha A}{c}=k-1$$

$$gx-m=y=e^{z}\qquad u=\frac{y}{\alpha\theta}$$

$$\alpha\theta=ue^{z}$$

$$\frac{2c}{A\alpha}\left[1+\frac{c}{A\alpha}-\frac{\cdots}{u}\right]\left(u+\frac{du}{dz}\right)^2=-[\;\cdot\;-\;\cdot$$

$$q=\int\left[2-\frac{1+\frac{c}{A\alpha}}{1+\frac{c}{A\alpha}-u}+\frac{\frac{c}{A\alpha}}{\frac{c}{A\alpha}-u}\right]e^{3u}\left(1+\frac{c}{\alpha A}-u\right)^{1+\frac{c}{A\alpha}}\left(\frac{c}{A\alpha}-u\right)^{\frac{c}{A\alpha}}du$$

$$=\frac{1}{1-p}\qquad=f(u)$$

$$\int\frac{du}{u\left[1-\frac{1}{f(u)}\right]}=\int dz=\int\frac{dy}{y}$$

$$\int u\,e^{u}\,du=2e^{u}$$

$$q_{max}=\int 2e^{3u}\,du=\frac{2}{3}\cancel{2e^{3u}u^2-\frac{4}{3}\int e^{3u}u\,du}$$

$$\cancel{=\frac{4}{9}\left(e^{3u}u-\frac{e^{3u}}{3}\right)}$$

$$\cancel{=e^{3u}\left[\frac{2}{3}u^2-\frac{4}{9}u+\frac{4}{27}\right]=\frac{2}{3}e^{3u}\left[u^2-\frac{2}{3}u+\frac{2}{9}\right]}$$

$$=\frac{2}{3}e^{3u}$$

$$\int\frac{du}{u\left[1-\frac{2}{2}e^{-3u}\right]}$$ dla wielkich wartości u

$$\int\frac{du}{u\left[1-\frac{e^{-u}}{2}\right]}$$ dla małych u

nie ~~[illegible]~~ może się zdarzyć

Uwzględniając wpływ energii kinetycznej ale nie tarcie:

1). $p = \alpha \theta \rho$ $\qquad\qquad$ $\rho\sigma = b$

2). $\sigma \frac{d\sigma}{dx} = -g - \frac{1}{\rho}\frac{\partial p}{\partial x} = -g - \alpha \frac{\partial \theta}{\partial x} + \frac{\alpha\,\theta}{\sigma}\frac{\partial \sigma}{\partial x}$ $\qquad$ $p = \frac{\alpha\,\theta\, b}{\sigma}$

3). $\frac{c}{A\theta}\frac{\partial\theta}{\partial x} + \alpha \frac{1}{\sigma}\frac{\partial\sigma}{\partial x} = 0$

$$\left(\frac{\theta}{\theta_0}\right)\left(\frac{\sigma}{\sigma_0}\right)^{\frac{\alpha A}{c}} = 1 \qquad \frac{\alpha A}{c} = k-1$$

$$\frac{\theta}{\theta_0} = \left(\frac{\sigma}{\sigma_0}\right)^{-(k-1)}$$

2 − θ3):
$$\sigma\frac{d\sigma}{dx} = -g - \left(\alpha + \frac{c}{A}\right)\frac{d\theta}{dx}$$

$$\frac{\sigma^2}{2} + gx + \left(\alpha + \frac{c}{A}\right)\theta = \text{const}$$

$$\frac{\sigma^2 - \sigma_0^2}{2} + g(x-a) + \alpha\frac{k}{k-1}(\theta - \theta_0) = 0$$

$$\frac{\sigma^2 - \sigma_0^2}{2} + g(x-a) + \frac{\alpha k}{k-1}\theta_0 \left[\left(\frac{\sigma}{\sigma_0}\right)^{-(k-1)} - 1\right] = 0$$

$$\frac{\sigma^2 - \sigma_0^2}{2} + g(x-a) + \alpha\frac{k}{k-1}\frac{\theta_0}{\sigma_0^{1-k}}\left[\sigma^{-(k-1)} - \sigma_0^{-(k-1)}\right] = 0$$

2D. $k-1 = \frac{1}{2}$

$$\sigma^2 - \sigma_0^2 + \frac{2k}{k-1}\frac{\alpha\theta_0}{\sigma_0^{1-k}}\left[\frac{1}{\sqrt{\sigma}} - \frac{1}{\sqrt{\sigma_0}}\right] = -g(x-a)$$

$$g(x-a) = \left(\sqrt{6} - \sqrt{6_0}\right) \left[\frac{2k}{k-1} \frac{\alpha \theta_0}{6_0^{1-k}} \frac{1}{\sqrt{6 6_0}} - (6+6_0)(\sqrt{6}+\sqrt{6_0})\right]$$

$$\frac{\alpha \theta_0 b}{6_0} = p_0$$

$$2g(x-a) = \frac{2k}{k-1} \frac{p_0 \, 6_0^{\frac{3}{2}}}{b} \left[\frac{1}{\sqrt{6_0}} - \frac{1}{\sqrt{6}}\right] - (6^2 - 6_0^2)$$

$$= \frac{6 p_0}{b} \, 6_0 \left[1 - \sqrt{\frac{6_0}{6}}\right] - (6^2 - 6_0^2)$$

} nie może nigdy ~~być~~ osiągnąć wartości $\frac{6 p_0 6_0}{b} = 6 \alpha \theta_0$

$$\frac{\partial \theta}{\partial x} = \frac{\theta_0 (1-k)}{6_0^{1-k}} \, 6^{-k} \frac{\partial 6}{\partial x}$$

$$\Big[\underbrace{6 + \left(\alpha + \tfrac{c}{A}\right) \frac{(1-k)\theta_0}{6_0}}_{=0} \left(\frac{6}{6_0}\right)^{-k}\Big] \quad + g \frac{\partial x}{\partial 6} = 0$$

$= 0$

max dla x:

$$\frac{6}{6_0} = \left[k \alpha \theta_0\right]^{\frac{1}{k+1}}$$

$$\theta = \theta_0 \left[k\alpha\theta_0\right]^{\frac{1-k}{1+k}} = \frac{\theta_0}{\left[k\alpha\theta_0\right]^{\frac{k-1}{k+1}}}$$

$$~~6^{k+1} + \left(\alpha + \tfrac{c}{A}\right)(1-k)\,\theta_0 \, 6_0^{-(k+1)} = 0~~$$

$$~~\left(\frac{6}{6_0}\right)^{k+1} = (k-1)\left(\alpha + \tfrac{c}{A}\right)\theta_0 = k \alpha \theta_0~~$$

$$\frac{6}{6_0} = \left[k\alpha\theta_0\right]^{\frac{2}{3}}$$

$$\frac{6_0}{6} = \left[k\alpha\theta_0\right]^{-\frac{2}{3}}$$

$$~~2g(x-a)_{max} = 6 \frac{p_0 6_0}{b} \left[1 - \left[k\alpha\theta_0\right]^{-\frac{1}{3}}\right] - 6_0^2 \left[(k\alpha\theta_0)^{\frac{4}{3}} - 1\right]~~$$

$$~~= 6 \alpha\theta_0 \left[1 - \left[k\alpha\theta_0\right]^{-\frac{1}{3}}\right] - 6_0^2 \left[(k\alpha\theta_0)^{\frac{4}{3}} - 1\right]~~$$

$$~~\boxed{= 6 \alpha \theta_0 + 6_0^2 - 6}~~$$

~~więc~~ tutaj granica jeszcze mniejsza o

$$6 \alpha\theta_0 \left[k\alpha\theta_0\right]^{-\frac{1}{3}} + 6_0^2 [\ ]$$

$$= -6_0^2 + (k\alpha\theta_0)^{-\frac{1}{3}} \left[6\alpha\theta_0 + (k\alpha\theta_0)^{\frac{5}{3}}\right]$$

Ow tego ~~bystry~~ dla $\theta_0 = 0$: $\quad 2g(x-a) = 2\frac{\alpha k}{k-1}\theta_0 = 6 \alpha \theta_0$

Równanie kompletne:

$$\begin{cases} \dfrac{\sigma^2}{2} = \cancel{g x} - g x + m - \left(\alpha + \dfrac{c}{A}\right)\theta + \dfrac{4}{3}\dfrac{\mu}{b}\,\theta\,\sigma\dfrac{d\sigma}{dx} \\ \dfrac{c}{A}\dfrac{1}{\theta}\dfrac{d\theta}{dx} + \dfrac{\alpha}{\sigma}\dfrac{d\sigma}{dx} = \dfrac{4}{3}\dfrac{\mu}{b}\left(\dfrac{d\sigma}{dx}\right)^2 \end{cases}$$

$$\frac{\sigma_0^2}{2} = m - \left(\alpha + \frac{c}{A}\right)\theta_0 + \frac{4}{3}\frac{\mu}{b}\,\theta_0\left(\sigma\frac{d\sigma}{dx}\right)_0$$

$$m = \left(\alpha + \frac{c}{A}\right)\theta_0 + \frac{\sigma_0^2}{2} - \frac{4}{3}\frac{\mu}{b}\,\theta_0\left(\sigma\frac{d\sigma}{dx}\right)_0$$

$$g x - m = y$$

$$\begin{cases} \dfrac{\sigma^2}{2} = -y - \left(\alpha + \dfrac{c}{A}\right)\theta + \dfrac{4}{3}\dfrac{\mu}{b}\,\theta\,\sigma\dfrac{d\sigma}{dy} \\ \dfrac{c}{A}\dfrac{1}{\theta}\dfrac{d\theta}{dy} + \dfrac{\alpha}{\sigma}\dfrac{d\sigma}{dy} = \dfrac{4}{3}\dfrac{\mu}{b}\left(\dfrac{d\sigma}{dy}\right)^2 \end{cases}$$

Zaniedbując σ^2:

$$\frac{4}{3}\frac{\mu}{b}\,\theta\,\sigma\frac{d\sigma}{dy} = y + \left(\alpha + \frac{c}{A}\right)\theta$$

$$\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dy}\,\sigma^2 + \alpha\,\sigma\frac{d\sigma}{dy} = \frac{4}{3}\frac{\mu}{b}\left(\sigma\frac{d\sigma}{dy}\right)^2 \quad \Big|\ \frac{4}{3}\frac{\mu}{b}$$

$$\frac{4}{3}\frac{\mu}{b}\,\sigma^2 = \frac{1}{\dfrac{c}{A}\dfrac{1}{\theta}\dfrac{d\theta}{dy}}\Bigg[\underbrace{-\alpha\left(\frac{y}{\theta} + \alpha + \frac{c}{A}\right) + \left(\frac{y}{\theta} + \alpha + \frac{c}{A}\right)^2}_{=\left(\alpha + \frac{c}{A} + \frac{y}{\theta}\right)\left(\frac{y}{\theta} + \frac{c}{A}\right)}\Bigg]$$

-1

$$2\left[\alpha + \frac{c}{A} + \frac{y}{\theta}\right] = \frac{d}{dy}\left[\frac{\left(\alpha + \frac{c}{A} + \frac{y}{\theta}\right)\left(\frac{c}{A} + \frac{y}{\theta}\right)}{\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dy}}\right]$$

$$\frac{2c}{A\alpha}\left[1+\frac{c}{A\alpha}+\frac{y}{\alpha\theta}\right]=\frac{d}{dy}\left[\frac{\left(1+\frac{c}{A\alpha}+\frac{y}{\alpha\theta}\right)\left(\frac{c}{A\alpha}+\frac{y}{\alpha\theta}\right)}{\frac{1}{\theta}\frac{d\theta}{dy}}\right]$$

$p\theta_0 = 0{,}00018$ [illegible]

$$\frac{c}{A\alpha}=\frac{1}{k-1}$$

$$\frac{2}{k-1}\left[\frac{k}{k-1}+\frac{y}{\alpha\theta}\right]=\frac{d}{dy}\left[\frac{\left(\frac{k}{k-1}+\frac{y}{\alpha\theta}\right)\left(\frac{1}{k-1}+\frac{y}{\alpha\theta}\right)}{\frac{1}{\theta}\frac{d\theta}{dy}}\right]$$

$$(k-1)y=e^z \qquad \frac{(k-1)y}{\alpha\theta}=u$$

$$\alpha\theta=\frac{1}{u}e^z$$

$$\log\theta=\cancel{\,}\,z-\log u$$

$$\frac{1}{\theta}\frac{d\theta}{dz}=1-\frac{1}{u}\frac{du}{dz}=\frac{1}{\theta}\frac{d\theta}{dy}\frac{dy}{dz}$$

$$=e^z\frac{1}{\theta}\frac{d\theta}{dy}$$

$$2(k+u)=\frac{d}{d(e^z)}\left[\frac{(k+u)(1+u)}{\left(1-\frac{1}{u}\frac{du}{dz}\right)e^{-z}}\right]$$

$$=e^{-z}\frac{d}{dz}\left[e^z\frac{(k+u)(1+u)}{\left(1-\frac{1}{u}\frac{du}{dz}\right)}\right]$$

$$(k+u)(1+u)=k+(1+k)u+u^2$$

$$2(k+u)=\frac{d}{dz}\left[\frac{(k+u)(1+u)}{1-\frac{1}{u}\frac{du}{dz}}\right]+\frac{(k+u)(1+u)}{1-\frac{1}{u}\frac{du}{dz}}$$

$$2(k+u)\left(1-\frac{1}{u}\frac{du}{dz}\right)^2=+(k+u)(1+u)\frac{d}{dz}\left(\frac{1}{u}\frac{du}{dz}\right)+\left(1-\frac{1}{u}\frac{du}{dz}\right)(1+k+2u)\frac{du}{dz}$$
$$+\left(1-\frac{1}{u}\frac{du}{dz}\right)(k+u)(1+u)$$

$$\frac{1}{u}\frac{du}{dz}=p$$

$$\cancel{2(k+u)(1-p)^2=(k+u)(1+u)\frac{dp}{dz}+p(1-p)(u+uk+2u^2)+(1-p)(k+u)(1+u)}$$

$$\cancel{(k+u)(1+u)\frac{dp}{dz}}-p^2(u+ku+2u^2 \qquad \frac{dp}{dz}=\frac{dp}{du}\cdot\frac{du}{dz}=p\frac{dp}{du}u$$

$$= u(1+u+k+u)$$

$$2(k+u)p^2 = -(k+u)(1+u)\frac{dp}{dx} + p(1-p)(u+uk+2u^2) + p(k+u)(1+u)$$

$$p\frac{dp}{dx} + \frac{p^2}{u}\left(\frac{2}{1+u} + \underbrace{\frac{u}{1+u} + \frac{u}{k+u}}_{=2-\frac{1}{1+u}-\frac{k}{k+u}}\right) - \frac{p}{u}\left(1 + \frac{u}{1+u} + \frac{u}{k+u}\right) = 0$$

$$\frac{1}{p}\frac{dp}{dx} + \frac{1}{u}\left(2 + \frac{1}{1+u} - \frac{k}{k+u}\right) - \frac{1}{p}\frac{1}{u}\left(3 - \frac{1}{1+u} - \frac{k}{k+u}\right) = 0$$

$$\frac{1}{p} = q$$

$$q\frac{dq}{dx} + \frac{q}{u}\left(3 - \frac{1}{1+u} - \frac{k}{k+u}\right) - \frac{1}{u}\left(2 + \frac{1}{1+u} - \frac{k}{k+u}\right) = 0$$

$$2(k+u)(1-p)^2 = (k+u)(1+u)u\,p\frac{dp}{du} + p(1-p)(u+uk+2u^2) + (1-p)(k+u)(1+u)$$

Jeżeli $\frac{4}{3}\pi b\frac{d\sigma}{dx} = \frac{\alpha}{\sigma}$ to mamy $\frac{d\theta}{dx} = 0$

i $\frac{4}{3}\pi b\frac{d\sigma}{dx}\sigma = \alpha$ więc: $g\,x = (k+\frac{c}{A})\theta_0 - \frac{c}{A}\theta$

więc różniczkując: $g = \frac{4}{3}\pi b\,\theta\,\frac{d}{dx}\left(\sigma\frac{d\sigma}{dx}\right)\Big|_{x=\dots}$

Gdy x większe niż $gx = (k+\frac{c}{A})\theta_0 - \frac{c}{A}\theta$

to $\frac{4}{3}\pi b\frac{d\sigma}{dx} > \frac{\alpha}{\sigma}$ zatem $\frac{d\theta}{dx} > 0$!

$$(u^2+4u+3k)$$

$$(k+u)(1+u)u\,p\frac{dp}{du} + p^2\left[u \mp uk \mp \cancel{2u^2} - 2k - \cancel{2u^2}\right] + p\left[u + uk + 2u^2 + 4k + 4u - k - ku - u - u^2\right] + \underbrace{k + u + ku + u^2 - 2k - 2u}_{u^2+ku-u-k}$$

$$-p^2\left[4u^2 - u(k+1) - 2k\right]$$

Tam gdzie $\frac{d\theta}{dx}=0$:

$\frac{4}{3}\frac{\varkappa}{b}\frac{d\sigma}{dx}=\frac{\alpha}{\sigma}$ więc:

$$\frac{\sigma^2}{2}=-y-\left(\alpha+\frac{c}{A}\right)\theta+\alpha\theta=-y-\frac{c}{A}\theta$$

Zaniedbując multiplicacyjny $=-gx+m-\frac{c}{A}\theta$

$$gx=\left(\alpha+\frac{c}{A}\right)\theta_0-\frac{c}{A}\theta$$

$$0=-gx+\left(\alpha+\frac{c}{A}\right)(\theta_0-\theta)+\frac{4}{3}\frac{\varkappa}{b}\,\theta\,\sigma\frac{d\sigma}{dx} \qquad \text{I).}$$

$$\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dx}+\frac{\alpha}{\sigma}\frac{d\sigma}{dx}=\frac{4}{3}\frac{\varkappa}{b}\left(\frac{d\sigma}{dx}\right)^2 \qquad \text{II).}$$

Najpierw $\frac{d\theta}{dx}<0$, zatem z II: $\frac{d\sigma}{dx}>0$

$\left[\frac{4}{3}\frac{\varkappa}{b}\frac{d\sigma}{dx}<\frac{\alpha}{\sigma}\right.$

~~Gdy $gx=\left(\alpha+\frac{c}{A}\right)(\theta_0-\theta)$ to z II). $\frac{d\sigma}{dx}>0$~~

ale z I): θ wciąż większe aniżeli gdyby $\sigma=0$ było

wtedy ono by się stało $=0$ przy $gx=\left(\alpha+\frac{c}{A}\right)\theta_0$

~~Gdy $gx<\left(\alpha+\frac{c}{A}\right)(\theta_0-\theta)$ to z I). : $\frac{d\sigma}{dx}=0$~~ nie możliwe

tutaj jednak przy $gx=\left(\alpha+\frac{c}{A}\right)\theta_0$ mamy

I). $\alpha+\frac{c}{A}=\frac{4}{3}\frac{\varkappa}{b}\,\sigma\frac{d\sigma}{dx}$ z czego wynika:

$$\left(\alpha+\frac{c}{A}\right)\frac{1}{\sigma}\frac{d\sigma}{dx}=\frac{4}{3}\frac{\varkappa}{b}\left(\frac{d\sigma}{dx}\right)^2=\frac{\alpha}{\sigma}\frac{d\sigma}{dx}+\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dx}$$

więc $\frac{1}{\sigma}\frac{d\sigma}{dx}=\frac{1}{\theta}\frac{d\theta}{dx}$ ~~nie możliwe~~

więc $\theta=0$?!

$$y\frac{dy}{dx} + yX_1 + X_2 = 0$$

$$\frac{1}{2}\frac{dz}{dx} + X_1\sqrt{z} + X_2 = 0$$

$$\frac{y^2}{2} + \int yX_1\,dx + \int X_2\,dx = 0$$

$$y + \int X_1\,dx + \int X_2\frac{dx}{y} = 0$$

$$\cancel{\frac{y^2}{2} + \int X_2\,dx + y\int X_1\,dx - \int dy\int X_1\,dx}$$

$$\cancel{= -\int dx\int X_1\,dx\left[X_1 + \frac{X_2}{y}\right]}$$

$$\cancel{= +\int X_1\int X_1\,dx\,dx + \int\frac{X_2}{y}\,dx\int X_1\,dx}$$

$$y = u + v$$

$$(u+v)\left(\frac{du}{dx} + \frac{dv}{dx}\right) + (u+v)X_1 + X_2 = 0$$

$$u\frac{du}{dx} + u\frac{dv}{dx} + v\frac{du}{dx} + v\frac{dv}{dx} + uX_1 + vX_1 + X_2 = 0$$

$$y = u + a$$

$$(y+a)\frac{dy}{dx} + (y+a)X_1 + X_2 = 0$$

$$y\frac{dy}{dx} + yX_1 + X_2 + a\left(\frac{dy}{dx} + X_1\right) = 0$$

$$y_1 \frac{dy_1}{dx} + y_1 X_1 \qquad = 0$$

$$\cancel{\frac{1}{2} \frac{d(y^2 - y_1^2)}{dx} + (y - y_1) X_1 + X_2 = 0}$$

$$\cancel{\frac{1}{2} \frac{d}{dx}\left[(y - y_1)(y + y_1)\right] +}$$

$$\cancel{(y - y_1)\left[\frac{1}{2} \frac{d}{dx}(y + y_1) + X_1\right] + \frac{1}{2}(y + y_1) \frac{d(y - y_1)}{dx} + X_2 = 0}$$

$$\cancel{(y - y_1) \frac{dy}{dx} + (y + y_1) \frac{d(y - y_1)}{dx} + 2X_2 = 0}$$

$$\cancel{y \frac{dy}{dx} - y \frac{dy_1}{dx} + y_1 \frac{dy}{dx} - y_1 \frac{dy_1}{dx}} + 2X_2 + y_1 X_1$$

$$\cancel{2 \frac{dy}{dx} y - y \frac{dy_1}{dx} + y_1 X_1 + 2X_2 = 0}$$

$$y_1 y \frac{dy}{dx} - y y_1 \frac{dy_1}{dx} + X_2 = 0$$

$$y y_1 \frac{d(y - y_1)}{dx} + X_2 = 0$$

$$(y - y_1 + y_1) y_1 \frac{d(y - y_1)}{dx} + X_2 = 0$$

$$(y - y_1) \frac{d(y - y_1)}{dx} + y_1^2 \frac{d(y - y_1)}{dx} + X_2 = 0$$

$$u \frac{du}{dx} + y_1^2 \frac{du}{dx} + X_2 = 0$$

$$\frac{dy}{dx} + X_1^E + \frac{1}{y} X_2^E = 0$$

$$\frac{d^2y}{dx^2} + X_1' + \frac{X_2'}{y} - \frac{X_2}{y^2}\frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} + X_1' + \frac{X_2'}{y} + \frac{X_1 X_2}{y^2} + \frac{X_2^2}{y^3} = 0$$

$$y = \frac{-X_2}{\frac{dy}{dx} + X_1} \qquad \frac{dy}{dx} = \frac{-X_2'}{\frac{dy}{dx} + X_1} + X_2 \frac{\frac{d^2y}{dx^2} + X_1'}{\left(\frac{dy}{dx} + X_1\right)^2}$$

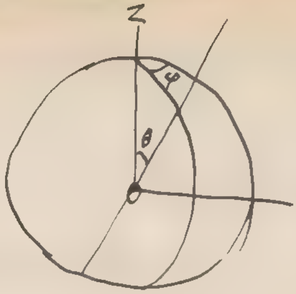
$$\frac{dy}{dx}\left(\frac{dy}{dx} + X_1\right)^2 = -X_2'\left(\frac{dy}{dx} + X_1\right) + X_2\left(\frac{d^2y}{dx^2} + X_1'\right)$$

$$p\,(p + X_1)^2 = -X_2'(p + X_1) + X_2\left(\frac{dp}{dx} + X_1'\right)$$

$$\frac{dp}{dx} + f_1(x)\,p^3 + f_2(x)\,p^2 + f_3(x)\,p + f_4(x) = 0$$

Spherical symmetry :

$\cancel{u = \sigma \cos\alpha = \frac{\sigma x}{r}}$



$x = r \sin\theta \cos\varphi$

$y = r \sin\theta \sin\varphi$

$z = r \cos\theta$

$u = \sigma \sin\theta \cos\varphi$

$v = \sigma \sin\theta \sin\varphi$

$w = \sigma \cos\theta$

$$\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = \frac{\partial \sigma}{\partial r}\left[\sin^2\theta \cos^2\varphi + \sin^2\theta \sin^2\varphi + \cos^2\theta\right]$$

$$+ \frac{\sigma}{r}\left[\cos^2\theta \cos^2\varphi + \cos^2\theta \sin^2\varphi + \sin^2\theta\right]$$

$$+ \frac{\sigma}{r}\left[\frac{+\sin\theta \sin^2\varphi}{\sin\theta} + \frac{\cos^2\varphi \sin\theta}{\sin\theta}\right]$$

$$= \frac{\partial \sigma}{\partial r} + \frac{2\sigma}{r}$$

$$\nabla^2 u = \frac{\partial^2 \sigma}{\partial r^2} \sin\theta \cos\varphi + \frac{2}{r}\frac{\partial \sigma}{\partial r} \sin\theta \cos\varphi + \frac{\sigma \cancel{\sin\theta}}{r^2 \sin^2\theta} \cos\varphi - \frac{\sigma}{r^2} \sin\theta \cos\varphi + \frac{\sigma \cos^2\theta \cos\varphi}{r^2 \sin\theta}$$

$$= \left[\frac{\partial^2 \sigma}{\partial r^2} + \frac{2}{r}\frac{\partial \sigma}{\partial r} - \frac{2\sigma}{r^2}\right] \sin\theta \cos\varphi$$

$$\nabla^2 w = \left[\frac{\partial^2 \sigma}{\partial r^2} + \frac{2}{r}\frac{\partial \sigma}{\partial r} - \frac{2\sigma}{r^2}\right] \cos\theta$$

$$\left(\frac{\partial u}{\partial x}\right) = \frac{\partial \sigma}{\partial r}\sin^2\theta\cos^2\varphi + \frac{\sigma}{r}\left(\cos^2\theta\cos^2\varphi + \sin^2\varphi\right)$$

$$= \left(\frac{\partial \sigma}{\partial r} - \frac{\sigma}{r}\right)\sin^2\theta\cos^2\varphi + \frac{\sigma}{r}$$

$$\frac{\partial u}{\partial y} = \frac{\partial \sigma}{\partial r}\sin^2\theta\sin\varphi\cos\varphi + \frac{\sigma}{r}\left(\cos^2\theta\sin\varphi\cos\varphi - \frac{\cos\varphi}{\cancel{\sin\theta}}\sin\varphi\right)$$

$$= \left(\frac{\partial \sigma}{\partial r} - \frac{\sigma}{r}\right)\sin^2\theta\sin\varphi\cos\varphi$$

$$\frac{\partial u}{\partial z} = \left(\frac{\partial \sigma}{\partial r} - \frac{\sigma}{r}\right)\sin\theta\cos\theta\cos\varphi$$

$$\frac{\partial v}{\partial x} = \frac{\partial \sigma}{\partial r}\sin^2\theta\sin\varphi\cos\varphi + \frac{\sigma}{r}\cos^2\theta\sin\varphi\cos\varphi - \frac{\sigma\sin\varphi\cos\varphi}{r\,\cancel{\sin\theta}}$$

$$= \left(\frac{\partial \sigma}{\partial r} - \frac{\sigma}{r}\right)\sin^2\theta\sin\varphi\cos\varphi$$

$$\frac{\partial v}{\partial y} = \frac{\partial \sigma}{\partial r}\sin^2\theta\sin^2\varphi + \frac{\sigma}{r}\left(\cos^2\theta\sin^2\varphi + \cos^2\varphi\right)$$

$$= \left(\frac{\partial \sigma}{\partial r} - \frac{\sigma}{r}\right)\sin^2\theta\sin^2\varphi + \frac{\sigma}{r}$$

$$\frac{\partial v}{\partial z} = \left(\frac{\partial \sigma}{\partial r} - \frac{\sigma}{r}\right)\sin\theta\cos\theta\sin\varphi$$

$$\frac{\partial w}{\partial x} = \left(\frac{\partial \sigma}{\partial r} - \frac{\sigma}{r}\right)\sin\theta\cos\theta\cos\varphi$$

$$\frac{\partial w}{\partial y} = \left(\frac{\partial \sigma}{\partial r} - \frac{\sigma}{r}\right)\sin\theta\cos\theta\sin\varphi$$

$$\frac{\partial w}{\partial z} = \frac{\partial \sigma}{\partial r}\cos^2\theta + \frac{\sigma}{r}\sin^2\theta = \left(\frac{\partial \sigma}{\partial r} - \frac{\sigma}{r}\right)\cos^2\theta + \frac{\sigma}{r}$$

$$\cancel{1-2\sin^2\varphi+\sin^4\varphi}$$

$$2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]=2\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)^2\left[\sin^4\theta\cos^4\varphi+\sin^4\theta\sin^4\varphi+\cos^4\theta\right]+$$

$$4\frac{\sigma}{r}\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)\left[\sin^2\theta\cos^2\varphi+\sin^2\theta\sin^2\varphi+\cos^2\theta\right]+6\left(\frac{\sigma}{r}\right)^2$$

$$\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)^2=4\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)^2\sin^2\theta\cos^2\theta\sin^2\varphi$$

$$\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2=4\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)^2\sin^2\theta\cos^2\theta\cos^2\varphi$$

$$\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2=4\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)^2\sin^4\theta\sin^2\varphi\cos^2\varphi$$

$$=2\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)^2\left[\cancel{\sin^4\theta\cos^4\varphi+\sin^4\theta\sin^4\varphi+\cos^4\theta}+\cancel{\sin^2\theta\cos^2\theta}+\right.$$

$$\left.+\cancel{\sin^4\theta\sin^2\varphi\cos^2\varphi}\right]$$

$$\cancel{+4\frac{\sigma}{r}\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)[\;]+6\left(\frac{\sigma}{r}\right)^2}$$

$$\cancel{1-2\sin^2\theta+\sin^4\theta}\qquad\cancel{-\cos^2\theta\sin^2\varphi}$$

$$\cancel{\sin^2\theta\left(\cos^2\varphi-\cos^2\varphi\cos^2\theta+\sin^2\theta\sin^2\varphi-\sin^2\theta\sin^2\varphi\cos^2\varphi\right)+\cos^2\theta}$$

$$1\quad-\cos^2\theta$$

$$=\cancel{\sin^4\theta\left[1-\sin^2\varphi\cos^2\varphi\right]=\sin^4\theta\left[1-\sin^2\varphi+\sin^4\varphi\right]}$$

$$=2\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)^2\left[\cancel{\sin^4\theta\cos^4\varphi+\sin^4\theta\sin^4\varphi+\cos^4\theta+2\sin^2\theta\cos^2\theta+2\sin^4\theta\sin^2\varphi\cos^2\varphi}\right]$$

$$\left.\cancel{}\right]+6\left(\frac{\sigma}{r}\right)^2$$

$$+4\frac{\sigma}{r}\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)\left[\cancel{\sin^2\theta\cos^2\varphi+\sin^2\theta\sin^2\varphi+\cos^2\theta}\right]$$

$$\Phi=2\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)^2+4\frac{\sigma}{r}\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)+6\left(\frac{\sigma}{r}\right)^2-\frac{2}{3}\left(\frac{\partial\sigma}{\partial r}+\frac{2\sigma}{r}\right)^2=\frac{4}{3}\left[\frac{d\sigma}{dr}-\frac{\sigma}{r}\right]^2$$

$+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}$

$$u\frac{\partial u}{\partial x} = -\frac{g a^2 x}{r^2 r} - \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{3\rho}\frac{\partial}{\partial x}(div) + \frac{\mu}{\rho}\nabla^2 u$$

$$u\frac{\partial u}{\partial r} = -\frac{g a^2}{r^2} - \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu}{3\rho}\partial \ldots$$

$$\cancel{\sigma\frac{\partial\sigma}{\partial r}\sin\theta\cos\varphi} = -\frac{g a^2}{r^2}\cancel{\sin^2\theta\ldots} - \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu}{3\rho}\frac{\partial}{\partial r}\left[\frac{\partial\sigma}{\partial r}+\frac{2\sigma}{r}\right] + \frac{\mu}{\rho}\left[\frac{\partial^2\sigma}{\partial r^2}+\frac{2}{r}\frac{\partial\sigma}{\partial r}-\frac{2\sigma}{r^2}\right] \sin\theta\cos\varphi$$

$$\cancel{\sigma\frac{\partial\sigma}{\partial r}\sin\theta\sin\varphi = -\frac{g a^2}{r^2}\sin\theta\sin\varphi - \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu}{3\rho}\frac{\partial}{\partial r}[\quad] + \frac{\mu}{\rho}[\quad]\sin\theta\ldots}$$

$$\cancel{\sigma\frac{\partial\sigma}{\partial r}\sin\theta\sin\varphi + \sin\theta\sin\varphi}$$

$$\sigma\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)\left(\sin\theta\cos\varphi + \cancel{\sin^3\theta\sin^2\varphi\cos\varphi} + \cancel{\sin\theta\cos^2\theta\cos\varphi}\right) + \frac{\sigma}{r}\sin\theta\cos\varphi =$$

$$= \left[\sigma\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)+\frac{\sigma^2}{r}\right]\sin\theta\cos\varphi =$$

$$\left[\sigma\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)+\frac{\sigma^2}{r}\right]\sin\theta\sin\varphi =$$

$$\cancel{\sigma\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)\sin^2\theta\cos\theta\cos^2\varphi + \sin^2\theta\cos\theta\sin^2\varphi + \cos^3\theta}$$

$$\left[\sigma\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)+\frac{\sigma^2}{r}\right]\cos\theta =$$

$\cancel{\sin^2\theta\cos\varphi}$

$\cancel{\sin\theta\sin\varphi}$

$\sin\theta\cos$

$$\underbrace{\left[\sigma\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)+\frac{\sigma^2}{r}\right]}_{=\sigma\frac{\partial\sigma}{\partial r}} = -\frac{g a^2}{r^2} - \frac{1}{\rho}\frac{\partial p}{\partial r} + \underbrace{\frac{\mu}{3\rho}\frac{\partial}{\partial r}\left[\frac{\partial\sigma}{\partial r}+\frac{2\sigma}{r}\right] + \frac{\mu}{\rho}\left[\frac{\partial^2\sigma}{\partial r^2}+\frac{2}{r}\frac{\partial\sigma}{\partial r}-\frac{2\sigma}{r^2}\right]}$$

$$\frac{\partial^2\sigma}{\partial r^2}+\frac{2}{r}\frac{\partial\sigma}{\partial r}-\frac{2\sigma}{r^2}+3\frac{\partial^2\sigma}{\partial r^2}+\frac{6}{r}\frac{\partial\sigma}{\partial r}-\frac{6\sigma}{r^2}$$

$$4\frac{\partial^2\sigma}{\partial r^2}+\frac{8}{r}\frac{\partial\sigma}{\partial r}-\frac{8\sigma}{r^2}$$

$$=\frac{4}{3}\frac{\partial}{\partial r}\left[\frac{\partial\sigma}{\partial r}+\frac{2\sigma}{r}\right]$$

$$\sigma \frac{\partial\sigma}{\partial r} = -g\frac{a^2}{r^2} - \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{4\mu}{3\rho}\frac{\partial}{\partial r}\left[\frac{\partial\sigma}{\partial r} + \frac{2\sigma}{r}\right]$$

$$\frac{c}{A}\rho\,\sigma\frac{\partial\theta}{\partial r} + \left(\frac{\partial\sigma}{\partial r} + \frac{2\sigma}{r}\right)p = \frac{4\mu}{3}\left[\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right]^2 \quad \Big|\; r^2$$

$$p = \alpha\rho\theta \qquad \mu = \gamma\theta$$

$$\rho\sigma r^2 = \rho_0\sigma_0 a^2 = b$$

$$\rho = \frac{b}{\sigma r^2} \quad \Big| \quad p = \cancel{\alpha\theta}\,\alpha b\,\frac{\theta}{\sigma r^2}$$

$$\left.\begin{aligned}\cancel{\sigma\frac{\partial\sigma}{}}\ \sigma\frac{d\sigma}{dr} &= -\frac{ga^2}{r^2} - \alpha\sigma r^2\frac{d}{dr}\left(\frac{\theta}{\sigma r^2}\right) + \frac{4\gamma\theta}{3b}\sigma r^2\frac{d}{dr}\left[\frac{d\sigma}{dr} + \frac{2\sigma}{r}\right]\\ \frac{c}{A}\frac{\partial\theta}{\partial r} + \alpha\frac{\theta}{\sigma r^2}\left(\frac{d\sigma}{dr} + \frac{2\sigma}{r}\right) &= \frac{4\gamma\theta}{3b}\left[\frac{d\sigma}{dr} - \frac{\sigma}{r}\right]^2\end{aligned}\right\}$$

$$\cancel{\frac{\partial}{\partial r}\left\{\sigma r^2\left[\frac{d\sigma}{dr} + \frac{2\sigma}{r}\right]\right\} = \left[\frac{d\sigma}{dr} + \frac{2\sigma}{r}\right]^2 + \sigma r^2\left[\frac{d^2\sigma}{dr^2} + \frac{2}{r}\frac{d\sigma}{dr} - \frac{2\sigma}{r^2}\right]}$$

$$\underline{\partial p_{xx} + \cdots} = -\frac{2}{3}\frac{\partial}{\partial x}(\mu\,\mathrm{div}) + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)\right]$$

$$+ 2\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)$$

$$= -\frac{2}{3}\frac{\partial\mu}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) - \frac{2}{3}\mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x\partial y} + \frac{\partial^2 w}{\partial x\partial z}\right) + \frac{\partial\mu}{\partial y}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) +$$

$$+ \mu\left(\frac{\partial^2 v}{\partial x\partial y} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{\partial\mu}{\partial z}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) + \mu\left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial z\partial x}\right) + 2\frac{\partial\mu}{\partial x}\frac{\partial u}{\partial x} + 2\mu\frac{\partial^2 u}{\partial x^2}$$

$$= \mu\left[-\frac{2}{3}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x\partial y} + \frac{\partial^2 w}{\partial x\partial z}\right) + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x\partial y} + \frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x\partial z} + 2\frac{\partial^2 u}{\partial x^2}\right]$$

$$+ \frac{\partial\mu}{\partial x}\left[-\frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + 2\frac{\partial u}{\partial x}\right] + \frac{\partial\mu}{\partial y}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) + \frac{\partial\mu}{\partial z}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)$$

$$= -\frac{2}{3}\frac{d}{dx}\left[\mu\left(\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}\right)\right] + \frac{d}{dx}\left(\mu\frac{du}{dx}\right)+\frac{d}{dy}\left(\mu\frac{du}{dy}\right)+\frac{d}{dz}\left(\mu\frac{du}{dz}\right)+$$

$$+\frac{d}{dx}\left(\mu\frac{du}{dx}\right)+\frac{d}{dy}\left(\mu\frac{dv}{dx}\right)+\frac{d}{dz}\left(\mu\frac{dw}{dx}\right)$$

$$-\frac{2}{3}\frac{\partial}{\partial x}\left[\mu\left(\frac{d\sigma}{dr}+\frac{2\sigma}{r}\right)\right] + 2\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)\sin^2\theta\cos^2\varphi+\frac{\sigma\mu}{r}\right] + 2\frac{\partial}{\partial y}\left[\mu(\quad)\ldots\right]+2\frac{\partial}{\partial z}[\mu\,,$$

$$= -\frac{2}{3}\frac{d}{dr}\left[\mu\left(\frac{d\sigma}{dr}+\frac{2\sigma}{r}\right)\right]\sin\theta\cos\varphi +$$

$$2\frac{d}{dr}\left[\mu\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\right]\left[\sin^3\theta\cos^3\varphi + \underbrace{\sin^3\theta\sin^2\varphi\cos\varphi + \sin\theta\cos^2\theta\cos\varphi}_{\sin\theta\cos\varphi}\right]$$

$$+ 2\frac{d}{dr}\left(\mu\frac{\sigma}{r}\right)\sin\theta\cos\varphi$$

$$= \frac{d}{dr}\left[-\frac{2}{3}\mu\left(\frac{d\sigma}{dr}+\frac{2\sigma}{r}\right)+2\mu\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)+2\mu\frac{\sigma}{r}\right]\sin\theta\cos\varphi$$

$$= \frac{d}{dr}\left[\mu\quad -\frac{2}{3}\frac{d\sigma}{dr}-\frac{4}{3}\frac{\sigma}{r}+2\frac{d\sigma}{dr}-\frac{2\sigma}{r}+\frac{2\sigma}{r}\right]$$

$$= \frac{d}{dr}\left[\mu\,\frac{4}{3}\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\right] = \frac{4}{3}\frac{d}{dr}\left[\mu\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\right]$$

$$+\frac{d}{d\varphi}\qquad\frac{d\varphi}{dx}\qquad+\frac{d}{d\theta}\frac{d\theta}{dx}$$

$$+\frac{d}{dr}\left[-\frac{2}{3}\mu\frac{d\sigma}{dr}+\frac{2}{3}\mu\frac{\sigma}{r}\right]$$

$$+\frac{d}{dr}2\left[\mu\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\right]+\ldots\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)$$

$$+\frac{4}{3}\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\frac{d\mu}{dr}$$

$$= \frac{d}{dr}\left[\frac{4}{3}\mu\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\right]+\frac{4\mu}{r}\left[\frac{d\sigma}{dr}-\frac{\sigma}{r}\right] = \frac{4\mu}{3}\left[\frac{d^2\sigma}{dr^2}-\frac{1}{r}\frac{d\sigma}{dr}+\frac{\sigma}{r^2}+\frac{3}{r}\frac{d\sigma}{dr}-\frac{3\sigma}{r^2}\right]+$$

$$= \frac{4}{3}\mu\left[\frac{d^2\sigma}{dr^2}+\frac{2}{r}\frac{d\sigma}{dr}-\frac{2\sigma}{r^2}\right]$$

$\nabla^4 + \frac{d}{dx}(\mathrm{div}) = \frac{d}{dr}\left(\frac{d\sigma}{dr}\right) - \cdots$

$\sin\theta\cos\varphi\left[\frac{d^2\sigma}{dr^2} + \frac{2}{r}\frac{d\sigma}{dr} - \frac{2\sigma}{r^2} + \frac{d}{dr}\left(\frac{d\sigma}{dr} + \frac{2\sigma}{r}\right)\right]$

$\frac{d^2\sigma}{dr^2} + \frac{2}{r}\frac{d\sigma}{dr} - \frac{2\sigma}{r^2}$

$\nabla^2 u = \frac{\partial}{\partial x}\frac{\partial u}{\partial x} + \frac{\partial}{\partial y}\frac{\partial u}{\partial y} + \frac{\partial}{\partial z}\frac{\partial u}{\partial z}$

$= \frac{d}{dr}\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)\sin^3\theta\cos^3\varphi + \frac{d}{dr}\left(\frac{\sigma}{r}\right)\sin\theta\cos\varphi$

$+ \quad \cdots \quad \cancel{\sin^3\theta\sin^2\varphi\cos\varphi}$

$\sin\theta\cos^2\theta\cos\varphi$

$= \sin\theta\cos\varphi\left[\frac{d}{dr}\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right) + \frac{d}{dr}\left(\frac{\sigma}{r}\right)\right]$

$= \sin\theta\cos\varphi\,\frac{d^2\sigma}{dr^2}$

$\mu\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)\sin^3\theta\cos^3\varphi +$

$\mu\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)\sin^2\theta\sin^2\varphi\cos\varphi$ $\quad \left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)\sin\theta\cos^2\theta\cos\varphi$

~~$d\sigma$~~

$= \frac{d}{dx}\left[-\frac{2}{3}\mu\left(\frac{d\sigma}{dr} + \frac{2\sigma}{r}\right)\right] + 2\left[\frac{d}{dx}\left(\mu\frac{du}{dx}\right) + \frac{d}{dy}\left(\mu\frac{du}{dy}\right) + \cdots \frac{d}{dz}\left(\mu\frac{du}{dz}\right)\right]$

$+ 2\mu\frac{\sigma}{r}$

$\frac{d}{dr}\left[\mu\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)\right]\sin^3\theta\cos^3\varphi + \mu\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)\frac{2\sin\theta\cos^2\theta\cos^3\varphi + 2\sin\theta\cos\varphi\sin^2\varphi}{r}$

$\cancel{\sin^3\theta\sin^2\varphi\cos\varphi}$ $\quad 2\sin\theta\cos^2\theta\sin^2\varphi\cos\varphi + \sin\theta\cos^3\varphi - \sin\theta\sin^2\varphi\cos\varphi$

$\cancel{\sin\theta\cos^2\theta\cos\varphi}$ $\quad - \sin\theta\cos\varphi(\cos^2\theta - \sin^2\theta)$

$= \sin\theta\left[2\cos^2\theta\cos\varphi + \cos\varphi - \cos^2\theta\cos\varphi + \sin^2\theta\cos\varphi\right]$

$= 2\sin\theta\cos\varphi$

$$\sigma \frac{d\sigma}{dr} = -g\frac{a^2}{r^2} - \alpha\sigma r^2 \frac{d}{dr}\left(\frac{\theta}{\sigma r^2}\right) + \frac{4\mu\theta}{3b}\sigma r^2 \frac{d}{dr}\left[\frac{d\sigma}{dr} + \frac{2\sigma}{r}\right] +$$

$$+ \frac{4\mu}{3b}\sigma r^2\left[\frac{d\sigma}{dr} - \frac{\sigma}{r}\right]\frac{d\theta}{dr}$$

$$\frac{c}{A}\frac{d\theta}{dr} + \frac{\alpha\theta}{\sigma}\left[\frac{d\sigma}{dr} + \frac{2\sigma}{r}\right] = \frac{4\mu\theta}{3b}r^2\left[\frac{d\sigma}{dr} - \frac{\sigma}{r}\right]^2$$

$$\frac{\sigma d\sigma}{dr}\cancel{\theta} = -g\frac{a^2}{r^2} - \alpha\frac{d\theta}{dr} + \frac{\alpha\theta}{\sigma}\left[\frac{d\sigma}{dr} + 2\frac{\sigma}{r}\right] + \frac{4\mu\theta}{3b}\dots$$

$$= \frac{d}{dr}\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right) + \frac{3}{r}\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)$$

$$\frac{\sigma d\sigma}{dr} = -\frac{ga^2}{r^2} - \left(\alpha + \frac{c}{A}\right)\frac{d\theta}{dr} + \frac{4\mu}{3b}\Big\{\theta\sigma r^2 \frac{d}{dr}\left[\frac{d\sigma}{dr} + \frac{2\sigma}{r}\right] + \sigma\frac{d\theta}{dr}r^2\left[\frac{d\sigma}{dr} - \frac{\sigma}{r}\right]$$

$$+ r^2\theta\left[\frac{d\sigma}{dr} - \frac{\sigma}{r}\right]^2\Big\}$$

$$\theta\left[r^2\sigma\frac{d^2\sigma}{dr^2} + \cancel{2r\sigma\frac{d\sigma}{dr}} - 2\sigma^2 + r^2\left(\frac{d\sigma}{dr}\right)^2 + \sigma^2 - \cancel{2\sigma r\frac{d\sigma}{dr}}\right]$$

$$= \theta\left[r^2\sigma\frac{d^2\sigma}{dr^2} + r^2\left(\frac{d\sigma}{dr}\right)^2 - \sigma^2\right]$$

$$= \frac{d}{dr}\left[\sigma r^2\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)\right] = \left(\frac{d\sigma}{dr}\right)^2 r^2 - \cancel{\sigma r\frac{d\sigma}{dr}} + \cancel{2r\sigma\frac{d\sigma}{dr}} - 2\sigma^2 + \sigma r^2\frac{d^2\sigma}{dr^2} - \cancel{\sigma r\frac{d\sigma}{dr}} + \sigma^2$$

$$\boxed{\begin{cases} \sigma\dfrac{d\sigma}{dr} = -g\dfrac{a^2}{r^2} - \left(\alpha + \dfrac{c}{A}\right)\dfrac{d\theta}{dr} + \dfrac{4\mu}{3b}\dfrac{d}{dr}\left[\theta\sigma r^2\left(\dfrac{d\sigma}{dr} - \dfrac{\sigma}{r}\right)\right] & \mathrm{I}_a \\ \dfrac{c}{A}\dfrac{d\theta}{dr} + \dfrac{\alpha\theta}{\sigma}\left[\dfrac{d\sigma}{dr} + \dfrac{2\sigma}{r}\right] = \dfrac{4\mu}{3b}\theta r^2\left[\dfrac{d\sigma}{dr} - \dfrac{\sigma}{r}\right]^2 & \mathrm{II}_a \end{cases}}$$

$$\frac{d\sigma}{dr} - \frac{\sigma}{r} = r\frac{d}{dr}\left(\frac{\sigma}{r}\right) = r\frac{d}{dr}\left(\frac{\sigma r^2}{r^3}\right) \qquad \frac{d\sigma}{dr} + \frac{2\sigma}{r} = \frac{1}{r^2}\frac{d}{dr}\left(\sigma r^2\right)$$

$\sigma r^2 = z$

$$\boxed{\begin{aligned} &\frac{\sigma^2}{2} = \frac{ga^2}{r} + m - \left(\alpha + \frac{c}{A}\right)\theta + \frac{4\mu}{3b}\,\theta\,\sigma\, r^2\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right) && \mathrm{I}_b \\ &\frac{c}{A}\frac{d\theta}{dr} + \frac{\alpha\theta}{\sigma}\left[\frac{d\sigma}{dr} + \frac{2\sigma}{r}\right] = \frac{4\mu}{3b}\,\theta\, r^2\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)^2 && \mathrm{II}_b \end{aligned}}$$

$\sigma r^2 = z = \frac{b}{\rho}$

$$\frac{1}{2}\frac{z^2}{r^4} = \frac{ga^2}{r} + m - \left(\alpha + \frac{c}{A}\right)\theta + \frac{4\mu}{3b}\,\theta\, z\, r\,\frac{d}{dr}\left(\frac{z}{r^3}\right) \qquad \mathrm{I}_c$$

$$\frac{c}{A}\frac{d\theta}{dr} + \frac{\alpha\theta}{z}\frac{dz}{dr} = \frac{4\mu}{3b}\,\theta\, r^4\left[\frac{d}{dr}\left(\frac{z}{r^3}\right)\right]^2 \qquad \mathrm{II}_c$$

$\frac{\sigma}{r} = u$

$$\frac{u^2 r^2}{2} = \frac{ga^2}{r} + m - \left(\alpha + \frac{c}{A}\right)\theta + \frac{4\mu}{3b}\,\theta\, r^4\, u\,\frac{du}{dr} \qquad \mathrm{I}_d$$

$$\frac{c}{A}\frac{d\theta}{dr} + \frac{\alpha\theta}{u r^3}\frac{d}{dr}(u r^3) = \frac{4\mu}{3b}\,\theta\, r^4\left(\frac{du}{dr}\right)^2 \qquad \mathrm{II}_d$$

żeby $\frac{d\theta}{dr} \gtreqless 0$ musi być:

$$\frac{3\alpha}{r} + \frac{\alpha}{u}\frac{du}{dr} \lesseqgtr \frac{4\mu}{3b}\, r^4\left(\frac{du}{dr}\right)^2$$

$$\frac{4\mu\alpha}{b}\frac{u^2}{r} + \frac{4\mu}{3b}\alpha\, u\frac{du}{dr} \lesseqgtr \left(\frac{4\mu}{3b}\right)^2 r^4\, u^2\left(\frac{du}{dr}\right)^2$$

$$\left(\frac{4\mu}{3b}\right) r^4\, u\,\frac{du}{dr} \gtreqless \frac{\alpha}{2} \pm \sqrt{\frac{\alpha^2}{4} + \frac{4\mu\alpha\, u^2 r^3}{b}}$$

$$\left.\begin{aligned} &\frac{ga^2}{r^2} + \frac{m}{r} - \left(\alpha + \frac{c}{A}\right)\frac{\theta}{r} + \frac{4\mu}{3b}\,\theta\, r^2\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)\frac{\sigma}{r} = 0 \\ &\left[\frac{ga^2}{r\sigma} + \frac{m}{\sigma} - \left(\alpha + \frac{c}{A}\right)\frac{\theta}{\sigma}\right]\frac{d\sigma}{dr} + \frac{4\mu}{3b}\,\theta\, r^2\left(\quad\right)\frac{d\sigma}{dr} = 0 \; — \end{aligned}\right\}$$

$$\frac{4}{3}\frac{\mu}{b}\,\theta\, r^2\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)^2 = \frac{d\sigma}{dr}\left[\frac{ga^2}{r\sigma}+\frac{m}{\sigma}-\left(\alpha+\frac{c}{A}\right)\frac{\theta}{\sigma}\right]-\frac{ga^2}{r^2}-\frac{m}{r}+\left(\alpha+\frac{c}{A}\right)\frac{\theta}{r}$$

$$= \frac{c}{A}\frac{d\theta}{dr}+\frac{\alpha\theta}{\sigma}\left(\frac{d\sigma}{dr}+\frac{2\sigma}{r}\right)$$

$$\underbrace{\frac{d\sigma}{dr}\left[\frac{ga^2}{r\sigma}+\frac{m}{\sigma}\right]-\left[\frac{ga^2}{r^2}+\frac{m}{r}\right]}_{\frac{1}{\sigma}\left(\frac{ga^2}{r}+m\right)\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)}+\underbrace{\left(\frac{c}{A}-\alpha\right)\frac{\theta}{r}-\left(\frac{c}{A}+2\alpha\right)\frac{\theta}{\sigma}\frac{d\sigma}{dr}}_{-\frac{\theta}{\sigma}\left(\frac{c}{A}+2\alpha\right)\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)-\frac{3\alpha\theta}{r}} = \frac{c}{A}\frac{d\theta}{dr}$$

$$\frac{1}{\sigma}\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\left[\frac{ga^2}{r}+m-\left(\frac{c}{A}+2\alpha\right)\theta\right] = \frac{3\alpha\theta}{r}+\frac{c}{A}\frac{d\theta}{dr}$$

$$\left[\frac{ga^2}{r}-ga+\left(\alpha+\frac{c}{A}\right)\theta_0-\left(\alpha+\frac{c}{A}\right)\theta-\alpha\theta\right] = \left[ga\left(\frac{a}{r}-1\right)+\left(\alpha+\frac{c}{A}\right)(\theta_0-\theta)-\alpha\theta\right]$$

$$I_{b)}:\quad \frac{\sigma_0^2}{2} = ga+m-\left(\alpha+\frac{c}{A}\right)\theta_0+\underbrace{\frac{4}{3}\frac{\mu}{b}\,\theta_0\sigma_0 a^2\left(\frac{d\sigma}{dr}-\frac{\sigma_0}{a}\right)}_{\varepsilon}$$

$$\frac{\sigma^2-\sigma_0^2}{2} = \underbrace{ga\left(\frac{a}{r}-1\right)}_{<0}\underbrace{-\left(\alpha+\frac{c}{A}\right)(\theta-\theta_0)}_{>0}+\frac{4\mu}{3b}\,\theta\sigma r^2\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)-\varepsilon$$

$$\frac{4\mu}{3b}\left\{\frac{c}{A}\frac{d\theta}{dr}\frac{1}{\theta}+\frac{\alpha}{\sigma}\left[\frac{d\sigma}{dr}+\frac{2\sigma}{r}\right]\right\} = \left[\frac{4\mu}{3b}\,r\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\right]^2$$

$$= \frac{1}{(\theta\sigma r)^2}\left[\frac{ga^2}{r}+m-\left(\alpha+\frac{c}{A}\right)\theta\right]^2$$

$$\frac{4\mu}{3b}\left\{\frac{c}{A}\frac{d\theta}{dr}\frac{1}{\theta}+\frac{\alpha}{\sigma}\frac{d\sigma}{dr}+\frac{2\alpha}{r}\right\} =$$

$$\frac{4\mu}{3b}\frac{d\sigma}{dr} = \frac{4\mu}{3b}\frac{\sigma}{r}-\frac{1}{\theta\sigma r^2}\left[\frac{ga^2}{r}+m-\left(\alpha+\frac{c}{A}\right)\theta\right]$$

$$\frac{4\mu}{3b}\left\{\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dr}+\frac{\alpha}{r}+\frac{2\alpha}{r}\right\} = \frac{\alpha}{\cancel{\theta\sigma r^2}}\left[\qquad\right]+\frac{1}{(\theta\sigma r)^2}\left[\qquad\right]^2$$

Jaka będzie wartość $\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right|_{r=a}$?

W przybliżeniu:

$$\frac{c}{A}\log\theta + \alpha(\log\sigma + \log r^2) = \text{const}$$

$$\theta^{\frac{c}{A}}(\sigma r^2)^{\alpha} = \text{const} = \theta_0^{\frac{c}{A}}\left(\frac{\sigma_0}{\rho_0}\right)^{\alpha}$$

$$\frac{ga^2}{r} + m = \left(\alpha + \frac{c}{A}\right)\theta$$

$$\underline{ga + m = \left(\alpha + \frac{c}{A}\right)\theta_0}$$

$$\left(\frac{d\theta}{dr}\right)_0 = -\frac{g\cancel{a}}{\left(\alpha+\frac{c}{A}\right)\cancel{a}}$$

$$ga\left(\frac{a}{r} - 1\right) = \left(\alpha + \frac{c}{A}\right)(\theta - \theta_0)$$

$$\frac{2\alpha\theta_0}{r} + \frac{\alpha\theta}{\sigma}\frac{d\sigma}{dr} - \frac{\frac{c}{A}g}{\alpha + \frac{c}{A}} = 0$$

$$\frac{\alpha\theta_0}{\sigma}\frac{d\sigma}{dr} = \frac{g}{1 + \frac{\alpha A}{c}} - \frac{2\alpha\theta_0}{a}$$

$$\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)_0 = \frac{\sigma}{\alpha\theta_0}\left[\frac{g}{1 + \frac{\alpha A}{c}} - \frac{2\alpha\theta_0}{a} - \frac{\alpha\theta_0}{a}\right]$$

$$= \sigma\left[\frac{g}{\underbrace{\alpha\theta_0\left(1 + \frac{\alpha A}{c}\right)}_{=k}} - \frac{3}{a}\right]$$

$$\frac{9{\cdot}8}{1{\cdot}4 \cdot 270 \cdot 200} - \frac{3}{6,366.200} \gg 0$$

$$\left(\frac{d\sigma}{dr}\right)_0 > \left(\frac{\sigma}{r}\right)_0$$

$$\left(\frac{1}{\sigma}\frac{d\sigma}{dr}\right)_0 > \left(\frac{1}{r}\right)_0$$

$$\left(\frac{d\log\sigma}{dr}\right)_0 > \left(\frac{1}{r}\right)_0$$

Zapewne zmniejsza się później ta różnica

Jeżeli $\frac{d\sigma}{dr}$ ~~stanie~~ się $= \frac{\sigma}{r}$ to:

$$\frac{ga^2}{r} + m - \left(\alpha + \frac{c}{A}\right)\theta = 0 \qquad ga\left(\frac{a}{r} - 1\right) - \left(\alpha + \frac{c}{A}\right)(\theta - \theta_0) = 0$$

$$\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dr} + \frac{3\alpha}{r} = 0 \qquad ga\left(1 - \frac{a}{r}\right) = \left(\alpha + \frac{c}{A}\right)(\theta_0 - \theta)$$

Powyższy wzgląd θ musiałoby być wciąż mniejsze aniżeli ~~gdy~~ bez tarcia

Jednak zanim $\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)$ stanie się $= 0$ to musi istnieć punkt gdzie:

$$\frac{\alpha}{\sigma}\left(\frac{d\sigma}{dr} + \frac{2\sigma}{r}\right) = \frac{4\mu}{3b} r^2 \left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)^2$$

Tam będzie $\frac{d\theta}{dx} = 0$!

Czy ten punkt będzie osiągnięty?

$$\frac{d\sigma}{dr} > \frac{\sigma}{r} \qquad \frac{\alpha}{\sigma}\frac{d\sigma}{dr} + \frac{2\alpha}{r} \gtrless \frac{4\mu}{3b} r^2\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)^2$$

$$\sigma^2 = s \qquad \frac{\alpha}{2}\frac{d(\sigma^2)}{dr} + \frac{2\alpha\sigma^2}{r} \gtrless \frac{4\mu}{3b}\frac{r^2}{\sigma^2}\left[\frac{1}{2}\frac{d(\sigma^2)}{dr} - \frac{\sigma^2}{r}\right]^2$$

$$\frac{\alpha}{2} s \frac{ds}{dr} + \frac{2\alpha s^2}{r} \gtrless \frac{4\mu}{3b}\left[\frac{1}{4} r^2 \left(\frac{ds}{dr}\right)^2 - rs\frac{ds}{dr} + s^2\right]$$

$$0 \gtrless \frac{4\mu}{3b}\left[\frac{r^2}{4}\left(\frac{ds}{dr}\right)^2\right] - s\frac{ds}{dr}\left[\frac{4\mu r}{3b} + \frac{\alpha}{2}\right] + s^2\left[\frac{4\mu}{3b} - \frac{2\alpha}{r}\right]$$

$$0 \gtrless \frac{4\mu}{3b}\frac{1}{4}\left(r\frac{ds}{dr}\right)^2 - r\frac{ds}{dr} s\left[\frac{4\mu}{3b} + \frac{\alpha}{2r}\right] + s^2\left[\frac{4\mu}{3b} - \frac{2\alpha}{r}\right]$$

$$0 \gtrless \left(\frac{r}{2}\frac{ds}{dr}\right)^2 - 2\left(\frac{r}{2}\frac{ds}{dr}\right)s\left(1 + \frac{3b}{4\mu}\frac{\alpha}{2r}\right) + s^2\left(1 - \frac{3b}{4\mu}\frac{2\alpha}{r}\right)$$

$$\left[\frac{r}{2}\frac{ds}{dr} - s\left(1+\frac{3b}{4\gamma}\frac{\alpha}{2r}\right)\right]^2 \lesseqgtr s^2\left[\frac{3b}{4\gamma}\frac{\alpha}{r} + \left(\frac{3b}{4\gamma}\frac{\alpha}{2r}\right)^2 + \frac{3b}{4\gamma}\frac{2\alpha}{r}\right]$$

$$s^2\,\frac{3b}{4\gamma}\frac{\alpha}{r}\left(3+\frac{3b}{4\gamma}\frac{\alpha}{4r}\right)$$

$$2\frac{\alpha}{r}\left[\left[s+\frac{1}{8}\frac{ds}{dr}\right]^2 - \frac{1}{64}\left(\frac{ds}{dr}\right)^2\right] \gtreqless \frac{4\gamma}{3b}\left[\frac{r}{2}\frac{ds}{dr} - s\right]^2$$

jeśli $\frac{d\sigma}{dr}$ wciąż się zmniejsza wtedy naturalnie musi być kiedyś osiągnięty

W przeciwnym wypadku (2) $\frac{d\theta}{dr}$ wszędzie będzie < 0

$$\frac{d\sigma}{dr} > \frac{\sigma}{r}$$

~~zatem mógłby być może punkt gdzie $\theta = 0$~~

$$(\theta_0-\theta)\left(\alpha+\frac{c}{A}\right) = +g a\left(1-\frac{a}{r}\right) - \frac{4\gamma}{3b}\theta\sigma r^2\left(\frac{\sigma}{r}-\frac{\sigma}{r}\right)$$

Więc mógłby być punkt gdzie $\theta=0$ jeśli 1). σ i $\frac{d\sigma}{dr}$ skończone a $\frac{d\theta}{dr}=0$

2).

1). wymagałoby według Ia także: $\sigma\frac{d\sigma}{dr} = -g\frac{a^2}{r^2}$! co nie możliwe bo

właśnie założeniem było że $\frac{d\sigma}{dx} > \frac{\sigma}{r}$

$$\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dr} + \frac{3\alpha}{r} + \frac{\alpha}{\sigma}\left[\frac{d\sigma}{dr} - \frac{\sigma}{r}\right] = \frac{4\mu}{3b}\, r^2\left[\frac{d\sigma}{dr} - \frac{\sigma}{r}\right]^2$$

$$\frac{4\mu}{3b} r^2\left[\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dr} + \frac{3\alpha}{r}\right] = \frac{\alpha}{\theta\sigma^2}\left[\frac{ga^2}{r} + m - \left(\alpha + \frac{c}{A}\right)\theta\right] + \left\{\frac{1}{\theta\sigma}\left[\frac{ga^2}{r} + m - \left(\frac{\alpha + c}{A}\right)\theta\right]\right\}^2$$

$$\frac{4\mu}{3b}\sigma^2 = \frac{\frac{\alpha}{\theta}\left[\frac{ga^2}{r} + m - \left(\alpha + \frac{c}{A}\right)\theta\right] + \frac{1}{\theta^2}\left[\frac{ga^2}{r} + m - \left(\frac{\alpha c}{A}\right)\theta\right]^2}{\cancel{\frac{4\mu}{3b}}\, r^2\left[\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dr} + \frac{3\alpha}{r}\right]}$$

$$0 = \underbrace{\frac{ga^2}{r} + m - \left(\alpha + \frac{c}{A}\right)\theta}_{v} + r^2\theta\left[\frac{1}{2}\frac{d}{dr}\left\{\frac{\frac{\alpha}{\theta}[\ \ ] + \frac{1}{\theta^2}[\ \ ]^2}{r^2\left[\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dr} + \frac{3\alpha}{r}\right]}\right\} - \frac{1}{r}\,\frac{\frac{\alpha}{\theta}[\ \ ] + \frac{1}{\theta^2}[\ \ ]^2}{r^2\left[\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dr} + \frac{3\alpha}{r}\right]}\right]$$

$$0 = v + \frac{r^2\theta}{2}\frac{d}{dr}\left\{\frac{\alpha v + \frac{1}{\theta}v^2}{r^2\left[\frac{c}{A}\frac{d\theta}{dr} + \frac{3\alpha\theta}{r}\right]}\right\} - \theta\,\frac{\alpha v + v^2}{r\left(\frac{c}{A}\frac{d\theta}{dr} + \frac{3\alpha\theta}{r}\right)}$$

Id:

$0 = \frac{ga^2}{r} + m - \left(\alpha + \frac{c}{A}\right)\theta + \frac{4\mu}{3b}\theta r^4 u \frac{du}{dr}$	$\frac{du}{dr}$
$\frac{c}{A}\frac{d\theta}{dr} + \frac{3\alpha\theta}{r} + \frac{\alpha\theta}{u}\frac{du}{dr} = \frac{4\mu}{3b}\theta r^4\left(\frac{du}{dr}\right)^2$	u

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$$\frac{c}{A}\frac{d\theta}{dr} + \frac{3\alpha\theta}{r} = \frac{c}{Ar}\left[r\frac{d\theta}{dr} + \frac{3\alpha}{\frac{c}{A}}\theta\right]$$

$$= \frac{c}{A}\frac{d}{dr}\left(\theta r^{\frac{3\alpha}{c/A}}\right) \cdot r^{-\frac{3\alpha}{c/A}}$$

$$\frac{c}{A}u\frac{d\theta}{dr} + \frac{3\alpha\theta u}{r} = +\frac{c}{A}\theta\frac{du}{dr} - m\frac{du}{dr} - \frac{ga^2}{r}\frac{du}{dr}$$

$$\frac{c}{A}\frac{d}{dr}\left(\frac{\theta}{u}\right) + \frac{3\alpha}{r}\frac{\theta}{u} - \left(m + \frac{ga^2}{r}\right)\frac{d\left(\frac{1}{u}\right)}{dr} = 0$$

$$\frac{4\pi}{36}\,\frac{r^2}{2}\,\frac{d(u^2)}{dr} = -\frac{v}{\theta r^2}$$

$$0 = v + \frac{4\pi}{36}\,\theta\,\frac{r^4}{2}\,\frac{d(u^2)}{dr}$$

$$u^2\left[\frac{c}{A}\frac{d\theta}{dr} + \frac{3\alpha\theta}{r}\right] + \frac{\alpha\theta}{2}\,\frac{d(u^2)}{dr} = \frac{4\pi}{36}\,\frac{\theta r^4}{4}\left[\frac{d(u^2)}{dr}\right]^2$$

$$\frac{4\pi}{36}\,u^2 = \frac{1}{\frac{c}{A}\frac{d\theta}{dr} + \frac{3\alpha\theta}{r}}\left(+\frac{v^2}{\theta r^4} + \frac{\alpha v}{r^4}\right)$$

$$0 = v + \frac{\theta r^4}{2}\,\frac{d}{dr}\left\{\frac{\frac{v^2}{\theta} + \alpha v}{r^4\left[\frac{c}{A}\frac{d\theta}{dr} + \frac{3\alpha\theta}{r}\right]}\right\}$$ to samo jak tam

$$\frac{1}{r^3} = z \qquad -\frac{3}{r^4}\,dr = dz \qquad \frac{d\theta}{dr} = -\frac{3}{r^4}\frac{d\theta}{dz}$$

$$\underbrace{\frac{ga^2}{r\theta} + \frac{m}{\theta} - \left(\alpha + \frac{c}{A}\right)}_{w} + \frac{r^4}{2}\,\frac{d}{dr}\left\{\frac{\left[\frac{ga^2}{r\theta} + \frac{m}{\theta} - \left(\alpha + \frac{c}{A}\right)\right]^2 + \alpha\left[\quad\right]}{r^4\left[\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dr} + \frac{3\alpha}{r}\right]}\right\}$$

$$2w = \frac{3}{\cancel{\ }}\,\frac{d}{dz}\left\{\frac{w^2 + w\alpha}{-3\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dz} + \frac{3\alpha}{z}}\right\} = \frac{d}{dz}\left\{\frac{w^2 + w\alpha}{\underbrace{\frac{\alpha}{z} - \frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dz}}}\right\}$$

$$= \frac{d}{dz}\log\left(z^{\alpha}\,\theta^{-\frac{c}{A}}\right)$$

$$\frac{1}{r} = t \qquad \frac{ga^2 t + m}{\alpha} = y \qquad \frac{dr}{r^2} = -dt$$

$$\frac{d}{dr} = -\frac{1}{r^2}\frac{d}{dt}$$

$$\frac{y}{\theta} - \frac{k}{k-1} - \frac{1}{2t^2}\,\frac{d}{dt}\left\{\frac{\left(\frac{y}{\theta} - \frac{k}{k-1}\right)^2 + \left(\frac{y}{\theta} - \frac{k}{k-1}\right)}{\frac{3}{t^3}\,\frac{1}{k-1}\,\frac{1}{\theta t^2}\frac{d\theta}{dt}}\right\}$$

$$\frac{1}{\theta}\left(\frac{ga^2}{r} + m\right) = w + \alpha + \frac{c}{A}$$

$$\log\left(\frac{ga^2}{r} + m\right) - \log\theta = \log\left(w + \alpha + \frac{c}{A}\right)$$

$$\frac{1}{\theta}\frac{d\theta}{dz} = \frac{-\frac{ga^2}{r^2}\frac{dr}{dz}}{\frac{ga^2}{r} + m} - \frac{\frac{dw}{dz}}{w + \alpha + \frac{c}{A}}$$

Uwzględniając tarcie tylko w I:

$$\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dr} + \frac{\alpha}{\sigma}\frac{d\sigma}{dr} + \frac{2\alpha}{r} = 0$$

$$\log\left(\theta^{\frac{c}{A}}\sigma^{\alpha} r^{2\alpha}\right) = \mathrm{const}$$

$$\left[\frac{\theta}{\theta_0}\right]^{\frac{c}{A\alpha}}\left(\frac{\sigma}{\sigma_0}\right)\left(\frac{r}{a}\right)^2 = 1$$

$$0 = -\frac{ga^2}{r^2}\underbrace{-\alpha\frac{d\theta}{dr} + \frac{2\alpha\theta}{r} + \frac{\alpha\theta}{\sigma}\frac{d\sigma}{dr}}_{-\left(\alpha+\frac{c}{A}\right)\frac{d\theta}{dr}} + \frac{4\mu}{3\varrho}\sigma r^2\left[\theta\frac{d}{dr}\left(\frac{d\sigma}{dr}+\frac{2\sigma}{r}\right)+\frac{d\theta}{dr}\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\right]$$

$$\frac{c}{A\alpha} = \frac{1}{k-1}$$

$$\frac{\theta}{\theta_0}\left(\frac{\sigma r^2}{\sigma_0 a^2}\right)^{(k-1)} = 1 \qquad \frac{\theta}{\theta_0} = \left(\frac{\sigma r^2}{\sigma_0 a^2}\right)^{(1-k)}$$

$$\frac{d\theta}{dr} = \frac{2(1-k)\theta_0}{(\sigma_0 a^2)^{(1-k)}}(\sigma r^2)^{-k}\frac{d}{dr}(\sigma r^2)$$

$$= 2(1-k)\frac{\theta}{\sigma r^2}\frac{d}{dr}(\sigma r^2)$$

$1 + 30\alpha$ 0.00367 1.110.14 184

1.414 14 1554 : 184 = 0.844

2.88081 844 2.0368

10.885

$\frac{760}{690} = 1.1$ mm 1.99455 0.844 2.8385

~~Przybliżenie stopniowe:~~

I $\cancel{\vartheta = g\frac{a}{r} - m - (\alpha + \frac{c}{A})\theta}$ $\quad g = -\frac{\alpha k}{k-1}\frac{d\theta}{dx} \quad \| \theta = \theta_0 - gx\frac{k-1}{\alpha k}$

$\cancel{ga\left(\frac{a}{r} - 1\right) = \alpha\frac{k}{k-1}\theta}$

$$\frac{\theta}{\theta_0} = \left(\frac{\sigma}{\sigma_0}\right)^{1-k} = \left(\frac{\rho_0}{\rho}\right)^{1-k} = \left(\frac{\rho}{\rho_0}\right)^{k-1} \qquad \frac{p}{p_0} = \frac{\theta}{\theta_0}\frac{\rho}{\rho_0} = \left(\frac{\theta}{\theta_0}\right)^{1+\frac{1}{k-1}} = \left(\frac{\theta}{\theta_0}\right)^{\frac{k}{k-1}}$$

$$gx + m = -(\alpha + \tfrac{c}{A})\theta \qquad \sigma = \sigma_0\left(\frac{\theta}{\theta_0}\right)^{\frac{1}{1-k}}$$

$$m = -(\alpha + \tfrac{c}{A})\theta_0 \qquad \frac{d\sigma}{dx} = \frac{\sigma_0}{1-k}\frac{\theta^{\frac{k}{1-k}}}{\theta_0^{\frac{1}{1-k}}}\frac{d\theta}{dx}$$

$$gx = (\alpha + \tfrac{c}{A})(\theta_0 - \theta)$$

$$gx = \alpha\frac{k}{k-1}(\theta_0 - \theta) \qquad = \frac{\sigma_0}{\cancel{(1-k)}\,\theta_0^{\frac{1}{1-k}}}\, g\frac{\cancel{k-1}}{\alpha k}\left(\theta_0 - gx\frac{k-1}{\alpha k}\right)^{\frac{k}{1-k}}$$

$$= \alpha\frac{k}{k-1}\theta_0\left(1 - \frac{\theta}{\theta_0}\right) \qquad = \frac{g\sigma_0}{\alpha k\,\theta_0^{\frac{1}{1-k}}}\left(\theta_0 - gx\frac{k-1}{\alpha k}\right)^{\frac{k}{1-k}}$$

$$\frac{d\sigma}{dx} = \frac{g\sigma_0}{\alpha k\theta_0}\left(1 - \frac{gx(k-1)}{\theta_0\alpha k}\right)^{\frac{k}{1-k}}$$

$$\frac{\theta}{\theta_0} = \frac{1}{5} \qquad \left(\frac{p}{p_0}\right) = \left(\frac{1}{5}\right)^{\frac{1.4}{0.4}} = \left(\frac{1}{5}\right)^{\frac{7}{2}}$$

$\theta = 56^\circ$ abs.

$= -217^\circ$

$$\lg\left(\frac{p}{p_0}\right) = -\frac{7}{2}\lg 5$$

0.69897
4 8 9 2 7 9
-2.44640
$= 0.55360 - 3$

$$\frac{p}{p_0} = 0.003578 . 760$$

$p =$ ca 2.5 mm

Nun aber nach Assmann, Hergesell etc.

in ca 14000 m : $-65^\circ = 208^\circ$ $\qquad \Delta = 80^\circ$

während nach obiger Formel sein sollte 140° $\qquad \Delta = 140^\circ$

1.99455
0.08192
2.08647

$$\frac{7}{2} : x = \cancel{14} : \cancel{8} \quad (7 : 4)$$

$$x = \frac{14}{7} = 2 = \frac{k}{k-1}$$

$$\frac{56}{208} = \frac{7}{26}$$

$$\left(\frac{p}{p_0}\right) = \left(\frac{208}{280}\right)^2\left(\frac{7}{26}\right)^{\frac{7}{2}} \qquad \frac{26}{35}$$

41497
54407
95904
-0.04096

84510
41497
43013
$-0.56987 \cdot \frac{7}{2}$
398909

7 mm

Krytyczny punkt leży na wysokości 27 km $=4gM.$ więc tam g o mniej niż 1%

zmniejszone a σ o tyle powiększone

więc zupełnie wystarczające są ~~w~~ równania przybliżone.

$$\text{I.)}\quad \sigma\frac{d\sigma}{dx} = -g-\left(\alpha+\frac{c}{A}\right)\frac{d\theta}{dx}+\frac{4}{3}\vartheta_b\Big[\sigma\overbrace{\Big\{\frac{d}{dx}\Big(\theta\frac{d\sigma}{dx}\Big)+\theta\Big(\frac{d\sigma}{dx}\Big)^2\Big\}}^{\frac{d}{dx}\left[\theta\sigma\frac{d\sigma}{dx}\right]}\Big]$$

$$\text{II.)}\quad \frac{\sigma^2}{2} = -gx+m-\left(\alpha+\frac{c}{A}\right)\theta+\frac{4}{3}\vartheta_b\,\theta\sigma\frac{d\sigma}{dx}$$

$$= \left[\sigma\frac{d^2\sigma}{dx^2}+\left(\frac{d\sigma}{dx}\right)^2\right]\theta+\sigma\frac{d\sigma}{dx}\frac{d\theta}{dx}$$

$$\text{III.)}\quad \frac{c}{A}\frac{d\theta}{dx}\frac{1}{\theta}+\frac{\alpha}{\sigma}\frac{d\sigma}{dx} = \frac{4}{3}\vartheta_b\left(\frac{d\sigma}{dx}\right)^2$$

$$\sigma\frac{d\sigma}{dx}+g-\frac{4}{3}\vartheta_b\left[\sigma\frac{d^2\sigma}{dx^2}+\left(\frac{d\sigma}{dx}\right)^2\right]\theta = \left[-\left(\alpha+\frac{c}{A}\right)+\frac{4}{3}\vartheta_b\,\sigma\frac{d\sigma}{dx}\right]\frac{d\theta}{dx}$$

$$\underbrace{\frac{1}{\theta}}_{=\dfrac{-\left(\alpha+\frac{c}{A}\right)+\frac{4}{3}\vartheta_b\,\sigma\frac{d\sigma}{dx}}{\frac{\sigma^2}{2}+gx-m}}\left[\sigma\frac{d\sigma}{dx}+g\right]-\frac{4}{3}\vartheta_b\left[\sigma\frac{d^2\sigma}{dx^2}+\left(\frac{d\sigma}{dx}\right)^2\right] = \left[-\left(\alpha+\frac{c}{A}\right)+\frac{4}{3}\vartheta_b\,\sigma\frac{d\sigma}{dx}\right]\underbrace{\frac{1}{\theta}\frac{d\theta}{dx}}_{=\frac{A}{c}\left[\frac{4}{3}\vartheta_b\left(\frac{d\sigma}{dx}\right)^2-\frac{\alpha}{\sigma}\frac{d\sigma}{dx}\right]}$$

$$\left[-\left(\alpha+\frac{c}{A}\right)+\frac{4}{3}\vartheta_b\,\sigma\frac{d\sigma}{dx}\right]\left[\sigma\frac{d\sigma}{dx}+g\right]-\left[\frac{\sigma^2}{2}+gx-m\right]\left[\sigma\frac{d^2\sigma}{dx^2}+\left(\frac{d\sigma}{dx}\right)^2\right]\frac{4}{3}\vartheta_b =$$

$$= \left[\frac{\sigma^2}{2}+gx-m\right]\left[\frac{4}{3}\vartheta_b\left(\frac{d\sigma}{dx}\right)^2-\frac{\alpha}{\sigma}\frac{d\sigma}{dx}\right]\left[-\left(\alpha+\frac{c}{A}\right)+\frac{4}{3}\vartheta_b\,\sigma\frac{d\sigma}{dx}\right]\frac{A}{c}$$

$$\left(p^2+\frac{a}{x}pq+bq=0\right)$$

$$\frac{c}{A}\frac{d\lg\theta}{dx}+\frac{\alpha}{\sigma}\frac{d\sigma}{dx} = \frac{4}{3}\vartheta_b\left(\frac{d\sigma}{dx}\right)^2$$

$$\frac{c}{A}\left(\frac{dx}{d\sigma}\right)^2+\frac{\alpha}{\sigma}\frac{dx}{d\sigma}\frac{dx}{d\lg\theta} = \frac{4}{3}\vartheta_b\frac{dx}{d\lg\theta}\quad\Big|\quad \frac{c}{A}p^2+\frac{\alpha}{x}pq = \frac{4}{3}\vartheta_b\, q$$

~~gives~~: $\frac{c}{A}\sigma\frac{d\theta}{dx}+\alpha\theta\frac{d\sigma}{dx}=\frac{4}{3}\sigma_b\,\theta\sigma\left(\frac{d\sigma}{dx}\right)^2$

$$\cancel{\frac{c}{A}\frac{d^2\theta}{dx^2}+\frac{d\theta}{dx}\frac{\alpha}{\sigma}\frac{d\sigma}{dx}+}$$

$$\cancel{\frac{c}{A}\sigma\frac{d^2\theta}{dx^2}+\frac{c}{A}\frac{d\sigma}{dx}\frac{d\theta}{dx}+\alpha\frac{d\theta}{dx}\frac{d\sigma}{dx}+\alpha\theta\frac{d^2\sigma}{dx^2}=\frac{4}{3}\sigma_b\left[\frac{d\theta}{dx}\sigma\left(\frac{d\sigma}{dx}\right)^2+\theta\left(\frac{d\sigma}{dx}\right)^3+2\theta\sigma\frac{d\sigma}{dx}\frac{d^2\sigma}{dx^2}\right]}$$

I + III): $\sigma\frac{d\sigma}{dx}=-g-\alpha\frac{d\theta}{dx}+\frac{\alpha}{\sigma}\frac{d\sigma}{dx}+\frac{4}{3}\sigma_b\left[\sigma\theta\frac{d^2\sigma}{dx^2}+\sigma\frac{d\sigma}{dx}\frac{d\theta}{dx}\right]$

$$\frac{1}{\theta}\frac{\sigma^2}{2}\frac{d\theta}{dx}=\frac{(m-gx)}{\theta}\frac{d\theta}{dx}-\left(\alpha+\frac{c}{A}\right)\frac{d\theta}{dx}+\frac{4}{3}\sigma_b\,\sigma\frac{d\sigma}{dx}\frac{d\theta}{dx}$$

$$\sigma\frac{d\sigma}{dx}-\frac{\sigma^2}{2}\frac{1}{\theta}\frac{d\theta}{dx}=-\frac{(m-gx)}{\theta}\frac{d\theta}{dx}+\frac{c}{A}\frac{d\theta}{dx}-g+\frac{\alpha}{\sigma}\frac{d\sigma}{dx}+\frac{4}{3}\sigma_b\,\sigma\theta\frac{d^2\sigma}{dx^2}$$

$$-\frac{c}{A}\frac{1}{\theta^2}\left(\frac{d\theta}{dx}\right)^2+\frac{c}{A}\frac{1}{\theta}\frac{d^2\theta}{dx^2}-\frac{\alpha}{\sigma^2}\left(\frac{d\sigma}{dx}\right)^2+\frac{\alpha}{\sigma}\frac{d^2\sigma}{dx^2}=2\frac{4}{3}\sigma_b\frac{d\sigma}{dx}\frac{d^2\sigma}{dx^2}$$

~~$\frac{dy}{dx}$~~

$$\frac{dy}{dx}+X_1y+X_2=0$$

$$y\frac{dy}{dx}+X_1y^2+X_2y+X_3=0$$

$$y=e^{-\int X_1dx}\left[c-\int X_2e^{\int X_1dx}dx\right]$$

$$e^{-\int X_1}\left[c-\int X_2e^{\int X_1}\right]\left[\cancel{-X_1e^{-\int X_1}\left(c-\int X_2e^{\int X_1}\right)}-\cancel{e^{-\int X_1}X_2e^{\int X_1}}+\frac{dc}{dx}e^{-\int X_1}\right]+$$

$$\left.\cancel{X_1e^{-\int X_1}\left(c-\int X_2e^{\int X_1}\right)+X_2}\right]$$

$$+X_3=0$$

$$\left[c-\int X_2e^{\int X_1dx}dx\right]\frac{dc}{dx}\cancel{e^{-\int X_1}}+X_3e^{2\int X_1}=0$$

$$c\frac{dc}{dx} + f_1(x)\frac{dc}{dx} + f_2(x) = 0$$

$$(c + f_1(x)) = y$$

$$\frac{dc}{dx} + f_1'(x) = \frac{dy}{dx}$$

$$c = \cancel{-\frac{f_2}{(1+f_1)\frac{dc}{dx}}} \quad \frac{-f_2 - f_1\frac{dc}{dx}}{\frac{dc}{dx}} = -f_1 - \frac{f_2}{\frac{dc}{dx}}$$

$$\frac{dc}{dx} = -\frac{df_1}{dx} - \frac{\frac{df_2}{dx}}{\frac{dc}{dx}} + \frac{f_2}{\left(\frac{dc}{dx}\right)^2}\frac{d^2c}{dx^2}$$

$$\frac{dc}{dx} = p \qquad f_2\frac{dp}{dx} = p^3 + \frac{df_1}{dx}p^2 + \frac{df_2}{dx}p$$

$$\frac{d\theta}{dx} = \frac{+\theta_0}{\theta_0^{\frac{1}{1-\kappa}}}\,\frac{\kappa}{1-\kappa}\,\frac{g\cancel{k}\cancel{(k-1)}}{\alpha k}\left[\theta_0 - \frac{g\,x\,(k-1)}{\alpha k}\right]^{\frac{\kappa}{1-\kappa}}$$

$$= \frac{\theta_0\, g k}{\alpha k\, \theta_0}\left[1 - \frac{g\,x\,(k-1)}{\alpha k\,\theta_0}\right]^{\frac{\kappa}{1-\kappa}}$$

Greca tarea prof/sec. 1cm³ } – =

$$y\frac{dy}{dx} + y^2X_1 + yX_2 + X_3 = 0$$

$$y + u = v \quad (u = f(x)) \qquad (y+u)\left(\frac{dy}{dx} + u'\right) + (y+u)^2X_1 + X = 0$$

$$v\frac{dv}{dx} + v^2X_1 + X = 0$$

$$y\frac{dy}{dx} + u\frac{dy}{dx} + y^2X_1 + y\underbrace{(u' + 2uX_1)}_{=X_2} + uu' + u^2X_1 + X = 0$$

Jeżeli jako pierwsze przybliżenie :

$$g x = \alpha \frac{k}{k-1}(\theta_0 - \theta) \qquad \frac{d\theta}{dx} = -\frac{g(k-1)}{\alpha k}$$

$$\sigma = \sigma_0 \left(\frac{\theta}{\theta_0}\right)^{\frac{1}{1-k}}$$

$$\frac{d\sigma}{dx} = \frac{g\sigma_0}{\alpha k \theta_0}\left(1 - \frac{gx(k-1)}{\theta_0 \alpha k}\right)^{\frac{k}{1-k}}$$

$$= \frac{g b}{k p_0}\left[1 - \frac{g x(k-1)}{\theta_0 \alpha k}\right]^{\frac{k}{1-k}}$$

$$\frac{4}{3}\mu \left(\frac{d\sigma}{dx}\right)^2 = \underbrace{\left(\frac{g b}{k p_0}\right)^2 \left[1 - \frac{g x(k-1)}{\theta_0 \alpha k}\right]^{\frac{2k}{1-k}} \frac{4}{3}\mu}$$

$$\int_0^x \underbrace{\left[1 - \frac{g x(k-1)}{\theta_0 \alpha k}\right]}_{=\frac{\theta}{\theta_0}}^{\frac{2k}{1-k}} dx \qquad \theta = \theta_0 - \frac{g x(k-1)}{\alpha k}$$

$$d\theta = -\frac{g(k-1)}{\alpha k} dx$$

$$= -\int_{\theta_0}^{\theta}\left(\frac{\theta}{\theta_0}\right)^{\frac{2k}{1-k}} \frac{\alpha k}{g(k-1)} d\theta = -\frac{\alpha k}{g(k-1)\theta_0^{\frac{2k}{1-k}}} \frac{1}{\frac{2k}{1-k}+1} \theta^{\frac{2k}{1-k}+1}$$

$$\text{Exp.} \quad = + \frac{\alpha k}{g(1+k)}\left[\left(\frac{\theta}{\theta_0}\right)^{\frac{2k}{1-k}} \theta - \theta_0\right] \frac{4}{3}\mu \left(\frac{g b}{k p_0}\right)^2$$

$$= \frac{4}{3}\mu \alpha \frac{1}{1+k} \frac{g b}{k p_0^2} \theta_0 \left[\left(\frac{\theta}{\theta_0}\right)^{\frac{1+k}{1-k}} - 1\right] = \frac{4}{3} \frac{\mu b g}{k(1+k)} \frac{1}{p_0 \rho_0}\left[\left(\frac{\theta}{\theta_0}\right)^{\frac{1+k}{1-k}} - 1\right]$$

$\mu\theta_0 = 0.00018$ (gram) 172

$g = 980$

$\rho_0 = 0.0013$

$p_0 = 980 \cdot 76 \cdot 13.6$

$$\frac{4}{3}b \quad \frac{\cancel{0.00018}}{\cancel{280}} \; 0.00006 \; 70 \quad \frac{\cancel{980}}{\cancel{980 \cdot 76 \cdot 13.6} \cdot 0.0013} \quad \underbrace{\frac{1}{1.4 \cdot 2.4}}_{=4.4}$$

24 · 96 = 336 · 1008 · 4.37

np. $b = 10$ cm

$$\frac{0.0006}{200}$$

np. $\frac{\theta}{\theta_0} = \frac{1}{5}$ $\quad \left(\frac{1}{5}\right)^6 = (125)^2$ 250 15000

$$\frac{0.0003}{\cancel{200}} \; 150$$

$= 0.045$

$\log e^{0.045} = 0.43426 \cdot 0.045$

21713.9

0.0195397

$e^{0.045} = 1.046$

$$\cancel{\frac{\sigma^2}{2} = -gx + m - \left(\alpha + \frac{c}{A}\right)\theta} + \frac{4}{3}\frac{\mu}{b}\,\sigma\frac{d\sigma}{dx} \qquad \Big| \; 2\sigma\frac{d\sigma}{dx} = \cancel{2}g - \cancel{2}\left(\alpha+\frac{c}{A}\right)\frac{d\theta}{dx} + \frac{4}{3}\frac{\mu}{b}\frac{d}{dx}\left(\theta\sigma\frac{d\sigma}{dx}\right)$$

$$\frac{c}{A}\sigma^2\frac{d\theta}{dx} + \alpha\theta\sigma\frac{d\sigma}{dx} \qquad = \frac{4}{3}\frac{\mu}{b}\theta\left(\sigma\frac{d\sigma}{dx}\right)^2$$

$$\left(\alpha+\frac{c}{A}\right) = \alpha\left(1+\frac{1}{k-1}\right) = \frac{\alpha k}{k-1}$$

$$\boxed{m = \left(\alpha+\frac{c}{A}\right)\theta_0 + \frac{\sigma_0^2}{2} - \frac{4}{3}\frac{\mu}{b}\theta_0\left(\sigma\frac{d\sigma}{dx}\right)_0}$$

$$\sigma^2 = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots\cdots$$

$$\frac{\theta}{\theta_0} = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + \cdots$$

$$2\sigma\frac{d\sigma}{dx} = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + 5a_5 x^4 + \cdots$$

$$\begin{aligned}
2\sigma\frac{\theta}{\theta_0}\frac{d\sigma}{dx} = {} & a_1 b_0 \\
& + (a_1 b_1 + 2a_2 b_0)x \\
& + (a_1 b_2 + 2a_2 b_1 + 3a_3 b_0)x^2 \\
& + (a_1 b_3 + 2a_2 b_2 + 3a_3 b_1 + 4a_4 b_0)x^3 \\
& + (a_1 b_4 + 2a_2 b_3 + 3a_3 b_2 + 4a_4 b_1 + 5a_5 b_0)x^4 \\
& +
\end{aligned}$$

$$2\sigma\frac{d\sigma}{dx} + 2g + 2\alpha\frac{k}{k-1}\frac{d\theta}{dx} + \frac{4}{3}\frac{\mu}{b}\frac{d}{dx}\left(\theta\, 2\sigma\frac{d\sigma}{dx}\right)$$

$$a_1 + 2g + \frac{2\alpha k}{k-1}\theta_0 b_1 + \underbrace{\left(\frac{4}{3}\frac{\mu}{b}\right)}_{=\varepsilon}(a_1 b_1 + 2a_2 b_0)\theta_0 = 0$$

$$x \quad 2a_2 + \frac{2\alpha k}{k-1}\theta_0\, 2b_2 + 2\,\frac{4}{3}\frac{\mu}{b}(a_1 b_2 + 2a_2 b_1 + 3a_3 b_0)\theta_0 = 0$$

$$x^2 \quad 3a_3 + \frac{2\alpha k}{k-1}\theta_0\, 3b_3 + 3\,\frac{4}{3}\frac{\mu}{b}(a_1 b_3 + 2a_2 b_2 + 3a_3 b_1 + 4a_4 b_0)\theta_0 = 0$$

$$x^3 \quad 4a_4 + \frac{2\alpha k}{k-1}\theta_0\, 4b_4 + 4\,\frac{4}{3}\frac{\mu}{b}(a_1 b_4 + 2a_2 b_3 + 3a_3 b_2 + 4a_4 b_1 + 5a_5 b_0)\theta_0 = 0$$

$$\frac{\alpha}{k-1}a_0 b_1 + \frac{\alpha}{2}a_1 b_0 = \frac{4}{3}\frac{\mu}{b}\frac{b_0 a_1^2}{4}$$

$$\frac{\alpha}{k-1}(2a_0 b_2 + a_1 b_1) + \frac{\alpha}{2}(2a_2 b_0 + a_1 b_1) = \frac{4}{3}\frac{\mu}{b}\frac{1}{4}\left(b_0\{4a_1 a_2\} + b_1 a_1^2\right)$$

$$\frac{\alpha}{k-1}(3a_0 b_3 + 2a_1 b_2 + a_2 b_1) + \frac{\alpha}{2}(3a_3 b_0 + 2a_2 b_1 + a_1 b_2) = \frac{4}{3}\frac{\mu}{b}\frac{1}{4}\Big[b_0(6a_1 a_3 + 4a_2^2) +$$
$$+ b_1\, 4a_1 a_2 + b_2 a_1^2\Big]$$

$$\frac{\alpha}{k-1}(4a_0 b_4 + 3a_1 b_3 + 2a_2 b_2 + a_3 b_1) + \frac{\alpha}{2}(4a_4 b_0 + 3a_3 b_1 + 2a_2 b_2 + a_1 b_3) =$$
$$= \frac{4}{3}\frac{\mu}{b}\frac{1}{4}\Big[b_0(8a_1 a_4 + 12a_2 a_3) + b_1(6a_1 a_3 + 4a_2^2) + b_2\, 4a_1 a_2 + b_3 a_1^2\Big]$$

Jakie jest $\frac{d^2\theta}{dx^2}$ w krytycznym punkcie?

$\frac{dI}{dx}$:

$$\frac{d}{dx}\left(6\frac{d6}{dx}\right) = \left(\frac{d6}{dx}\right)^2 + 6\frac{d^26}{dx^2} = -\left(\alpha+\frac{c}{A}\right)\frac{d^2\theta}{dx^2} + \frac{4}{3}\frac{k}{b}\left\{\cancel{\left[6\frac{d^26}{dx^2}+\left(\frac{d6}{dx}\right)^2\right]\frac{d\theta}{dx}} + \theta\frac{d}{dx}\left[6\frac{d^26}{dx^2}+\left(\frac{d6}{dx}\right)^2\right] + 6\frac{d6}{dx}\frac{d^2\theta}{dx^2} + \cancel{\frac{d}{dx}\left(6\frac{d6}{dx}\right)\frac{d\theta}{dx}}\right\}$$

$$0 =$$

$$\frac{4}{3}\frac{k}{b}\,6\frac{d6}{dx} = \alpha$$

$$\frac{c}{A}\frac{d^2\theta}{dx^2} = \frac{4}{3}\frac{k}{b}\theta\frac{d}{dx}\left[6\frac{d^26}{dx^2}+\left(\frac{d6}{dx}\right)^2\right]$$

$$\underline{g = \frac{4}{3}\frac{k}{b}\theta\left[\left(\frac{d6}{dx}\right)^2 + 6\frac{d^26}{dx^2}\right]}$$

$$\frac{4}{3}\frac{k}{b}\,g\,6^2 = \alpha^2\theta + \left(\frac{4}{3}\frac{k}{b}\right)^2\theta\,6^3\frac{d^26}{dx^2} \quad \mathrm{II} \qquad 2(\mathrm{I}) \qquad \times\left(\frac{4}{3}\frac{k}{b}\right)^2 6^3$$

$$\left(\frac{d}{dx}\mathrm{II}\right)$$

$$\frac{c}{A}6\frac{d^2\theta}{dx^2} + \cancel{\frac{c}{A}\frac{d6}{dx}\frac{d\theta}{dx}} + \cancel{\alpha\frac{d6}{dx}\frac{d\theta}{dx}} + \alpha\theta\frac{d^26}{dx^2} = \frac{4}{3}\frac{k}{b}\left[\cancel{\frac{d\theta}{dx}6\left(\frac{d6}{dx}\right)^2} + \theta\left(\frac{d6}{dx}\right)^3 + 2\theta 6\frac{d6}{dx}\frac{d^26}{dx^2}\right]$$

$$\frac{c}{A}6\frac{d^2\theta}{dx^2}\frac{4}{3}\frac{k}{b}6^3 \quad + \cancel{\alpha\left[\left(\frac{4}{3}\frac{k}{b}\right)^2 g6^2-\alpha^2\theta\right]} = \theta\,\alpha^3 + \alpha\left(\frac{4}{3}\frac{k}{b}g6^2-\alpha^2\theta\right)$$

$$\cancel{\frac{4}{3}\frac{k}{b}}\frac{c}{A}6^4\frac{d^2\theta}{dx^2} = \cancel{\frac{4}{3}\frac{k}{b}}g6^2\alpha$$

$$\boxed{\frac{d^2\theta}{dx^2} = \frac{g\alpha}{\frac{c}{A}6^2}}$$

$$>0$$

Inny sposób rozwinięcia w szereg:

$$p = \frac{1}{s} = \frac{1}{u}\frac{du}{dz} \qquad dz = \frac{s}{u}\,du \qquad y = \frac{e^2}{k-1}$$

$$\beta = \frac{e^2}{\alpha u} = \frac{k-1}{\alpha}\frac{y}{u}$$

$$\frac{dp}{du} = -\frac{1}{s^2}\frac{ds}{du}$$

$$-(k+u)(1+u)u\frac{1}{s^3}\frac{ds}{du} - \frac{1}{s^2}\left[4u^2 - u(1+k) - 2k\right] + \frac{1}{s}\left[u^2+4u+3k\right] + u^2 + ku - u - k = 0$$

$$(k+u)(1+u)\,u\,\frac{ds}{du} + \left[4u^2 - u(1+k) - 2k\right]s - \left[u^2+4u+3k\right]s^2 - \left[u^2+ku-u-k\right]s^3 = 0$$

$$s = a_0 + a_1 u + a_2 u^2 + a_3 u^3 + a_4 u^4 + \dots$$

$$-2k a_0 - 3k a_0^2 + k a_0^3 = 0$$

$$k a_0 \left[2 + 3a_0 - a_0^2\right] = 0$$

Ad Ritter

$$-g = \frac{1}{\rho} \frac{dp}{dz}$$

$\rho =$ d. e Vol. ~) $\quad$ 1 kg Wolke : $\frac{1}{\rho}$ Vol. (Eis + Wasser)

" " " x kg Eis, $(1-x)$ kg Wasser

Eis spec. Vol. ω : $\rho_d = \frac{1}{\omega}$

Wasser spec. Vol. $v = 1$: $\rho_w = 1$

1 kg Wolke : Vol. $\omega x + 1 - x$

$$\therefore \quad \rho = \frac{1}{\omega x + 1 - x} \neq \frac{\rho_d}{x} \qquad \frac{1}{\rho} = \frac{x}{\rho_d}$$

$$\frac{x r}{T} - \frac{x_0 r_0}{T_0} = c \lg \frac{T_0}{T} \qquad \frac{r}{T} = A \omega \frac{dp}{dT}$$

$$A \omega x \frac{dp}{dT} = \frac{x_0 r_0}{T_0} + c \lg \frac{T_0}{T} = A \frac{x}{\rho_d} \frac{dp}{dT} = A \frac{1}{\rho} \frac{dp}{dT} = -A g \frac{dz}{dT}$$

$$-A g z = \frac{r_0}{T_0} T + c T \lg T_0 - c T \lg T + c T \Big|_{T_0}^{T}$$

$$+ A g H = r_0 + c T_0$$

Dabei vorausgesetzt dass c von Temp. [illegible]

1. Równ. II rzędu względem $\theta \; \rho \; \sigma$

I " względem $\theta \; \sigma$

I " $\rho \; \sigma$

$$p = \alpha \rho \theta = \frac{\alpha \theta b}{\sigma} \qquad \frac{dp}{dx} = \alpha b \left[\frac{1}{\sigma} \frac{d\theta}{dx} - \frac{\theta}{\sigma^2} \frac{d\sigma}{dx} \right]$$

$$= \alpha b \frac{1}{\sqrt{v}} \left[\frac{d\theta}{dx} - \frac{\theta}{2} \frac{dv}{dx} \right]$$

$$p - p_0 = \int_0^\theta \alpha b \left[\frac{1}{\sqrt{v}} \frac{d\theta}{dx} - \frac{\theta}{2} \frac{dv}{dx} \right] = f_1(\theta_0 v_0 \, b \, m)$$

$$\log p = \log \alpha b + \log \theta - \log \sigma$$

$$\frac{1}{p} \cdot \frac{dp}{dx} = \frac{1}{\theta} \frac{d\theta}{dx} - \frac{1}{2v} \frac{dv}{dx}$$

$$p_0 + \int_0^\theta = \frac{\alpha \theta b}{\sigma}$$

$$\frac{p_0}{\alpha b} + \underbrace{\int_0^\theta \left[\frac{1}{\sqrt{v}} \frac{d\theta}{dx} - \frac{\theta}{2} \frac{dv}{dx} \right]}_{\frac{\theta}{\sigma} - \frac{\theta_0}{\sigma_0}} = \frac{\theta}{\sigma}$$

$$v = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$$

$$\frac{1}{\sqrt{v}} = v^{-\frac{1}{2}} =$$

$$\frac{p_0}{\alpha b} = \frac{\theta_0}{\sigma_0}$$

$$\log p = A \log \frac{\theta}{\sigma}$$

Izotermiczny rozkład:

$$\rho = \frac{p}{\sigma}$$

$$0 = -g + \alpha\theta \frac{1}{\sigma}\frac{d\sigma}{dx} + \frac{4}{3}\frac{\mu}{b}\,\sigma \frac{d^2\gamma}{dx^2}$$

$$\frac{1}{\rho}\frac{d\rho}{dx} + \frac{1}{\sigma}\frac{d\sigma}{dx} = 0$$

Czy nie wystarczy podanie warunków w dwuch miejscach?

trzeba by podać $\sigma_1\ \sigma_2\ \rho_1$

$$p = \alpha\theta\rho$$

$$\frac{\partial p_1}{\partial r} + \rho_1 g + a\rho_1\rho_2(u_1 - u_2) = 0$$

$$\frac{dp_2}{dr} + \rho_2 g + a\rho_1\rho_2(u_2 - u_1) = 0$$

$p_1 = \alpha_1\rho_1\theta$ $\quad \frac{dp_1}{dr} = \alpha_1\theta\frac{d\rho_1}{dr}$ (jeśli θ = const)

$p_2 = \alpha_2\rho_2\theta$

$$\alpha_1\theta\frac{1}{\rho_1}\frac{d\rho_1}{dr} + g + a\rho_2(u_1 - u_2) = 0$$

$$\alpha_2\theta\frac{1}{\rho_2}\frac{d\rho_2}{dr} + g + a\rho_1(u_2 - u_1) = 0$$

$$\frac{d}{dr}\overbrace{(p_1 + p_2)}^{P} + (\rho_1 + \rho_2)g = 0$$

$$\left[\alpha_1\theta\frac{d\rho_1}{dr} + \rho_1 g\right] = -\left[\alpha_2\theta\frac{d\rho_2}{dr} + \rho_2 g\right]$$

$$\frac{d}{dx}(\rho_1 u_1) = 0 \qquad \frac{d}{dx}(\rho_2 u_2) = 0$$

$u = u + \mu$

$$\rho_1 u_1 = c_1 \qquad \rho_2 u_2 = c_2$$

Dla $r = 0$: $u_1 = u_2 = u_0$

$$\frac{\rho_{10}}{\rho_{20}} = \frac{c_1}{c_2}$$

$$\frac{\rho_1}{\rho_2} = \frac{c_1}{c_2}\frac{u_2}{u_1}$$

$$\cancel{\alpha_1\theta\frac{1}{\rho_1}\frac{d\rho_1}{dr} - g + a\left(\frac{\rho_2 c_1}{\rho_1} - c_2\right) = 0}$$

$= \frac{c_1^2}{\rho_1^2}\frac{d\rho_1}{dr} +$

$$\alpha_1\theta\frac{d\rho_1}{dr} + \rho_1 g + a(c_1\rho_2 - c_2\rho_1) = 0$$

$$\alpha_2\theta\frac{d\rho_2}{dr} + \rho_2 g + a(c_2\rho_1 - c_1\rho_2) = 0$$

$$\rho_1\rho_2(u_1 - u_2) = c_1\rho_2 - c_2\rho_1 = \rho_2 c_1\left[1 - \frac{c_2}{c_1}\frac{\rho_1}{\rho_2}\right]$$

$c_1 = \rho_{10}u$

$c_2 = \rho_{20}u$

$$= \rho_2 c_1\left[1 - \frac{\rho_{20}}{\rho_{10}}\frac{\rho_1}{\rho_2}\right]$$

$$= \rho_1 c_1\left[\frac{\rho_2}{\rho_1} - \frac{\rho_{20}}{\rho_{10}}\right]$$

$$\alpha_1\theta\frac{d^2\rho_1}{dr^2}+(g\pm ac_2)\frac{d\rho_1}{dr}+\cdots\frac{}{dr}=0$$

$$\alpha_2\theta\frac{d\rho_2}{dr}=(-g+ac_1)\rho_2-ac_2\rho_1$$

$$=\frac{(-g+ac_1)}{ac_1}\left[\rho_1(-g+ac_2)-\alpha_1\theta\frac{d\rho_1}{dr}\right]-ac_2\rho_1$$

$$\alpha_1\alpha_2\theta^2\frac{d^2\rho_1}{dr^2}+\alpha_2\theta(g\mp ac_2)\frac{d\rho_1}{dr}+(-g+ac_1)\left[\rho_1(-g+ac_2)-\alpha_1\theta\frac{d\rho_1}{dr}\right]-a^2c_1c_2\rho_1=0$$

$$\alpha_1\alpha_2\theta^2\frac{d^2\rho_1}{dr^2}+\left[\alpha_2(g\mp ac_2)+\alpha_1(g\mp ac_1)\right]\theta\frac{d\rho_1}{dr}+\left[(g\mp ac_1)(g\mp ac_2)-a^2c_1c_2\right]\rho_1=0$$

$\theta = t$

$$\rho_1=\cancel{A\,e^{\beta r}}\ A_1e^{-\beta r}\left[+Be^{-\gamma r}\right.$$

$$\alpha_1\alpha_2\theta^2\beta^2-\left[\alpha_2(g\mp ac_2)+\alpha_1(g\mp ac_1)\right]\theta\beta+(g\mp ac_1)(g\mp ac_2)-a^2c_1c_2=0$$

$$\left[\alpha_1\theta\beta-(g+ac_2)\right]\left[\alpha_2\theta\beta-g+ac_1\right]=a^2c_1c_2\qquad=\frac{a^2c_1c_2-(g-ac_1)(g-ac_2)}{\alpha_1\alpha_2}$$

$$(\theta\beta)^2-\left[\frac{g\pm ac_2}{\alpha_1}+\frac{g\pm ac_1}{\alpha_2}\right]\theta\beta+\cancel{\frac{g\pm ac}{\alpha_1}\frac{g\pm ac_1}{\alpha_2}}+\frac{g^2\pm g\,a(c_1+c_2)}{\alpha_1\alpha_2}$$

$$\theta\beta=+\frac{g\mp ac_2}{2\alpha_1}+\frac{g\mp ac_1}{2\alpha_2}\pm\sqrt{-\frac{(g\mp ac_1)(g\mp ac_2)}{2\alpha_1\alpha_2}+\frac{a^2c_1c_2}{\alpha_1\alpha_2}+}$$

$$\frac{(g+ac_1)^2}{4\alpha_2^2}+\frac{(g+ac_2)^2}{4\alpha_1^2}+g\,\cancel{\frac{\alpha_1)\ldots}{2\alpha_1\alpha_2}}$$

$$\alpha_1\alpha_2\theta^2\beta^2-g(\alpha_1+\alpha_2)\theta\beta+g^2=0\qquad \theta\beta=\frac{g}{\alpha_1}$$

$$(\alpha_1\theta\beta-g)(\alpha_2\theta\beta-g)=0\qquad \theta\beta=\frac{g}{\alpha_2}$$

$$\cancel{-2\alpha_1\alpha_2(g+ac_1)(g+ac_2)+4a^2c_1c_2\alpha_1\alpha_2+\alpha_1^2(g+ac_1)^2+\alpha_2^2(g+ac_2)^2}$$

$$\cancel{-2\alpha_1\alpha_2g^2\pm 2\alpha_1\alpha_2g(ac_1+ac_2)+2\alpha_1\alpha_2a^2c_1c_2+\alpha_1^2g^2+2\alpha_1^2gac_1+\alpha_1^2a^2c_1^2}$$

$$+4\qquad \cancel{+\alpha_2^2g^2+2\alpha_2^2gac_2+\alpha_2^2a^2c_2^2}$$

$$+2\alpha_1\alpha_2a^2c_1c_2$$

$$=\cancel{\sqrt{\frac{g+ac_1}{2\alpha_1}+\frac{ac_1}{}}}$$

$$g^2(\alpha_1-\alpha_2)^2+2ga(\alpha_1^2c_1+\alpha_2^2c_2-2\alpha_1\alpha_2c_1-2\alpha_1\alpha_2c_2)+a^2(\alpha_1^2c_1^2+\alpha_2^2c_2^2)$$

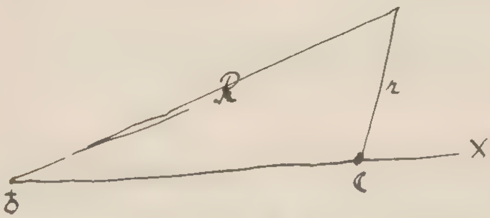
Izotermiczny rozkład dwuch gazów n.p. między ziemią i księżycem.

$u = u_1 = 0$

$$\left.\begin{aligned} \frac{\partial p_1}{\partial x} &= \rho_1 X_1 & \qquad \frac{\partial p_2}{\partial x} &= \rho_2 X_2 \\ \frac{\partial p_1}{\partial y} &= \rho_1 Y_1 & \qquad \frac{\partial p_2}{\partial y} &= \rho_2 Y_2 \\ \frac{\partial p_1}{\partial z} &= \rho_1 Z_1 & \qquad \frac{\partial p_2}{\partial y} &= \rho_2 Z_2 \end{aligned}\right\} \quad \cancel{\frac{\partial p_1}{\partial r} = \rho_1 F_1 \qquad \frac{\partial p_2}{\partial r} = \rho_2 F_2}$$

$$X_1 = X_2 = \cancel{\frac{kMx}{R^3}} \quad \frac{\partial U}{\partial x}$$

etc.



$$\int \frac{1}{\rho_1} dp_1 = U = \int \frac{1}{\rho_2} dp_2 \qquad U = \frac{kM}{R} + \frac{km}{r}$$

$p_1 = \alpha_1 \rho_1 \theta \qquad p_2 = \alpha_2 \rho_2 \theta$

$\lg p_1 = \lg \alpha_1 \theta_1 + \lg \rho_1$

$$U = \alpha_1 \theta \lg \rho_1 + \text{const} = \alpha_2 \theta \lg \rho_2 + \text{const} = \frac{kM}{R} + \frac{km}{r}$$

$$\alpha_1 \theta \lg p_1 + \text{const} = \alpha_2 \theta \lg p + \text{const} -$$

$$\alpha_1 \theta \log p_{10} + \text{const} = \frac{kM}{A}$$

$$\alpha_1 \theta \log \frac{p_1}{p_{10}} = \left[\frac{kM}{R} + \frac{km}{r} - \frac{kM}{A} - \cdot\right] \quad \Bigg\| \quad \alpha_2 \theta \lg \frac{p_2}{p_{20}} = \left[\cdot - \quad\right]$$

$$\alpha_1 \log \frac{p_1}{p_{10}} = \alpha_2 \log \frac{p_2}{p_{20}}$$

$$\left(\frac{p_1}{p_{10}}\right)^{\alpha_1} = \left(\frac{p_2}{p_{20}}\right)^{\alpha_2}$$

$$\frac{p_1}{p_{10}} = \left(\frac{p_2}{p_{20}}\right)^{\frac{\alpha_2}{\alpha_1}}$$

$$\frac{p_1}{p_2} = \frac{p_{10}}{p_{20}^{\frac{\alpha_2}{\alpha_1}}}\, p_2^{\left(\frac{\alpha_2}{\alpha_1}-1\right)}$$

Więc żeby stosunkowa gęstość dwuch gazów była niezmienna, $\frac{\alpha_2}{\alpha_1}-1$ musiałoby być $=0$, więc $\alpha_1=\alpha_2$

Inaczej ów gaz który ma większe α n.p. $\alpha_2>\alpha_1$ będzie miał stosunkowo ~~większe~~ mniejsze ciśnienie parcyalne tam gdzie p jest większe n.p. $\frac{\text{wodór}}{\text{hel}}$ ~~większe~~ mniejsze w bliskości

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible] $= -\frac{3}{2}$ [illegible]

$b = -\frac{1}{2}$ [illegible]

Izotermiczna masa gazu działająca na siebie grawitacyą i ciśnieniem

$$p = \alpha\rho\theta \qquad \theta = \text{const}$$

$$\rho = f.(r)$$

$$R = -\frac{k}{r^2}\int_0^r r^2\pi\rho\,dr = -\frac{k}{r^2}\int_0^r f(r)\,dr\,r^2\pi$$

$$R = \frac{1}{\rho}\frac{dp}{dr}$$

$$-\frac{k}{r^2}\int_0^r r^2\pi\rho\,dr = \frac{1}{\rho}\,\alpha\theta\,\frac{d\rho}{dr}$$

$$\cancel{\frac{\rho}{\rho}\int = \frac{\alpha\theta}{k}r^2\frac{d\rho}{dr}}$$

$$\cancel{\frac{d\rho}{dr}\int + \rho^2 = \frac{\alpha\theta}{k}\left(2r\frac{d\rho}{dr} + r^2\frac{d^2\rho}{dr^2}\right)}$$

$$-k\,\rho\,r^2\pi = \alpha\theta\,\frac{d}{dr}\left(r^2\frac{1}{\rho}\frac{d\rho}{dr}\right)$$

$$-\frac{k\pi}{\alpha\theta}r^2\rho = \frac{2r}{\rho}\frac{d\rho}{dr} - \frac{r^2}{\rho^2}\left(\frac{d\rho}{dr}\right)^2 + \frac{r^2}{\rho}\frac{d^2\rho}{dr^2}$$

$$E_{gr} = \int \rho U\,dr \qquad U = \frac{k\pi}{r}\int_0^r r^2\rho\,dr$$

$$= \cancel{\int} k\pi\int_0^r \frac{r^2\pi\,dr}{r}\,\rho\int_0^r r^2\rho\,dr$$

$$= k\pi^2\int_0^r r\rho\,dr\int_0^r r^2\rho\,dr = -k\pi^2\int_0$$

$$J_v = \int\left[u\frac{Du}{Dt} + v\frac{Dv}{Dt} + w\frac{Dw}{Dt} - (uX + vY + wZ) + c\frac{D\theta}{Dt}\right]dm +$$

$$J_s = \iint\left\{\begin{array}{l}(p_{xx}\cos nx + p_{xy}\cos ny + p_{xz}\cos nz)u + \\ (p_{yx}\cos nx + p_{yy}\cos ny + p_{yz}\cos nz)v \\ (p_{zx}\cos nx + p_{zy}\cos ny + p_{zz}\cos nz)w\end{array}\right\}dS = 0$$

$$J_s = -\iiint \frac{\partial[(p_{xx}u) + (p_{xy}v) + (p_{xz}w)]}{\partial x} + \frac{\partial}{\partial y}(p_{xy}u + p_{yy}v + p_{zy}w) + \frac{\partial}{\partial z}(p_{xz}u + --)$$

$$= -\iiint\Big[u\left(\frac{\partial p_{xx}}{\partial x} + \frac{\partial p_{xy}}{\partial y} + \frac{\partial p_{xz}}{\partial z}\right) + v\left(\frac{\partial p_{xy}}{\partial x} + \frac{\partial p_{yy}}{\partial y} + \frac{\partial p_{yz}}{\partial z}\right) + w(\quad)$$

$$+ p_{xx}\frac{\partial u}{\partial x} + p_{yy}\frac{\partial v}{\partial y} + p_{zz}\frac{\partial w}{\partial z} + p_{xy}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) + p_{yz}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) + p_{zx}(\quad)\Big]$$

$$+J_v = u\rho X + v\rho Y + w\rho Z + u\left[\frac{\partial p_{xx}}{\partial x} + \frac{\partial(p_{xy})}{\partial y} + \frac{\partial(p_{xz})}{\partial z}\right]$$

$$+ v\left[\frac{\partial p_{xy}}{\partial x} + \frac{\partial p_{yy}}{\partial y} + \frac{\partial p_{zy}}{\partial z}\right]$$

$$+ w\left[\frac{\partial p_{xz}}{\partial x} + \frac{\partial p_{yz}}{\partial y} + \frac{\partial p_{zz}}{\partial z}\right]$$

$$-(u\rho X + v\rho Y + w\rho Z) + c\rho\frac{D\theta}{Dt} \Big] d\tau = 0$$

$$= \iiint c\rho\frac{D\theta}{Dt} \pm p_{xx}\frac{\partial u}{\partial x} + \cdots \quad \pm p_{xy}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \pm p_{yz}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) + p_{zx}(\quad)$$

$$= \iiint c\rho\frac{D\theta}{Dt} + p\,\mathrm{div} + \frac{2}{3}\mu(\mathrm{div})^2 - 2\mu\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] - \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 - \mu(\quad)^2 - \mu(\quad)^2$$

$$\boxed{c\rho\frac{D\theta}{Dt} + p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = -\frac{2}{3}\mu(\mathrm{div})^2 + 2\mu[\quad] + \mu[\,\cdot\,—\,]}$$

$$A_{ex} = \int -p\,\sigma\, dS - \int \frac{2}{3}\mu \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2 - \frac{2}{3}\mu \begin{Bmatrix} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z} + \\ \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y \partial z} \\ + \frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 v}{\partial y \partial z} + \frac{\partial^2 w}{\partial z^2} \end{Bmatrix}$$

$$+ \mu \cancel{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2}$$

$$\cancel{+ \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2}$$

$$+ u\nabla^2 u + v\nabla^2 v + w\nabla^2 w$$

$$+ \cancel{2\frac{\partial v}{\partial x}\frac{\partial u}{\partial y} + 2\frac{\partial w}{\partial y}\frac{\partial v}{\partial z} + 2\frac{\partial u}{\partial z}\frac{\partial w}{\partial x}}$$

$$+ v\frac{\partial^2 u}{\partial x \partial y} + w\frac{\partial^2 u}{\partial x \partial z} + w\frac{\partial^2 v}{\partial y \partial z} + u\frac{\partial^2 v}{\partial y \partial x} + u\frac{\partial^2 w}{\partial z \partial x} + v\frac{\partial^2 w}{\partial z \partial y}$$

$$= 2\mu\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2 + \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + (\quad)^2$$

$$-\frac{2}{3}\mu\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2$$

$$\sigma p = b$$

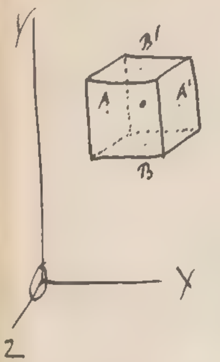
$$p = \alpha\theta\rho$$

$$= \frac{\alpha\theta b}{\sigma}$$

$$c\rho\,\sigma\frac{\partial\theta}{\partial x} + p\frac{\partial\sigma}{\partial x} = \frac{4}{3}\mu\left(\frac{\partial\sigma}{\partial x}\right)^2$$

$$c\frac{\partial\theta}{\partial x} + \frac{\alpha\theta}{\sigma}\frac{\partial\sigma}{\partial x} = \frac{4}{3}\frac{\mu}{b}\left(\frac{\partial\sigma}{\partial x}\right)^2$$

Direct method:



6 (triples of) forces:

~~path~~ of the centre : $u\Delta t$, $v\Delta t$, $w\Delta t$

path of point A : $u_{(x-\frac{\delta}{2}, y, z)}\Delta t$, $v_{(x-\frac{\delta}{2}, y, z)}\Delta t$, $w_{(x-\frac{\delta}{2}, y, z)}\Delta t$

A' : $u(x+\frac{\delta}{2}, y, z)\Delta t$, $v(x+\frac{\delta}{2}, y, z)\Delta t$ ~~ete~~

but the forces are not $p_{xx}\,\delta^2$ ~~dt~~. during the whole time,

but varying from $p_{xx\,(x-\frac{\delta}{2}, y, z)}\,\delta^2$ to $p_{xx\,(x+u\Delta t-\frac{\delta}{2}, y, z)}\left[\delta^2 + \frac{\partial(\delta^2)}{\partial u}\Delta t\right]$

$u(x+\frac{\delta}{2}, y, z)\Delta t.\ p_{xx}(x+\frac{\delta}{2}, y, z)\,\delta^2 - u_{(x-\frac{\delta}{2}, y, z)}\Delta t$ - - - -

$+ u(x, y+\frac{\delta}{2}, z)\Delta t\ p_{xy}(x, y+\frac{\delta}{2}, z)$

$$A_{ex} = \int -(p + \tfrac{2}{3}\mu\,\mathrm{div})\,6\,dS + \cancel{2\mu\left[u\frac{\partial u}{\partial x}\,dy\,dz + v\frac{\partial v}{\partial y}\,dx\,dy + w\frac{\partial w}{\partial z}\,dx\,dz\right]}$$

$$+\mu\left\{\left[u\frac{\partial u}{\partial x} + \left(u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} + w\frac{\partial w}{\partial x}\right)\right]dy\,dz + \left(v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)dy\,dz\right\}$$

$$+\mu\left[v\frac{\partial v}{\partial y} + \left(u\frac{\partial u}{\partial y} + v\frac{\partial v}{\partial y} + w\frac{\partial w}{\partial y}\right)\right]dz\,dx + \left(w\frac{\partial v}{\partial z} + u\frac{\partial v}{\partial x}\right)dz\,dx\ ($$

$$+\mu\left[w\frac{\partial w}{\partial z} + (\qquad)\right]dx\,dy + \left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y}\right)dx\,dy$$

$$\int \mu\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right]dx\,dy\,dz + \mu\left[u\frac{\partial^2 u}{\partial x^2} + v\frac{\partial^2 v}{\partial y^2} + w\frac{\partial^2 w}{\partial z^2}\right]dx\,dy\,dz$$

$+$ [illegible, struck out] $+$ [illegible, struck out]

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2 + u\frac{\partial^2 u}{\partial x^2} + v\frac{\partial^2 v}{\partial x^2} + w\frac{\partial^2 w}{\partial x^2} \quad + \frac{\partial v}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial u}{\partial z} + v\frac{\partial^2 u}{\partial x\partial y} + w\frac{\partial^2 u}{\partial x\partial z}$$

$$+\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 + u\frac{\partial^2 u}{\partial y^2} + v\frac{\partial^2 v}{\partial y^2} + w\frac{\partial^2 w}{\partial y^2} \quad \frac{\partial w}{\partial y}\frac{\partial v}{\partial z} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}$$

$$+ \quad + \quad + \quad + - \qquad \frac{\partial u}{\partial z}\frac{\partial w}{\partial x} + \frac{\partial v}{\partial z}\frac{\partial w}{\partial y}$$

$$-\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right]=\frac{1}{\rho}\left[u\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial y}+w\frac{\partial \rho}{\partial z}+\frac{\partial \rho}{\partial t}\right]$$

$$\int \cancel{p_{xx} u + p_{xy} v + p_{xz}}$$

$$A_{xx}=\iint\Big[(p_{xx}\cos nx+p_{xy}\cos ny+p_{xz}\cos nz)\,u+(p_{yx}\cos nx+p_{yy}\cos ny+p_{yz}\cos nz)\,v+ \quad (\qquad)\,w\Big]\,dS$$

$$\int U\Delta^2 V\,d\tau=-\int\left(\frac{\partial U}{\partial x}\frac{\partial V}{\partial x}+\frac{\partial U}{\partial y}\frac{\partial V}{\partial y}+\frac{\partial U}{\partial z}\frac{\partial V}{\partial z}\right)d\tau+\int U\frac{\partial V}{\partial n}\,dS$$

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2} \qquad =\int U\left(\frac{\partial V}{\partial x}\cos nx+\ \cdots\ \right)dS$$

$$\cancel{A_{xx}=\int p_{xx}u+p_{xy}}$$

$$=\int -\overbrace{p(u\cos nx+v\cos ny+w\cos nz)}^{6}\,dS-\frac{2}{3}\mu\,\mathrm{div}\,(u\cos nx+v\cos ny+w\cos nz)\,dS+$$

$$+2\mu\left(u\frac{\partial u}{\partial x}\cos nx+v\frac{\partial v}{\partial y}\cos ny+w\ \ \right)dS$$

$$+\mu\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)(w\cos ny+v\cos nz)+\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)(u\cos nz+w\cos nx)+\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)(v\cos nx+u\cos ny)=0$$

$$\mu\left[2\frac{\partial u}{\partial x}u+\frac{\partial w}{\partial y}v+\frac{\partial v}{\partial z}v+\frac{\partial w}{\partial x}w+\frac{\partial u}{\partial z}w\right]\frac{dy\,dz}{\cancel{\cos nx}}+-$$

$$\left[2\frac{\partial v}{\partial y}v+\frac{\partial w}{\partial y}w+\frac{\partial w}{\partial z}v+\frac{\partial v}{\partial y}u+\frac{\partial v}{\partial z}u\right]\frac{dx\,dz}{\cancel{\cos ny}}+$$

$$\left[2\frac{\partial w}{\partial z}w+\frac{\partial u}{\partial y}u+\frac{\partial v}{\partial x}u+\frac{\partial u}{\partial z}v+\frac{\partial w}{\partial x}v\right]\frac{dy\,dx}{\cancel{\cos nz}}$$

$$\rho \frac{Du}{Dt} = \rho X - \frac{\partial p}{\partial x} + \frac{\mu}{3}\frac{\partial\, div}{\partial x} + \mu \nabla^2 u \qquad X = -\frac{\partial U}{\partial x}$$

$$\rho \frac{Dv}{Dt} = \rho Y - \frac{\partial p}{\partial y} + \frac{\mu}{3}\frac{\partial\, div}{\partial y} + \mu \nabla^2 v$$

$$\rho \frac{Dw}{Dt} = \rho Z - \quad \ldots \quad - \qquad + \int^{D} p\, 6\, dS$$

$$\frac{D}{Dt} \int \left[\rho\frac{(u^2+v^2+w^2)}{2}\, d\tau + \rho U\, d\tau + \rho c\,\theta\, d\tau\right] = 0$$

$$\rho\, d\tau = dm$$

$$\frac{D}{Dt} \int\left[\frac{u^2+v^2+w^2}{2} + U + c\,\theta\right] dm + \int = 0 \qquad \frac{D\, dm}{Dt} = 0$$

$$\int\left[u\frac{Du}{Dt} + v\frac{Dv}{Dt} + w\frac{Dw}{Dt} + \underbrace{\frac{DU}{Dt}}_{= u\frac{\partial U}{\partial x} + v\frac{\partial U}{\partial y} + w\frac{\partial U}{\partial z}} + c\frac{D\theta}{Dt}\right] dm + \int = 0$$

jsi-li $\frac{\partial U}{\partial t} = 0$

i $\frac{\partial c}{\partial t} = \frac{\partial c}{\partial x} = \ldots = 0$

$$= -(uX + vY + wZ)$$

$$\int^{D} p\, 6\, dS + \int^{D} p_{xx}\, u\, dS + \ldots$$

$$\int\left\{-\frac{1}{\rho}\left[u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right] + \frac{\mu}{3\rho}\left(u\frac{\partial\, div}{\partial x} + v\frac{\partial\, div}{\partial y} + w\frac{\partial\, div}{\partial z}\right) + \frac{\mu}{\rho}\left(u\nabla^2 u + v\nabla^2 v + w\nabla^2 w\right)\right.$$

$$\left. + c\left(\frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right)\right] dm = 0$$

$$\int^{D} p\, 6\, dS + v$$

$$\int -\left[u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + \quad\right] + \frac{\mu}{3}\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\ .\ div\right) + \mu\left(u\nabla^2 u + v\nabla^2 v + .\ \right) +$$

$$+ c\rho\left(\frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial x} + -\quad\right)\Big] d\tau = 0$$

$$\int \mu\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) - \frac{\mu}{3}\left[(div)^2 + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 + \frac{\partial v}{\partial x}\frac{\partial u}{\partial x} + \left(\frac{\partial v}{\partial y}\right)^2 + \frac{\partial v}{\partial z}\frac{\partial w}{\partial z} + \right.$$

$$+ \frac{\partial w}{\partial x}\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial y} + \left(\frac{\partial w}{\partial z}\right)^2$$

$$\frac{4}{3}\,\mathrm{div}^2 + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \cdots \;\cancel{+}\; \frac{2}{3}\,\mathrm{div}^2 - 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right]$$

$$-\left[\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 + (\quad)^2\right]$$

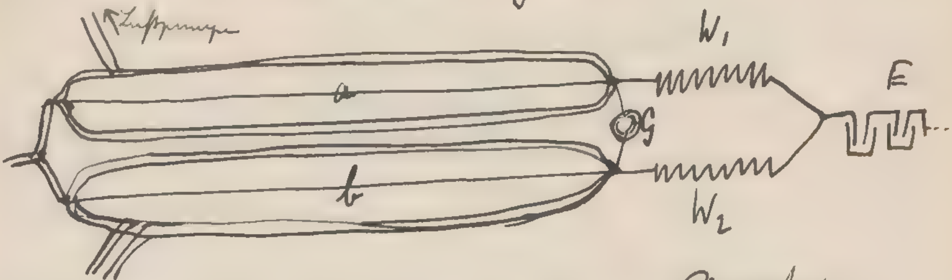
$$\mathrm{div}^2 - \left[\left(\frac{\partial u}{\partial x}\right)^2 + \cancel{\left(\frac{\partial u}{\partial y}\right)^2} + \cancel{\left(\frac{\partial u}{\partial z}\right)^2} + \cancel{2}\,\frac{\partial u}{\partial y}\frac{\partial v}{\partial x} + \cancel{2}\,\frac{\partial u}{\partial z}\frac{\partial w}{\partial x} - \cancel{\left(\frac{\partial u}{\partial y}\right)^2} - \cancel{\left(\frac{\partial u}{\partial z}\right)^2}\right]$$

$$-\left[\left(\frac{\partial v}{\partial y}\right)^2 + \frac{\partial u}{\partial y}\frac{\partial v}{\partial x} + \frac{\partial w}{\partial y}\frac{\partial v}{\partial z}\right]$$

$$-\left[\qquad\qquad\qquad\right]$$

$$= 2\left[\frac{\partial v}{\partial y}\frac{\partial u}{\partial x} - \cancel{2}\,\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\right] + 2\left(\frac{\partial u}{\partial x}\frac{\partial w}{\partial z} - \frac{\partial u}{\partial z}\frac{\partial w}{\partial x}\right) + 2(\qquad)$$

Verfahren zur Messung der Leitfähigkeit, Temperatur Coeff., derselben und Temperatursprung von Gasen.
(Ähnlich schon von Schleiermacher gemacht



a wird mit Gas gefüllt, b mit gut leitender Flüssigkeit
die Widerstands Änderung von a, ~~welche~~ Ausschlag in G ~~bewirkt~~ bewirkt;
entweder diesen messen oder Nullmethode; Compensirung durch
Änderung von W_1 oder Nebenschluss in a Leitung mit zu messendem
Widerstand; Strommessung ~~entweder~~ im W_1-Kreise durch Dorsol.
oder besser in W_1 und W_2 (oder auch ~~im~~ am besten gemeinsamen Kreise) durch
Silbervoltameter. Widerstände von a, b, G möglichst groß; E ebenso.
Anwendung von Drähten verschiedener Dicke (gleichzeitig) und
verschiedenem Material; bei dickerem Draht wird ~~W~~ Wärmeleitung
gemessen, bei ~~dünn~~ sehr dünnem (Wollaston $\frac{1}{20}$ mm) Temperatur-
sprung

$$-\frac{d\omega}{dt} = \frac{8\pi\mu a^3\omega_0}{\frac{4}{3}a^3\pi \cdot \frac{2a^2}{5}} = \frac{15\mu}{a^2}\omega_0 = \frac{15\mu \cdot 2\pi}{a^2 t} \qquad K\frac{d\omega}{dt} = P$$

$$t\frac{d\omega}{dt} = \frac{30\pi\mu}{a^2}$$ = zmiana ω ~~w sekundach~~ podczas dnia

$$\frac{365 \cdot 30 \cdot \pi\mu}{a^2} =$$ " " 1 roku

$$\frac{35 \cdot \cancel{10}^{3} \cdot 0{\cdot}00017}{36 \cdot 10^{16}} = 1{\cdot}7 \cdot 10^{-17} \cdot \frac{t}{2\pi} = 1{\cdot}7$$

$$\frac{\cancel{365} \cdot \cancel{30} \cdot \cancel{\pi} \cdot \cancel{0{\cdot}00017}^{50}}{\cancel{36}\ 10^{16}} \cdot \frac{86400}{2\cancel{\pi}} \qquad \frac{43 \cdot 5}{10^{15}} = 2 \cdot 10^{-13}$$

$$\Delta p = \frac{2}{3}\ \frac{2000000 \cdot 0{\cdot}00017}{6\ 10^{8}} \qquad \frac{4}{800}$$

$$p = 980 \cdot 76 \cdot 13{\cdot}6 = 10^{6}$$

$$\frac{\Delta p}{p} = \frac{10^{-6}}{10^{6}} = 10^{-12} = 10^{-9}\ \text{mm}$$

$$P = \frac{a^2\cancel{\pi}}{\cancel{5{\cdot}6} \cdot \frac{f}{3} a^3\cancel{\pi}} \Delta p \cdot 86400 \cdot 365 =$$ śr. γ = 1 m

$$= \frac{10^{-9} \cdot 3 \cdot 10^{7}}{15\ 7{\cdot}5 \cdot 3 \cdot 10^{8}} = \frac{10^{-10}}{15}$$

ony i
nacisk

jeszcze za
by motywó
uwałę z 28
tego sztanda-
...ków,
e.
di.

prze-
sz wi-

egiej
czad-

...k długo jeszcze
... Ale idealem byłoby
że ... naprawialiśmy, gdyby naród nasz nie ciężył na decyzyach w każdej fazie, jaką przynoszą zawikłania orężne i dyplomatyczne. Dlatego oczekiwać będziemy w tej chwili głosu G... su nietylko ..., ale po... go, który powinien odezwać się z ... łą przekonania, że przyszłość na... wymaga złączenia Galicyi z powstającą niepodległą Polską. Mamy nadzieję, ... się odezwie, i że w rokowaniu ... padnie z takim naciskiem, jakiego ... magają obecne stosunki i dobro Ojczyzny.

... żądań ...
innych grup narod...
„Zwieszcza ...
przez ...

5. ... ...

130

Thermal effect of effusion:

Natanson Wied. Ann. 37 p. 341 (1889)

ciepło udzielone naczyniu

$$\int \kappa \frac{\partial \vartheta}{\partial n}\, dF\, dt = \iiint \kappa\, \Delta^2 \vartheta\, dv\, dt = \left[\frac{c}{A} \frac{\partial \rho}{\partial t} + \rho \left(u \frac{\partial \vartheta}{\partial x} + v \frac{\partial \vartheta}{\partial y} + w \frac{\partial \vartheta}{\partial z}\right) + p \operatorname{div} - \Phi\right] dt\, dv$$

$$0 = \frac{\partial p_{xx}}{\partial x} + \frac{\partial p_{xy}}{\partial y} + \frac{\partial p_{xz}}{\partial z} = -\frac{\partial p}{\partial x} + \frac{\mu}{3}\frac{\partial \operatorname{div}}{\partial x} + \mu \nabla^2 u$$

$$\cancel{0 = -\frac{\partial p}{\partial x}}$$

$$\frac{\partial p}{\partial x} = \mu\left(\frac{1}{3}\frac{\partial \operatorname{div}}{\partial x} + \nabla^2 u\right)$$

$$\frac{\partial p}{\partial y} = \mu\left(\frac{1}{3}\frac{\partial \operatorname{div}}{\partial y} + \nabla^2 v\right)$$

$$\frac{\partial p}{\partial z} = \mu\left(\frac{1}{3}\frac{\partial \operatorname{div}}{\partial z} + \nabla^2 w\right)$$

$$p = R\rho\theta$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

$$\frac{c}{A}\rho\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right) + p \operatorname{div} = \Phi$$

Praca zewnętrzna

$$W_I = -\iint p(ul + vm + wn)\, d\sigma = -\iiint\left[\frac{\partial}{\partial x}(pu) + \frac{\partial}{\partial y}(pv) + \frac{\partial}{\partial z}(pw)\right] dv$$

$$W_{II} = \iiint \Phi\, dv$$

$$W_I = -R\iiint \theta\left[\frac{\partial}{\partial x}(\rho u) + \dots\right] + \rho\left[\frac{\partial}{\partial x}(\theta u) + \dots\right]$$

$$= -\iiint R\rho\left[u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + \dots\right] + p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

$$-W_I = \iiint \Phi + \left(R - \frac{c}{A}\right)\underbrace{\iiint \rho\left(u\frac{\partial\theta}{\partial x} + \dots\right)}_{\iint \theta\rho(ul + vm + wn)\, d\sigma}$$

$$= \left(1 - \frac{c}{AR}\right)\iint p(ul + vm + wn)\, d\sigma$$

$$-W_I = (k-1)\, W_{II}$$

$$-W_I = \iiint \left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right) + p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

2ᵈ term:

$$\underbrace{\iiint R\rho\left(u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z}\right) dv}_{} = \iiint \left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right) dv$$

$$\underbrace{\iint \underbrace{R\rho}_{p}\,\theta(ul + vm + \ldots) - \iiint R\theta\frac{\partial(\rho u)}{\partial x} + \ldots}_{= -W_I}$$

2ᵈ term 2 ; $\iiint p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) dv = 0$

$$-W_I = \iiint \left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right) dv =$$

$$= \mu \iiint \left(u\nabla^2 u + v\nabla^2 v + w\nabla^2 w\right) dv + \frac{1}{3}\left(u\frac{\partial\, \mathrm{div}}{\partial x} + v\frac{\partial\, \mathrm{div}}{\partial y} + w\frac{\partial\, \mathrm{div}}{\partial z}\right) dv$$

$$= \mu \iint u\left(\frac{\partial u}{\partial x} l + \frac{\partial u}{\partial y} m + \frac{\partial u}{\partial z} n\right) + v\left(\frac{\partial v}{\partial x} l + \cdots\right) + \cdots \quad dS +$$

$$+ \frac{1}{3}\iiint \mathrm{div}.(ul + vm + wn)\, dS$$

$$- \iiint \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \cdots \qquad + \frac{1}{3}(\mathrm{div})^2\right] dv$$

$$= \mu \iint \frac{\partial}{\partial n}\left(\frac{u^2+v^2+w^2}{2}\right) dS + \frac{\mu}{3}\iint v_n\, \mathrm{div}\, dS - \mu \iiint \uparrow$$

Z drugiej strony $W_{II} = \iiint \Phi \, dv =$

$$\mu \iiint \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \cdots \qquad + \frac{1}{3} (div)^2$$

$$+ 2\left[\frac{\partial w}{\partial y}\frac{\partial v}{\partial z} + \frac{\partial u}{\partial z}\frac{\partial w}{\partial x} + \frac{\partial v}{\partial x}\frac{\partial u}{\partial y} - \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial v}{\partial y}\frac{\partial w}{\partial z} - \frac{\partial w}{\partial z}\frac{\partial u}{\partial x}\right] dv$$

$$\iiint \left[\frac{\partial w}{\partial y}\frac{\partial v}{\partial z} - \frac{\partial v}{\partial y}\frac{\partial w}{\partial z}\right] dv = \iint w\left(m\frac{\partial v}{\partial z} - n\frac{\partial v}{\partial y}\right) dS - \cancel{\iiint w \frac{\partial^2 v}{\partial y \partial z} - w\frac{\partial^2 v}{\partial y\partial z}} \, dv$$

$$\iiint \cdots \qquad = \iint v\left(n\frac{\partial w}{\partial y} - m\frac{\partial w}{\partial z}\right) dS$$

$$W_{II} = \mu \iiint \left(\frac{\partial u}{\partial x}\right)^2 + \cdots \quad \frac{1}{3}(div)^2 + \mu \iint \Big\{ u\left(m\frac{\partial w}{\partial x} - l\frac{\partial w}{\partial z}\right) + w\left(l\frac{\partial u}{\partial z} - n\frac{\partial u}{\partial x}\right) +$$

$$\pm l u \frac{\partial u}{\partial x} \qquad \pm m v \frac{\partial v}{\partial y} \qquad \pm n w \frac{\partial w}{\partial z}$$

$$+ v\left(l\frac{\partial u}{\partial y} - m\frac{\partial u}{\partial x}\right) + u\left(m\frac{\partial v}{\partial x} - l\frac{\partial v}{\partial y}\right) + w\left(m\frac{\partial v}{\partial z} - n\frac{\partial v}{\partial y}\right) + v\left(n\frac{\partial w}{\partial y} - m\frac{\partial w}{\partial z}\right)\Big\} dS$$

zatem jeżeli można zaniedbać całki powierzchniowe to będzie

$W_{II} = T_I$ co z tamtym rezultatem. Tylko wtedy się zgadzać może

jeżeli $W_{II} = W_I = 0$

Albo też: uwzględniając $\iiint \rho \, dv = 0$

$$\iiint \Phi \, dv = \frac{c}{A}\iiint \rho\left(u\frac{\partial \vartheta}{\partial x} + \cdots\right) = -\frac{c}{AR} W_I$$ to samo co przedtem

$$W_{II} = \mu \iiint \left(\frac{\partial u}{\partial x}\right)^2 + \cdots \quad \frac{1}{3} div^2 + \mu \iint \left[\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\vartheta_n - div.\right] dS$$

$$= \cancel{+2\mu \iint l\left(v\frac{\partial u}{\partial y} - u\frac{\partial w}{\partial z}\right) + m \cdots \pm v\frac{\partial v}{\partial x} \pm u}$$

patrz Lamb p. 519

Czy możliwe rozwiązanie formy $\theta = f(p)$

$$\rho = \varphi(p) = \frac{p}{R\,f(p)}$$

$$\frac{\partial p}{\partial x} = \mu\left(\nabla^2 u + \frac{\partial\,\mathrm{div}}{\partial x}\right)$$

$$\frac{\partial p}{\partial y} =$$

$$\frac{\partial p}{\partial z} =$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

$$\text{II}).\quad \frac{c}{A}\rho\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right) + p\left(\frac{\partial u}{\partial x} + \dots\right) = \Phi$$

$$\left[\mu\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = -\left[u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right] + R\,\rho\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right)\right.$$

$$\frac{C}{A}\rho\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right) = \Phi + \left[u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right]$$

$$\frac{\partial\theta}{\partial x} = f'\frac{\partial p}{\partial x}$$

$$\frac{C}{AR}\frac{p f'}{f}\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right) = \Phi + \left[\quad -- \quad\right]$$

$$\frac{C}{AR} = \frac{k}{k-1}$$

Jeśli się zgadza równanie adiabatycznego: $p = c\rho^k$

$$\rho = c' p^{\frac{1}{k}} = \frac{p}{R f(p)}$$

$$f = p^{1-\frac{1}{k}}$$

$$f \sim p^{\frac{k-1}{k}}$$

$$f' \sim \frac{k-1}{k} p^{-\frac{1}{k}} \qquad \frac{p f'}{f} = \frac{k-1}{k}$$

zatem redukuje się $\frac{C}{AR}\frac{p f'}{f} \equiv 1$

Więc tylko możliwe jeśli $\Phi = 0$

~~Na prędkość ...~~

Jeśli $v = w = 0$:

$$\frac{\partial p}{\partial x} = \mu\left[\frac{1}{3}\frac{\partial^2 u}{\partial x^2} + \nabla^2 u\right] \qquad \frac{\partial(\rho u)}{\partial x} = 0$$

$$\frac{\partial p}{\partial y} = \mu\frac{1}{3}\frac{\partial^2 u}{\partial x \partial y} \qquad \frac{c}{A}\rho u\frac{\partial \theta}{\partial x} + p\frac{\partial u}{\partial x} = \Phi_u$$

$$\frac{\partial p}{\partial z} = \mu\frac{1}{3}\frac{\partial^2 u}{\partial x \partial z}$$

Jednowymiarowe zadanie:

$x = 0$: p_0, ρ_0

$x = l$: p_1

$$\frac{\partial p}{\partial x} = \frac{4}{3}\mu\frac{\partial^2 u}{\partial x^2} \qquad \rho u = \rho_0 u_0$$

$$\frac{c}{A}\rho u\frac{\partial \theta}{\partial x} + p\frac{\partial u}{\partial x} = \frac{4}{3}\mu\left(\frac{\partial u}{\partial x}\right)^2 \qquad \frac{c\rho\theta}{A} = \frac{1}{k-1}p$$

$$\frac{c}{A}\underbrace{\rho u\frac{\partial \theta}{\partial x}}_{=\frac{\partial}{\partial x}(\rho u \theta)} + \frac{\partial}{\partial x}(pu) = \frac{4}{3}\mu\left[u\frac{\partial^2 u}{\partial x^2} + \left(\frac{\partial u}{\partial x}\right)^2\right] = \frac{4}{3}\mu\frac{\partial}{\partial x}\left(u\frac{\partial u}{\partial x}\right)$$

$$p = R\rho\theta$$

$$\frac{c}{A}\rho u\theta + pu = \frac{4}{3}\mu\frac{\partial u}{\partial x}$$

$$p = \frac{4}{3}\mu\frac{du}{dx} + p_0 \qquad \frac{c}{A}\rho u\frac{\partial \theta}{\partial x} + \cancel{\frac{4}{3}\mu\left(\frac{\partial u}{\partial x}\right)^2} + p_0\frac{\partial u}{\partial x} = \cancel{\frac{4}{3}\mu\left(\frac{\partial u}{\partial x}\right)^2}$$

$$\frac{c}{A}\rho u\theta + p_0 u = [redacted] = a$$

$$\frac{c}{AR}pu + p_0 u = \Big(\underbrace{\frac{c}{AR}}_{\frac{1}{k-1}} + 1\Big)p_0 u + \frac{4}{3}\mu\frac{c}{AR}\,u\frac{du}{dx} = a$$

$$\frac{4}{3}\mu\,u\frac{du}{dx} + k p_0 u = (k-1)a$$

$$\frac{4}{3}\mu\, u \frac{du}{dx} + k p_0 u = (k-1) a \qquad \frac{\frac{4}{3} \mu u\, du}{(k-1)a - k p_0 u} = dx$$

Substytucya: $u = \alpha + z$

$$\frac{4}{3}\mu\,\alpha \frac{dz}{dx} + \frac{4}{3}\mu\, z \frac{dz}{dx} + k p_0 \alpha + k p_0 z = (k-1) a$$

$$\alpha = \frac{(k-1)a}{k p_0}$$

$$\frac{4}{3}\mu\, z \frac{dz}{dx} + \frac{4}{3}\mu\alpha \frac{dz}{dx} + k p_0 z = 0$$

$$\frac{4}{3}\mu \frac{dz}{dx} + \frac{4}{3}\mu\alpha \frac{d \log z}{dx} + k p_0 = 0$$

$$\frac{4}{3}\mu z + \frac{4}{3}\mu\alpha \log z + k p_0 x = b$$

$$\frac{4}{3}\mu (u-\alpha) + \frac{4}{3}\mu\alpha \log (u-\alpha) + k p_0 x = b$$ zamiast a lepiej używać α

$$\frac{4}{3}\mu \log\left[e^{u-\alpha} \cdot (u-\alpha)^{\alpha}\right] = b - k p_0 x$$

$$\frac{p u}{k-1} + p_0 u = \frac{k p_0}{k-1}\alpha$$

$$p = p_0 + \frac{4}{3}\mu \frac{du}{dx}$$

$$e^{u-\alpha}(u-\alpha)^{\alpha} = e^{\frac{b - k p_0 x}{\frac{4}{3}\mu}}$$

$$e^{u}(u-\alpha)^{\alpha} = B\, e^{-\frac{3}{4}\frac{k p_0}{\mu} x}$$

$$e^{u + \frac{3}{4}\frac{k p_0}{\mu} x} (u-\alpha)^{\alpha} = B$$

Dokonaj eliminacji: $\theta = \frac{p}{R\rho}$

$$\rho \frac{du}{dx} + u \frac{d\rho}{dx} = 0$$

$$-\frac{1}{\rho} \frac{d\rho}{dx} = + \frac{1}{u} \frac{du}{dx}$$

$$\frac{c}{AR} \rho u \left[\frac{1}{\rho} \frac{dp}{dx} - \frac{p}{\rho^2} \frac{d\rho}{dx} \right] + p \frac{du}{dx} = \frac{4}{3} \mu \left(\frac{du}{dx} \right)^2$$

$$\left. \begin{aligned} \frac{c}{AR} \left[u \frac{dp}{dx} + p \frac{du}{dx} \right] + p \frac{du}{dx} &= \frac{4}{3} \mu \left(\frac{du}{dx} \right)^2 \\ \frac{dp}{dx} &= \frac{4}{3} \mu \frac{d^2u}{dx^2} \end{aligned} \right\}$$

Zatem 3 stałe do oznaczenia p, u a jedna jeszcze do θ lub ρ

Stałe można by określić w ten sposób [illegible]:

$u = u_1 \quad : \quad x = 0$

$u = u_2 \quad : \quad x = l$

$$b = \frac{4}{3} \mu \log e^{u_1 - \alpha} (u_1 - \alpha)^\alpha$$

$$\frac{4}{3} \mu \log \left[e^{u - u_1} \cdot \left(\frac{u - \alpha}{u_1 - \alpha} \right)^\alpha \right] = -k p_0 x$$

$$k p_0 x = \frac{4}{3} \mu \log \left[e^{u_1 - u} \left(\frac{u_1 - \alpha}{u - \alpha} \right)^\alpha \right]$$

$$k p_0 l = \frac{4}{3} \mu \log \left[e^{-u_1 - u_2} \left(\frac{u_1 - \alpha}{u_2 - \alpha} \right)^\alpha \right]$$

$$\frac{x}{l} = \frac{\log}{\log} = \frac{(u_1 - u) + \alpha \log(u_1 - \alpha) - \alpha \log(u - \alpha)}{(u_1 - u_2) + \alpha \log(u_1 - \alpha) - \alpha \log(u_2 - \alpha)}$$

$$\left[e^{(u_1 - u_2)} \left(\frac{u_1 - \alpha}{u_2 - \alpha} \right)^\alpha \right]^x = e^{(u_1 - u) l} \left(\frac{u_1 - \alpha}{u - \alpha} \right)^{\alpha l}$$

$\alpha =$ prędkość dla $x = \infty$

$$p_\infty \alpha + (k-1) p_0 \alpha = k p_0 \alpha$$

$$p_\infty = p_0$$

$$\left(\frac{du}{dx} \right)_\infty = 0$$

$$p = p_\infty \frac{k\alpha - (k-1)u}{u} = p_\infty\left(k\frac{\alpha}{u} - k + 1\right) = p_\infty\left[1 + k\left(\frac{\alpha}{u} - 1\right)\right]$$

$u \leqq \alpha$

$x = 0 : u = u_1$

$x = \infty : u = \alpha$

$$k p_\infty x = \frac{4}{3}\mu \log\left[e^{u_1 - u}\left(\frac{\alpha - u_1}{\alpha - u}\right)^\alpha\right]$$

$x = \infty : p = p_\infty$

$p u = \mathrm{const}$

$$\frac{p}{\theta} u = \mathrm{const} = \frac{p_\infty \alpha}{\theta_\infty}$$

$$\theta = \frac{p u}{p_\infty \alpha}\theta_\infty = \theta_\infty \cdot \frac{u + k(\alpha - u)}{\alpha} = \theta_\infty\left[k - (k-1)\frac{u}{\alpha}\right]$$

$$\frac{4}{3}\mu\frac{du}{dx} = p - p_0$$

Więc istotnie: jeśli $u_1 < u < \alpha$

to: $0 < x < \infty$

$$p_\infty\left[1 + k\left(\frac{\alpha}{u_1} - 1\right)\right] > p > p_\infty$$

$$> \frac{du}{dx} > 0$$

$$\theta_\infty\left[k - (k-1)\frac{u_1}{\alpha}\right] > \theta > \theta_\infty$$

Więc dane muszą być 4 wielkości!

Wyznaczę związek $p\ \theta$:

$$\frac{1}{k}\frac{p - p_\infty}{p_\infty} = \left(\frac{\alpha}{u} - 1\right)$$

$$\frac{u}{\alpha} = \frac{1}{1 + \frac{1}{k}\left(\frac{p}{p_\infty} - 1\right)}$$

$\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3}$

$$\frac{p}{p_0} = \frac{u_0}{u}$$

$$x = \frac{\frac{3}{2}(u^2 - u_0^2)}{p_0 u}$$

$$p\frac{u}{\theta} = \mathrm{const} = \frac{p_0 u_0}{\theta} \quad \| \quad \theta = \mathrm{const}$$

$$x = \frac{3}{2}\frac{u}{p_0}\left[1 - \left(\frac{u_0}{u}\right)^2\right]$$

$$= \frac{3}{2}\frac{u_0}{p}\left[1 - \left(\frac{u_0}{u}\right)^2\right] = \frac{3}{2}\frac{u_0}{p}\left[1 - \left(\frac{p}{p_0}\right)^2\right]$$

$$= \frac{3}{2}\frac{u_0}{p_0}\left[\frac{p_0}{p} - \frac{p}{p_0}\right]$$

$$\theta = \theta_\infty \left[k - \frac{k-1}{1 + \frac{1}{k}\left(\frac{p}{p_\infty} - 1\right)} \right] = \theta_\infty \frac{k + \frac{p}{p_\infty} - 1 - k + 1}{1 - \frac{1}{k} + \frac{1}{k}\frac{p}{p_\infty}}$$

$$= \theta_\infty \frac{p}{p_\infty} \frac{k}{k - 1 + \frac{p}{p_\infty}}$$

Zatem: $\rho_\infty = \rho \dfrac{k}{k-1+\frac{p}{p_\infty}}$

$$\rho = \rho_\infty \left[\frac{k-1}{k} + \frac{p}{k\,p_\infty} \right]$$

$u_0 = 30000$ $\quad p_0 = 1\text{ atm.} = 10^6$

$\frac{p}{p_0} = \frac{1}{2}$ $\quad x = \frac{9}{4}\,\frac{3}{100} = \frac{27}{400} = 0{\cdot}0675$ cm

$\frac{p}{p_0} = \frac{1}{10}$ $\quad x = \frac{9}{4}\,\frac{10}{100} = \frac{9}{40} = 0{\cdot}225$ cm

---

Zamiast tego można także: $\boxed{p_0} = p_\infty \left[1 + k\left(\frac{\alpha}{u_0} - 1\right) \right]$

$\boxed{p_\infty}$

$\boxed{u_0}$

$\boxed{\theta_0}$

Jeśli $\lim p_\infty = 0$: u_0 = skończone przy $\alpha = \infty$

$$\frac{p_0}{p} = \lim \frac{1 + k\left(\frac{\alpha}{u_0} - 1\right)}{1 + k\left(\frac{\alpha}{u} - 1\right)} = \frac{u}{u_0} \quad \Big\| \quad \lim_{\alpha=\infty} \log \left(\frac{\alpha - u_1}{\alpha - u}\right)^\alpha = \log \left(\frac{1 - \frac{u_1}{\alpha}}{1 - \frac{u}{\alpha}} \right)^\alpha -$$

$$= \log \left(1 + \frac{u - u_1}{\alpha} \right)^\alpha = (u - u_1)$$

$\cancel{k\,p_\infty x = \frac{4}{3} = (u_1 - u) + \ldots}$

$$p_\infty = \frac{p_0}{1 + k\left(\frac{\alpha}{u} - 1\right)} \quad \Big\| \quad k\,p_0 x = \frac{4}{3}\mu \left[1 + k\left(\frac{\alpha}{u} - 1\right) \right] \left\{ u_1 - u + \alpha \left[\log(\alpha - u_1) - \log(\alpha - u) \right] \right\}$$

$\lim \alpha = \infty$

$$\cancel{\log \frac{k\,p_0 x}{\frac{4}{3}\mu}} \quad \frac{u_1 - u + \alpha\left[\log \ldots\right.}{\frac{1}{1 + k\left(\frac{\alpha}{u} - 1\right)}} = \frac{\log \frac{\alpha - u_1}{\alpha - u} + \alpha \left(\frac{1}{\alpha - u_1} - \frac{1}{\alpha - u} \right)}{- \frac{\frac{k}{u}}{\left[1 + k\left(\frac{\alpha}{u} - 1\right)\right]^2}}$$

$$= - \frac{-\frac{u_1}{\alpha} + \frac{1}{2}\left(\frac{u_1}{\alpha}\right)^2 + \frac{u}{\alpha} - \frac{1}{2}\left(\frac{u}{\alpha}\right)^2 + 1 + \frac{u}{\alpha} + \left(\frac{u_1}{\alpha}\right)^2 - 1 - \frac{u}{\alpha} - \left(\frac{u_1}{\alpha}\right)^2}{\frac{k}{u}} \left(\frac{k}{u}\right)^2 \alpha^2$$

$$= -\frac{k}{u}\,\frac{3}{2}\left(u_1^2 - u^2\right)$$

Powolne

Ruchy gazów rozrzedzonych, zaniedbując: ρ, p itd.

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\mu}{3}\frac{\partial \operatorname{div}}{\partial x} + \mu\nabla^2 u \qquad \frac{\partial}{\partial x} \qquad \frac{\partial}{\partial y}$$

$$\rho\frac{\partial v}{\partial t} = \qquad \frac{\partial}{\partial y} \qquad -\frac{\partial}{\partial x}$$

$$\rho\frac{\partial w}{\partial t} =$$

$$\frac{\mu}{3}\nabla^2 \operatorname{div} v + \mu\nabla^2 \operatorname{div} v = 0$$

$$\nabla^2 \operatorname{div} v = 0$$

$$\underbrace{\nabla^2\xi = \nabla^2\eta = \nabla^2\zeta = 0}_{\nabla^2 \operatorname{curl} v = 0}$$

$\cancel{\nabla^2 \operatorname{curl} v = 0}$

$\cancel{v = \nabla U + \operatorname{curl} V}$ $\qquad \cancel{\nabla^2\nabla^2 U = 0}$

Jeśli przestrzeń wielka, tak że $\operatorname{div} v = 0$ i $\operatorname{curl} v = 0$ w wielkiej odległości, wtedy: ruch taki jak cieczy nieściśliwych z potencyałem prędkości:

[nie ma zjawisk akustycznych!
ruch ustanawia się natychmiastowo

$$u = \frac{\partial \varphi}{\partial x} \qquad v = \frac{\partial \varphi}{\partial y} \qquad w = \frac{\partial \varphi}{\partial z}$$

$$\operatorname{div} v = \frac{\partial u}{\partial x} + \cdots = \nabla^2\varphi = 0$$

$$\nabla^2 u = \frac{\partial}{\partial x}\nabla^2\varphi = 0$$

N. p. ~~Kula w przestrzeni~~

$$\cancel{\varphi = \frac{a}{r} \qquad u = -\frac{x}{r^3}a \qquad v = -\frac{y}{r^3}a \qquad w = -\frac{z}{r^3}a}$$

$$\frac{\partial u}{\partial x} = \left(-\frac{1}{r^3} + \frac{3x^2}{r^5}\right)a = \frac{a}{r^3}\left(-1 + \frac{3x^2}{r^2}\right)$$

To byłoby niemożliwe, bo na powierzchni byłyby składowe ruchu ślizganie. Ale ów wniosek przedwczesny: bo $\operatorname{curl} v$ nie będzie $= 0$ na powierzchni

$\operatorname{div} v$ ini potencyał

rota

Równanie ciepła tutaj nie można użyć, bo wtedy lim ρ=0 tutaj zatem lim θ = ∞ ażeby uwzględnić ilość ciepła wytworzonego przez tarcie.

Równowaga ustanowi się wskutek przewodnictwa ciepła które tutaj dopiero wchodzi znacznie w rachuby, tak że będzie:

$$\kappa \nabla^2 \theta = \Phi$$

linie wkonujące
kierunek curl na powierzchni!

~~N. p. kula w przestrzeni~~

$$\cancel{\varphi = \frac{Ca^2 x}{r^3}} \qquad \cancel{\frac{\partial\varphi}{\partial x} = \frac{Ca^2}{r^3}\left[1-\frac{3x^2}{r^2}\right] = u}$$

$$\cancel{\frac{\partial\varphi}{\partial y} = 3Ca^2\frac{xy}{r^5} = v}$$

$$\operatorname{div} v = -\,\cancel{(v\nabla \log)} \qquad \cancel{\frac{\partial\varphi}{\partial z} = 3Ca^2\frac{xz}{r^5} = w}$$

$$-\{v \cdot \nabla \log \rho\}$$

$$v = \nabla\varphi + \nabla\int \frac{\operatorname{div} v}{r}\,dv + \operatorname{curl}\int\frac{\operatorname{curl} v}{r}\,dv$$

$$\nabla^2 \operatorname{div} v = \operatorname{div}\nabla^2 v \quad ? \qquad \nabla^2 \operatorname{curl} v = \operatorname{curl}\nabla^2 v$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\cdots\right)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial}{\partial z}\right) \Big| \ \frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\cdots$$

zatem Resultat: $\operatorname{div}\nabla^2 v = 0$

$\operatorname{curl}\nabla^2 v = 0$

$$\therefore \quad \nabla^2 v = \nabla U \qquad U = -\frac{1}{3}\operatorname{div} v$$

bezwzględnie!

$$\underbrace{\nabla^2 v}_{\nabla \operatorname{div} v - \operatorname{curl}^2 v} = -\frac{1}{3}\nabla \operatorname{div} v$$

$$\operatorname{curl}^2 v = \frac{4}{3}\nabla\operatorname{div} v$$

N_{γ}. [illegible] $v = w = 0$ $dw = 0$

$$\frac{1}{3}\frac{\partial^2 u}{\partial x^2} + \cancel{0} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}} = 0$$

$$\frac{1}{3}\frac{\partial^2 u}{\partial x \partial y} = 0$$

$$\frac{1}{3}\frac{\partial^2 u}{\partial z \partial x} = 0$$

$\frac{\partial u}{\partial x} = f(x)$

[illegible] $x = c$

$\frac{\partial u}{\partial x} = 0$

$$u = \frac{y}{\delta} c \qquad \frac{\partial u}{\partial y} = \frac{c}{\delta} \qquad \Phi = \mu \left(\frac{\partial u}{\partial y}\right)^2 = \mu \frac{c^2}{\delta^2}$$

$$-k \frac{\partial^2 \theta}{\partial y^2} = \mu \frac{c^2}{\delta^2}$$

$$\frac{d\theta}{dy} = -\frac{\mu}{k}\frac{c^2}{\delta^2} y + a$$

$$\theta = -\frac{\mu}{k}\frac{c^2}{\delta^2}\frac{y^2}{2} + ay + \theta_0$$

$$\cancel{\theta_0} = -\frac{\mu}{k}\frac{c^2}{\delta^2}\frac{\delta^2}{2} + a\delta + \cancel{\theta_0}$$

$\cancel{\theta - \theta_0 = \frac{\mu}{k}\frac{c^2}{\delta^2}\frac{y^2 \delta^2}{2}}$

$$a = -\frac{\mu}{k}\frac{c^2}{\delta^2}\frac{\delta}{2}$$

$$\theta = \frac{\mu}{k}\frac{c^2}{\delta^2}\frac{y(\delta - y)}{2} + \theta_0$$

Maximum: $y = \frac{\delta}{2}$ | $\theta = \frac{\mu}{k}\frac{c^2}{8} + \theta_0$

Niezależnie od grubości warstwy N_p. $\mu = 0{\cdot}00017$

$k = 42 . 10^6 . \; 0{\cdot}000057$

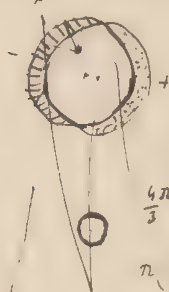
$$\frac{\mu}{8k} = \frac{1{\cdot}7 . 10^{-4}}{8 . 42 . \; 0{\cdot}57 . 10^2} = 10^{-8}$$

więc dopiero przy $q = 10^4 \frac{cm}{s}$

[illegible]

Wszystkie ~~ppp~~ wyrazy terminy będą bardzo nieznaczne przy zwykłych przedmiotach

$$\iint \rho \frac{dS \cos nx}{r}$$



$$\frac{1}{3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)+\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$$

$$\frac{1}{3}\frac{\partial}{\partial y}\left(\qquad\right)+\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}=0$$

$$\frac{1}{3}\frac{\partial}{\partial z}\left(\qquad\right)+\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}=0$$

Obierając dowolnie $\operatorname{div} v = U = f(xyz)$ otrzyma się

$$\frac{\partial}{\partial x}\iiint \rho \frac{dv}{r} = \dots$$

$$w = -\frac{1}{3}\int \frac{\frac{\partial U}{\partial z}}{r}\, dv + A_3$$

$$v = -\frac{1}{3}\int \frac{\frac{\partial U}{\partial y}}{r}\, dv + A_2$$

$$u = -\frac{1}{3}\int \frac{\partial U}{\partial x}\frac{dv}{r} + A_1$$

$$v = -\frac{1}{3}\int \frac{\nabla U}{r}\, dv + \mathfrak{A} \qquad \nabla^2 \mathfrak{A} = 0 \qquad \operatorname{div} \mathfrak{A} = 0$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} = -\frac{1}{3}\int \frac{\overbrace{\nabla^2 U}^{=0}}{r}\, dv + \frac{\partial A_1}{\partial x}+\frac{\partial A_2}{\partial y}+\frac{\partial A_3}{\partial z} \qquad + \iint \frac{\partial U}{\partial x}\frac{dS \cos nx}{r}$$

$$\nabla^2 u = -\frac{1}{3}\int \frac{\partial(\nabla^2 U)}{\partial x}\frac{dv}{r} \qquad \frac{\partial A_1}{\partial x}+\frac{\partial A_2}{\partial y}+\frac{\partial A_3}{\partial z} = c = 0?$$

Ad. —

wyglądających inertia terms:

dodać należy: $W_I = \iiint u \frac{\partial p}{\partial x} + \quad \ldots$

$$\iiint \frac{\rho}{\mu} u \left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}\right] + v \left[u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}\right] + w \left[u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z}\right] dv = \iiint \frac{\rho}{\mu}\left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}\right) \cdot \frac{u^2+v^2+w^2}{2}\, dv$$

$$= -\iint \frac{u^2+v^2+w^2}{2} \frac{\rho}{\mu}(ul + vm + wn)\, dS + \cancel{\iiint \frac{u^2+v^2+w^2}{2} \frac{1}{\mu}\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right]}$$

Więc dowód się stosuje tak samo

Całka powierzchniowa

$$\mu \iint u \left(n \frac{\partial w}{\partial x} - l \frac{\partial w}{\partial z} + m \frac{\partial v}{\partial x} - l \frac{\partial v}{\partial y}\right) + \ldots$$
$$\pm l \frac{\partial u}{\partial x}$$

$$= \iint u \frac{\partial}{\partial x}(lu + mv + nw) - lu\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$
$$+ v \frac{\partial}{\partial y} \qquad - mv(\qquad)$$
$$+ w \frac{\partial}{\partial z} \qquad - nw(\qquad)$$

$$= \iint \left[\left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}\right) v_n - v_n \cdot \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)\right] dS$$

zatem w całości:

$$W_{II} = W_I + \mu \iint \left[\frac{\partial}{\partial n}\left(\frac{u^2+v^2+w^2}{2}\right) + \frac{1}{3} v_n \operatorname{div} + \left(u\frac{\partial}{\partial x} + \cdots\right) v_n - v_n \operatorname{div} \right] dS$$

$$W_{II} = W_I + \mu \iint \left[\frac{\partial}{\partial n}\left(\frac{v^2}{2}\right) + (v \nabla) \mathfrak{S}(v\, n) - \frac{2}{3} \operatorname{div} v\, \mathfrak{S}(v\, n) \right] dS$$

$$W_{II} = \frac{-W_I}{k-1}$$

$$0 = W_I \left(1 + \frac{1}{k-1}\right) + \mu \iint$$

$$W_I = -\frac{k-1}{k} \mu \iint \cdots$$

$$W_{II} = \frac{1}{k} \mu \iint \frac{\partial}{\partial x} \left[u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial x} + w \frac{\partial w}{\partial x} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - \right.$$

$$\left. - \frac{2}{3} u \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right] \right]$$

$$= \frac{\mu}{k} \iint \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial x}\right)^2 + \frac{\partial v}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial u}{\partial z} - \frac{2}{3}\frac{\partial u}{\partial x}\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right]$$

$$+ \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \frac{\partial w}{\partial y}\frac{\partial v}{\partial z} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y}\left[\qquad \right]$$

$$+ \left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 + (\quad) + \left(\frac{\partial w}{\partial z}\right)^2 + \frac{\partial u}{\partial z}\frac{\partial w}{\partial x} + \frac{\partial v}{\partial z}\frac{\partial w}{\partial y} - \frac{2}{3}\frac{\partial w}{\partial z}\left[\qquad \right]$$

$$+ \underline{\underline{u\, 2\frac{\partial^2 u}{\partial x^2}}} + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y}\right) + w\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial z}\right) - \frac{2}{3} u \underline{\underline{\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z}\right)}}$$

$$+ v\, 2\frac{\partial^2 v}{\partial y^2} + w\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 v}{\partial y \partial z}\right) + u \underline{\underline{\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial y \partial x}\right)}} - \frac{2}{3} v \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y \partial z} + \frac{\partial^2 u}{\partial y \partial x}\right)$$

$$+ w\, 2\frac{\partial^2 w}{\partial z^2} + u \underline{\underline{\left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x \partial z}\right)}} + v\left(\frac{\partial^2 v}{\partial z^2} + \frac{\partial^2 w}{\partial y \partial z}\right) - \frac{2}{3} w \left(\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 v}{\partial y \partial z}\right)$$

$$u\left[\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\frac{1}{3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\right] = \frac{2}{3}\,u\,\frac{\partial}{\partial x}\left(\quad\right)$$

$$= u\,\frac{\partial p}{\partial x}$$

$$\frac{1}{k}\left[\Phi + u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}\right] = \Phi$$

$$u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z} = (k-1)\,\Phi$$

Czy to ogólny rezultat?

Two dimensional prob.

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial \chi}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \qquad \frac{\partial}{\partial y} \qquad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial \chi}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \qquad \frac{\partial}{\partial x} \qquad \frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=\zeta$$

$$u\frac{\partial \zeta}{\partial x}+v\frac{\partial \zeta}{\partial y}=\mu\left(\frac{\partial^2 \zeta}{\partial x^2}+\frac{\partial^2 \zeta}{\partial y^2}\right)$$

$$\cancel{\frac{\partial u}{\partial x}\frac{\partial \zeta}{\partial x}+\frac{\partial v}{\partial x}\frac{\partial \zeta}{\partial y}+u\frac{\partial^2 \zeta}{\partial x^2}+v\frac{\partial^2 \zeta}{\partial x\partial y}=\mu\frac{\partial}{\partial x}(\nabla^2\zeta)}$$

$$\cancel{\frac{\partial u}{\partial y}\frac{\partial \zeta}{\partial x}+\frac{\partial v}{\partial y}\frac{\partial \zeta}{\partial y}+u\frac{\partial^2 \zeta}{\partial x\partial y}+v\frac{\partial^2 \zeta}{\partial y^2}=\mu\frac{\partial}{\partial y}(\nabla^2\zeta)}$$

$$\frac{\partial v}{\partial x}\frac{\partial \zeta}{\partial y}-\frac{\partial v}{\partial y}\frac{\partial \zeta}{\partial x}+u\frac{\partial^2 \zeta}{\partial x^2}+v\frac{\partial^2 \zeta}{\partial x\partial y}=\mu\frac{\partial \nabla^2\zeta}{\partial x}$$

$$v=\nabla\cdots+\operatorname{curl}\cdots$$

$$\frac{\partial u}{\partial x}\frac{\partial \zeta}{\partial x}+\frac{\partial u}{\partial y}\frac{\partial \zeta}{\partial y}+\zeta\frac{\partial \zeta}{\partial y}+u\frac{\partial^2 \zeta}{\partial x^2}+v\frac{\partial^2 \zeta}{\partial x\partial y}=$$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \cancel{\frac{\partial}{\partial z}} \\ F & G & H \end{vmatrix}$$

$$\cancel{u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}}-v\zeta=-\frac{\partial \chi}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\cancel{u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}}\quad u\zeta=-\frac{\partial \chi}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

$$u=\frac{\partial H}{\partial x}+\frac{\partial H}{\partial y}$$

$$v=\frac{\partial H}{\partial y}-\frac{\partial G}{\partial x}$$

~~by method~~:

$$u=\frac{\partial \psi}{\partial y}$$

$$v=-\frac{\partial \psi}{\partial x}$$

$$\cancel{\frac{\partial^2 \varphi}{\partial x^2}+\frac{\partial^2 \varphi}{\partial y^2}=0}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0:\ \cancel{\nabla^2 H+\frac{\partial^2}{\partial x\partial y}(H-G)=0}$$

$$\cancel{\zeta=-\left(\frac{\partial^2 G}{\partial x^2}+\frac{\partial^2 H}{\partial y^2}\right)}$$

$$\zeta=-\left(\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2}\right)$$

$$\frac{\partial \psi}{\partial y}\frac{\partial}{\partial x}\left(\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2}\right)-\frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}\left(\quad\right)=\mu\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2}\right)$$

Dla μ bardzo małych lewa strona będzie prawie $=0$

zatem $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = f(\psi)$

zatem prawa strona:

$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) f(\psi) = 0$ Nieprawda! dowolne!

Ale można spróbować rozwinąć:

$\psi = \psi_0 + \mu \left(\frac{\partial \psi}{\partial \mu}\right)_0$ $\quad \psi = f(x, y, \mu)$

$\psi(x,y,\mu) = \psi(x, y, \mu=0) + \mu \left[\frac{\partial \psi(x,y,\mu)}{\partial \mu}\right]_{\mu=0}$

$v = w = 0$

$$\frac{\partial p}{\partial x} = \mu\left[\frac{1}{3}\frac{\partial^2 u}{\partial x^2} + \nabla^2 u\right]$$

$$\frac{\partial p}{\partial y} = \mu\frac{1}{3}\frac{\partial^2 u}{\partial x \partial y}$$

$$\frac{\partial p}{\partial z} = \mu\frac{1}{3}\frac{\partial^2 u}{\partial x \partial z}$$

$$\frac{\partial}{\partial x}(\rho u) = 0 \qquad \underline{\rho u = f(y,z)} \qquad u\frac{\partial \rho}{\partial x} + \rho\frac{\partial u}{\partial x} = 0$$

$$\frac{c}{A}\rho u \frac{\partial \theta}{\partial x} + p\frac{\partial u}{\partial x} = \Phi_u$$

$p = R\rho\theta$

$\theta = \frac{p}{R\rho}$

$$\frac{\partial^2 p}{\partial x \partial y}$$

$$\left.\begin{aligned} &\frac{\partial}{\partial y}(\nabla^2 u) = 0 \\ &\frac{\partial}{\partial z}(\nabla^2 u) = 0 \\ &\underline{\nabla^2 p = \frac{4}{3}\mu\frac{\partial}{\partial x}(\nabla^2 u)} \end{aligned}\right\} \qquad \underline{\nabla^2 u = f.(x)}$$

$$\nabla^2 p = \frac{4}{3}\mu\nabla^2\frac{\partial u}{\partial x}$$

$$\frac{c}{AR}\rho u\left[\frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{p}{\rho^2}\frac{\partial \rho}{\partial x}\right] + p\frac{\partial u}{\partial x} = \Phi$$

$$\frac{c}{AR}\left[u\frac{\partial p}{\partial x} - \underbrace{\frac{pu}{\rho}\frac{\partial \rho}{\partial x}}_{+p\frac{\partial u}{\partial x}}\right] + p\frac{\partial u}{\partial x} = \Phi$$

$$\frac{c}{AR}u\frac{\partial p}{\partial x} + \left[\frac{c}{AR} + 1\right]p\frac{\partial u}{\partial x} = \Phi$$

$$\underline{u\frac{\partial p}{\partial x} + kp\frac{\partial u}{\partial x} = (k-1)\Phi}$$

$$p = \frac{\mu}{3}\frac{\partial u}{\partial x} + \varphi(x)$$

$$\cancel{\frac{\mu}{3}\frac{\partial^2 u}{\partial x^2}} + \frac{\partial \varphi}{\partial x} = \cancel{\frac{\mu}{3}\frac{\partial^2 u}{\partial x^2}} + \mu\nabla^2 u$$

$$\nabla^2 p = \frac{\mu}{3}\frac{\partial}{\partial x}(\nabla^2 u) + \nabla^2\varphi = \frac{4}{3}\mu\frac{\partial}{\partial x}(\nabla^2 u)$$

$$\nabla^2\varphi = \frac{\partial^2\varphi(x)}{\partial x^2} = \mu\frac{\partial}{\partial x}(\overbrace{\nabla^2 u}^{=f(x)})$$

$$\frac{\partial\varphi(x)}{\partial x} = \mu\nabla^2 u + \cancel{\text{const}}$$

$$\varphi(x) = \mu\int\nabla^2 u\, dx$$

Spurdne ...

$$p = \frac{4}{3}\mu\frac{\partial u}{\partial x} + c$$

$$\frac{4}{3}\mu u \frac{\partial^2 u}{\partial x^2} + k \frac{4}{3}\mu \left(\frac{\partial u}{\partial x}\right)^2 + kc \frac{\partial u}{\partial x} = (k-1)\mu \left[\frac{4}{3}\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right]$$

$$\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = f(x)$$

$$\cancel{u = a_0 + a_1 x + a_2 x^2 + b_1 y} + \cancel{b_2 yx}$$

$$\cancel{+ b_2 y^2}$$

$v = w = 0$ Jednowymiarowe

$$\rho u \frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x}$$

$$\rho u = \text{const}$$

$$\frac{c}{A} \rho u \frac{\partial \theta}{\partial x} + p \frac{\partial u}{\partial x} = 0$$

$$p = R\rho\theta$$

$$\frac{c}{A} \rho_0 u_0 \frac{d\theta}{dx} + (p_0 - u \rho_0 u_0) \frac{du}{dx} = 0$$

$$\frac{c}{A} \rho_0 u_0 \theta + p_0 u - \rho_0 u_0 \frac{u^2}{2} = a$$

$$\frac{c}{AR} u p$$

$$= \frac{c}{AR} u (p_0 - \rho_0 u_0 u)$$

$$u^2 \left[\frac{c}{AR} + \frac{1}{2}\right] \rho_0 u_0 - u \left[\frac{c}{AR} + 1\right] p_0 = a$$

$$\frac{1}{k-1}$$

$$u^2 (k+1) \rho_0 u_0 - 2u k p_0 = A$$

$$p \frac{\partial u}{\partial x} = \frac{c}{A} \frac{\partial p}{\partial x}$$

$$\rho_0 u_0 \frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x}$$

$$\rho_0 u_0 u + p = p_0$$

$$\frac{c}{AR} \frac{d\theta}{\theta} + \frac{du}{u} = 0$$

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^k$$

$$p = p_0 \left(\frac{u_0}{u}\right)^k$$

$$\rho_0 u_0 \cdot u + p_0 \left(\frac{u_0}{u}\right)^k = p_0$$

?

$u = \text{const.}\,!$

$$\frac{dp}{dx} = \frac{4}{3}\mu \frac{d^2u}{dx^2} \qquad \frac{d}{dx}(\rho u) = 0$$

$$\frac{c}{A}\rho u \frac{d\theta}{dx} + p\frac{du}{dx} = \frac{4}{3}\mu\left(\frac{du}{dx}\right)^2 \qquad \mu = \mu_0 \theta$$

Wypływ kulisty

$$u = s\,\frac{x}{r} \qquad v = s\,\frac{y}{r} \qquad w = s\,\frac{z}{r}$$

$$= x\varphi \qquad = y\varphi \qquad = z\varphi$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 3\varphi + \varphi' r$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 4\varphi'\frac{x}{r} + \varphi'' x$$

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \ldots\right) = 4\varphi'\frac{x}{r} + \varphi'' x$$

$$\frac{\partial p}{\partial x} = \frac{dp}{dr}\,\frac{x}{r} = \frac{4}{3}\mu\left[4\varphi'\frac{x}{r} + \varphi'' x\right]$$

$$= \frac{4}{3}\mu\,\frac{d}{dr}\left[3\varphi + r\varphi'\right]\cdot\frac{x}{r}$$

I). $$p = \frac{4}{3}\mu\left[3\varphi + r\varphi'\right] + p_0$$

II). $$4\pi r^2 \varphi r \rho = \text{const}$$

$$\varphi\rho r^3 = \text{const}$$

$$\rho\, d(\varphi r^3) + \varphi r^3\, d\rho = 0$$

$$\rho\left(\varphi' r^3 + 3r^2\varphi\right) + \varphi r^3 \frac{d\rho}{dr} = 0$$

$$\left(3\varphi + \varphi' r\right) + \varphi r\,\frac{1}{\rho}\frac{d\rho}{dr} = 0$$

III). $$\frac{c}{A}\rho\left[x\varphi\frac{x}{r}\frac{d\vartheta}{dr} + y\varphi\frac{y}{r}\frac{d\vartheta}{dr} + \ldots\right] + p\left[3\varphi + \varphi' r\right] = \Phi$$

$$\Phi =$$

$$\frac{\partial u}{\partial x} = \varphi + \varphi'\frac{x^2}{r}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{3x}{r}\varphi' - \varphi'\frac{x^3}{r^3} + \varphi''\frac{x^3}{r^2}$$

$$\frac{\partial u}{\partial y} = \varphi'\frac{xy}{r}$$

$$\frac{\partial^2 u}{\partial y^2} = \varphi'\left(\frac{x}{r} - \frac{xy^2}{r^3}\right) + \varphi''\frac{xy^2}{r^2}$$

$$\frac{\partial^2 u}{\partial z^2} = \varphi'\left(\frac{x}{r} - \frac{xz^2}{r^3}\right) + \varphi''\frac{xz^2}{r^2}$$

$\frac{\partial u}{\partial x} = \varphi + \varphi' \frac{x^2}{r}$	$\frac{\partial v}{\partial x} = \varphi' \frac{xy}{r}$	$\frac{\partial w}{\partial x} = \varphi' \frac{xz}{r}$
$\frac{\partial u}{\partial y} = \varphi' \frac{xy}{r}$	$\frac{\partial v}{\partial y} = \varphi + \varphi' \frac{y^2}{r}$	$\frac{\partial w}{\partial y} = \varphi' \frac{yz}{r}$
$\frac{\partial u}{\partial z} = \varphi' \frac{xz}{r}$	$\frac{\partial v}{\partial z} = \varphi' \frac{yz}{r}$	$\frac{\partial w}{\partial z} = \varphi + \varphi' \frac{z^2}{r}$

$$\frac{\Phi}{\mu} = -\frac{2}{3}\left[3\varphi + r\varphi'\right]^2 + 2\left[\left(\varphi + \frac{x^2}{r}\varphi'\right)^2 + \left(\varphi + \frac{y^2}{r}\varphi'\right)^2 + \left(\varphi + \frac{z^2}{r}\varphi'\right)^2\right] +$$

$$+ 4\varphi'^2\left[\frac{y^2z^2}{r^2} + \frac{z^2x^2}{r^2} + \frac{x^2y^2}{r^2}\right]$$

$$= -\frac{2}{3}\left[3\varphi + r\varphi'\right]^2 + 2\left[3\varphi^2 + 2\varphi\varphi' r + \varphi'^2 \frac{x^4 + y^4 + z^4 + 2x^2y^2 + \cdots}{r^2}\right]$$

$$= -\frac{2}{3}\left[3\varphi + r\varphi'\right]^2 + 2\left[3\varphi^2 + 2r\varphi\varphi' + r^2\varphi'^2\right]$$

$$= -\frac{2}{3} r^2\varphi'^2 + 2r^2\varphi'^2 = \frac{4}{3} r^2 \varphi'^2$$

III). $\frac{c}{A}\rho\varphi r \frac{d\theta}{dr} + p\left[3\varphi + r\varphi'\right] = \frac{4}{3}\mu r^2\varphi'^2$

$$\theta = \frac{p}{R\rho}$$

$$\frac{c}{AR}\left(\varphi r \frac{dp}{dr} - \frac{\varphi r p}{\rho}\frac{d\rho}{dr}\right) = \frac{c}{AR}\left[\varphi r \frac{4}{3}\mu\left(7\varphi' + r\varphi''\right) + \left[\frac{4}{3}\mu\left(3\varphi + r\varphi'\right) + p_0\right]\left[3\varphi + \varphi' r^{2}\right]\right] +$$

$$+ \left[\frac{4}{3}\mu\left(3\varphi + r\varphi'\right) + p_0\right]\left[3\varphi + \varphi' r\right] = \Phi$$

$$\frac{c}{AR}\left(4\varphi'+r\varphi''\right)\varphi r+\left(1+\frac{c}{AR}\right)\left(3\varphi+r\varphi'\right)\left(3\varphi+r\varphi'+P_0\right)=r^2\varphi'^2$$

$$\boxed{P_0=\frac{p_0}{\frac{4}{3}\mu}}$$

$$(4\varphi'+r\varphi'')\varphi r-(k-1)\,r^2\varphi'^2+k\,(3\varphi+r\varphi')(3\varphi+r\varphi'+P_0)=0$$

$$r^2\varphi\varphi''+\varphi'^2\left[r^2(1-k)+kr^2\right]+\varphi'\left[4r\varphi+6kr\varphi+P_0kr\right]+$$

$$+9k\varphi^2+3k\varphi P_0=0$$

$$r^2\underbrace{(\varphi\varphi''+\varphi'^2)}_{\frac{d}{dr}(\varphi\varphi')}+\varphi'\left[2r\varphi(2+3k)+krP_0\right]+3k\varphi\left[3\varphi+P_0\right]=0$$

$$\underbrace{r^4\frac{d}{dr}(\varphi\varphi')+\varphi\varphi'.4r^3}_{\frac{d}{dr}(\varphi\varphi'r^4)}+k\,(6\varphi\varphi'r^3+9\varphi^2r^2)+$$

$$+3k\left[\underbrace{2r^3\varphi\varphi'+3r^2\varphi^2}_{\frac{d}{dr}(r^3\varphi^2)}\right]\qquad kP_0\,(\underbrace{\varphi'r^3+3\varphi r^2}_{\frac{d}{dr}(\varphi r^3)})=0$$

$$\varphi\varphi'r^4+3k\varphi^2r^3+kP_0\,\varphi r^3=a$$

$$\varphi\varphi'+3k\frac{\varphi^2}{r}+kP_0\frac{\varphi}{r}=\frac{a}{r^4}$$

$$\varphi^2=2z$$

$$\varphi\varphi'=\frac{dz}{dr}$$

$$\frac{dz}{dr}+6k\frac{z}{r}+kP_0\sqrt{2}\,\frac{\sqrt{z}}{r}=\frac{a}{r^4}$$

$$r\varphi=y$$

$$r\varphi'+\varphi=\frac{dy}{dr}$$

$$r^2\varphi'=r\frac{dy}{dr}-y$$

$$ry\left(r\frac{dy}{dr}-y\right)+3kry^2+kP_0r^2y=a$$

$$r^2y\frac{dy}{dr}+(3k-1)ry^2+kP_0r^2y=a$$

$$y\frac{dy}{dr}+(3k-1)\frac{y^2}{r}+kP_0y=\frac{a}{r^2}$$

$$\frac{dy}{dr} + 3(k-1)\frac{y}{r} + kP_0 = 0$$

$$y = uv$$

$$\frac{dy}{dr} = u\frac{dv}{dr} + v\frac{du}{dr}$$

$$u^2 v\frac{dv}{dr} + u v^2\frac{du}{dr} + (3k-1)\frac{u^2v^2}{r} + kP_0\, uv = \frac{a}{r^2}$$

$$u^2\underbrace{\left[v\frac{dv}{dr} + (3k-1)\frac{v^2}{r}\right]}_{=0} + u\frac{du}{dr}\cdot v^2 + kP_0\, u v = \frac{a}{r^2}$$

$$\lg v = -(3k-1)\lg r$$

$$v = b\, r^{1-3k}$$

$$u\frac{du}{dr} + uR_1 = R_2 \qquad \Bigg\| \qquad \cancel{u = x + y}$$

$$\cancel{x\frac{dx}{dr} + x\frac{dy}{dr} + y\frac{dx}{dr} + y\frac{dy}{dr} + xR_1 + yR_1 = R_2}$$

$$\cancel{u = xy}$$

$$\cancel{y^2 x\frac{dy}{dr} + xy^2\frac{dx}{dr} + xyR_1 = R_2}$$

Wenn $a = 0$:

$$u = -kP_0\int\frac{dr}{v} = -\frac{kP_0}{b}\int\frac{dr}{r^{1-3k}} = -\frac{kP_0}{b}\int\left(r^{3k-1}\right)dr$$

$$u = -\frac{kP_0}{b}\,\frac{r^{3k}}{3k} + c$$

$$y = c\, r^{1-3k} - \frac{P_0}{3}\, r = r\varphi = s$$

$$\varphi = c\, r^{-3k} - \frac{P_0}{3}$$

$$\varphi' = -3kc\, r^{-3k-1}$$

I). $p = \frac{4}{3}\mu\left[3c\, r^{-3k} - P_0 - 3kc\, r^{-3k} + P_0\right] = 4\mu\, c(1-k)\, r^{-3k}$

II). $\rho\left[c\, r^{3(1-k)} - \frac{P_0}{3} r^3\right] = \text{const}$ 2 ote to numerator!

$$\varphi r^3 = \frac{c}{\rho} \qquad \varphi = \frac{c}{\rho r^3}$$

$$\varphi' = \frac{-3c}{\rho r^4} - \frac{c}{\rho^2 r^3}\frac{d\rho}{dr}$$

$$\left[-\frac{3c^2}{\rho^2 r^7} - \frac{c^2}{\rho^3 r^6}\frac{d\rho}{dr}\right] r^4 + 3k\frac{c^2}{\rho^2 r^6} r^3 + kP_0\frac{c}{\rho} = a$$

$$\frac{c^2}{r^2}\frac{1}{\rho^3}\frac{d\rho}{dr} + \underbrace{\frac{3c^2}{\rho^2 r^3} + \frac{3k\, c^2}{\rho^2 r^3}} + kP_0\frac{c}{\rho} = a$$

$$\frac{d\rho}{dr} + \rho\cdot\frac{3(k+1)}{r} + \frac{kP_0}{c}\frac{\rho^2}{r^2} = \frac{a}{c^2}\rho^3 r^2$$

$$\rho = u\,v$$

$$u\left[\frac{dv}{dr} + v\frac{3k+1}{r} + \frac{kP_0}{c}\frac{u\, v^2}{r^2}\right] + v\frac{du}{dr} = \frac{a}{c^2} u^3 v^3 r^2$$

$$\frac{y}{r}\frac{dy}{dr} + (3k-1)\frac{y^2}{r^2} + kP_0\frac{y}{r} = \frac{a}{r^3}$$

Aerodynamiczne dwurozmiarowe, powolne :

$$\mathrm{I}\begin{cases} \dfrac{\partial p}{\partial x} = \dfrac{\mu}{3}\dfrac{\partial}{\partial x}\left(\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}\right) + \mu\left(\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}\right) \\[2ex] \dfrac{\partial p}{\partial y} = \dfrac{\mu}{3}\dfrac{\partial}{\partial y}\left(\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}\right) + \mu\left(\dfrac{\partial^2 v}{\partial x^2}+\dfrac{\partial^2 v}{\partial y^2}\right) \end{cases}$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)=0$$

$$\mathrm{II}).\ \frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0$$

$$+\frac{c}{A}\rho\frac{\partial\theta}{\partial t}$$

$$\mathrm{III}).\ \frac{c}{A}\rho\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right)+p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=\cancel{\mu}\,\Phi$$

$$\theta=\frac{p}{R\rho}$$

$$\frac{c}{AR}\rho\left[\frac{1}{\rho}\frac{\partial p}{\partial t}-\frac{p}{\rho^2}\frac{\partial\rho}{\partial t}\right]$$

$$\mathrm{III}).\ \frac{c}{AR}\rho\left[u\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{v}{\rho}\frac{\partial p}{\partial y}-u\frac{p}{\rho^2}\frac{\partial\rho}{\partial x}-v\frac{p}{\rho^2}\frac{\partial\rho}{\partial y}\right]+\dots$$

$$\frac{1}{k-1}\left[\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}-\frac{p}{\rho}\left(\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial x}+v\frac{\partial\rho}{\partial y}\right)\right]+p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=\Phi$$

$$\frac{\partial\rho}{\partial t}+\rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+u\frac{\partial\rho}{\partial x}+v\frac{\partial\rho}{\partial y}=0$$

$$\mathrm{III}).\ \frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+(k-1)\,p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=(k-1)\,\Phi$$

$$\mathrm{III}).\ \frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+k\,p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=(k-1)\,\Phi$$

Jeśli istnieje potencjał prędkości! $\quad u=\dfrac{\partial\varphi}{\partial x} \quad v=\dfrac{\partial\varphi}{\partial y}$

$$p=\frac{\mu}{3}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\mu\,\cancel{\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\int u\,dx+v\,dy}\qquad \varphi$$

$$\nabla^2\varphi=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}$$

Eliminując $p = R\rho\theta$

~~$R\left(\theta \frac{\partial \rho}{\partial x} + \rho \frac{\partial \theta}{\partial x}\right) = \frac{\mu}{3}\frac{\partial}{\partial}$~~

Wtedy: $\frac{\partial p}{\partial x} = \frac{\mu}{3} \frac{\partial}{\partial x}\left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2}\right) + \mu \frac{\partial}{\partial x}\left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2}\right) = \frac{4\mu}{3} \frac{\partial}{\partial x}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\varphi$

$$\frac{\partial p}{\partial y} = \frac{4\mu}{3} \frac{\partial}{\partial y}\left(\quad \right)\varphi$$

I). $p = p_0 + \frac{4\mu}{3}\left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2}\right)$

III). $\frac{\partial \varphi}{\partial x}\frac{\partial p}{\partial x} + \frac{\partial \varphi}{\partial y}\frac{\partial p}{\partial y} + k p \nabla^2\varphi = (k-1)\Phi$

$$\Phi = -\frac{2}{3}\mu\left(\nabla^2\varphi\right)^2 + \underbrace{2\mu\left[\left(\frac{\partial^2 \varphi}{\partial x^2}\right)^2 + \left(\frac{\partial^2 \varphi}{\partial y^2}\right)^2\right]}_{\cancel{\left[\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)\left(\frac{\partial \varphi}{\partial x} + \frac{\partial \varphi}{\partial y}\right)\right]^2}} + 4\left(\frac{\partial^2 \varphi}{\partial x \partial y}\right)^2$$

$$= \frac{4\mu}{3}\left[\left(\frac{\partial^2 \varphi}{\partial x^2}\right)^2 + \left(\frac{\partial^2 \varphi}{\partial y^2}\right)^2 - \frac{\partial^2 \varphi}{\partial x^2}\frac{\partial^2 \varphi}{\partial y^2} + 3\left(\frac{\partial^2 \varphi}{\partial x \partial y}\right)^2\right]$$

I). $\nabla^2 p = \frac{\mu}{3}\nabla^2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \mu \nabla^2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = \frac{4}{3}\mu \nabla^2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$

$p = \frac{4\mu}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + A \quad \| \nabla^2 A = 0$ } Ogólny rezultat dla ruchu powolnego

Założenie że $A = p_0 = \text{const}$:

$\frac{4}{3} u\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y}\right) + \frac{4}{3} v\left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2}\right) + \ldots =$

$(k-1)\left[-\frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2 + 2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right] - \frac{4}{3} A\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2 - A\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$

$$u\left[\frac{4}{3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\cancel{\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}}\right]+v\left[\frac{4}{3}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\cancel{\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}}\right]=$$

$$k\left[-2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)^2+2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right]+$$

$$+\left[\frac{2}{3}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)^2-2\left(\frac{\partial u}{\partial x}\right)^2-2\left(\frac{\partial v}{\partial y}\right)^2-\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right]$$

$$=k\left[-4\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right]+\left[\frac{2}{3}(\quad)-\quad\right]$$

Jeśli specjalny rodzaj ruchu potencjalnego: $\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=0$

$\frac{\partial u}{\partial x}=\frac{\partial^2\varphi}{\partial x^2}$ | $\frac{\partial u}{\partial y}=\frac{\partial^2\varphi}{\partial x\partial y}=\frac{\partial v}{\partial x}$

$$\cancel{\frac{\partial\varphi}{\partial x}\left[\frac{1}{3}\frac{\partial}{\partial x}\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}\right)+\frac{\partial^3\varphi}{\partial x^3}+\right.}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\cancel{\frac{\partial\varphi}{\partial x}\left[\frac{\partial^3\varphi}{\partial x^3}+\frac{\partial^3\varphi}{\partial x^2\partial y}\right.}$$

$$0=(k-1)\left[2\left(\frac{\partial^2\varphi}{\partial x^2}\right)^2+2\left(\frac{\partial^2\varphi}{\partial y^2}\right)^2+4\left(\frac{\partial^2\varphi}{\partial x\partial y}\right)^2\right]$$

$$0=\left(\frac{\partial^2\varphi}{\partial x^2}\right)^2+2\left(\frac{\partial^2\varphi}{\partial x\partial y}\right)^2+\left(\frac{\partial^2\varphi}{\partial y^2}\right)^2$$

Możliwe tylko jeśli zawsze

$$\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2$$

$$\frac{\partial u}{\partial x}=\frac{\partial u}{\partial y}=\frac{\partial v}{\partial x}=\frac{\partial v}{\partial y}=0$$

$v = \varphi . u$ ~~Na powierzchniach musi być równocześnie~~ $u = 0$, $\cancel{v = 0}$

~~t.zn. tam musi~~

Hypoteza:

$v = u(a - u) = au - u^2$

$$\cancel{\frac{\partial v}{\partial x} = u \frac{\partial \varphi}{\partial x} + \varphi \frac{\partial u}{\partial x}}$$

σ

$$d\left[p - \frac{\mu}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)\right] = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy + \frac{\partial \psi}{\partial z} dz$$

$$= \mu\left(\nabla^2 u \, dx + \nabla^2 v \, dy + \nabla^2 w \, dz\right)$$

$$= \mu \, S(\nabla^2 \mathfrak{v} . d\sigma)$$ ~~niezależne od drogi~~

~~Nad krzywą zamkniętą:~~

$$\cancel{\int_\sigma d[p \quad] = 0 \qquad \int_\sigma S(\nabla^2 \mathfrak{v} . d\sigma) = 0}$$

~~Twierdzenie Stokesa:~~ $\cancel{\iint S(N . \operatorname{curl} U) \, dS = \int_\sigma S(U . d\sigma)}$

$$\cancel{\iint S(N . \operatorname{curl} \nabla^2 \mathfrak{v}) \, dS = 0}$$

$$\operatorname{curl} \nabla^2 \mathfrak{v} = \nabla^2 \operatorname{curl} \mathfrak{v}$$

$$\operatorname{curl}^2 \mathfrak{v} = \nabla \operatorname{div} - \nabla^2 \mathfrak{v}$$

$$\operatorname{curl} \nabla^2 \mathfrak{v} = -\operatorname{curl}^3 \mathfrak{v} - \cancel{\operatorname{curl} \nabla \operatorname{div} \mathfrak{v}}$$

$$= \nabla^2 \operatorname{curl} \mathfrak{v} - \cancel{\nabla \operatorname{div} \operatorname{curl} \mathfrak{v}}$$

~~więc~~ $\cancel{\nabla^2 \mathfrak{v} = \nabla U = \nabla \operatorname{div} - \operatorname{curl}^2 \mathfrak{v}}$

~~tzn.~~

$$\cancel{\int (\nabla^2 u \, dx + \nabla^2 v \, dy + \nabla^2 w \, dz) = \nabla^2 \int u \, dx + v \, dy + w \, dz}$$?

Szczególny rodzaj: $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{v}{r}=0$

$$\begin{cases}\frac{\partial p}{\partial x}=\mu\nabla^2 u & p=A \quad \| \nabla^2 A=0\\ -- \end{cases}$$

$$u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}=(k-1)\mu\left[2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2+2\left(\frac{v}{r}\right)^2+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2+\ -\ \right]$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$u\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+v\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)=(k-1)\left[\underbrace{2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2}_{\cancel{\left(\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}\right)^2-4\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}}}\right]$$

$$\cancel{u\frac{\partial^2 u}{\partial x^2}+\left(\frac{\partial u}{\partial x}\right)^2+\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}+u\frac{\partial^2 v}{\partial x\partial y}=0}$$

$$\cancel{u\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}+\frac{\partial u}{\partial y}\frac{\partial v}{\partial y}+u\frac{\partial^2 v}{\partial y^2}=0}$$

$$\cancel{v\frac{\partial^2 v}{\partial y^2}+\left(\frac{\partial v}{\partial y}\right)^2+\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}+v\frac{\partial^2 u}{\partial x\partial y}=0}$$

~~Rozkłady stożek~~ Dla każdej powierzchni jeżeli $u=v=0$

otrzymuje się z III: składające u w kierunku stycznym

v normalnym

$$\frac{\partial u}{\partial x}=0 \qquad \text{[co dozwolone powoduje tylko pierwsze pochodne]}$$

$$k\,p\,\frac{\partial v}{\partial y}=(k-1)\mu\left[\underbrace{-\frac{2}{3}\left(\frac{\partial v}{\partial y}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2}_{\frac{4}{3}\left(\frac{\partial v}{\partial y}\right)^2}+\left(\frac{\partial u}{\partial y}\right)^2\right]+(k-1)\,\kappa\,\nabla^2\theta$$

z II wypływa zresztą : $\frac{\partial v}{\partial y}=0$ z czego wypływałoby $\frac{\partial u}{\partial y}=0$ || ?? !

co prawda : ciepło zostanie wyrzucane dopóki $\frac{\partial u}{\partial y} \gtrless 0$, podczas gdy $u=0$ przy powierzchni, zatem prawdopodobnie nastąpi [illegible] – czyli że $\frac{\partial u}{\partial y}=0$

lub też : uwzględnienie przewodnictwa ciepła !

ważnem n.p. dla przepływu gazu przez rurki uwzględnić przewodnictwo niezbędne

ważnem tylko dla prędkich ruchów

Zatem adiabatyczna teorya będzie niemożliwa n.p. dla ruchu ~~[illegible]~~

Powinnoby się wogóle okazać wpływ przewodnictwa cieplnego materyału rurki na przepływ ! Co prawda, że znaczenie może być tylko n.p. dla oporu waty u dośw. Kelvina

Zły przewodnik będzie oporą więcej znaczący niż dobry

Obliczyć zatem różnicę przepływu izotermicznego i adiabat.!

$$\cancel{u = u_0 + a\rho + b\rho^2}$$

$$\left.\begin{aligned} \frac{\partial u}{\partial \rho} &= a + 2b\delta = 0 \\ &u_0 + a\delta + b\delta^2 = 0 \end{aligned}\right\}$$

$$u_0 + \delta(a + 2b\delta) - b\delta^2 = 0$$

$$b = \frac{u_0}{\delta^2}$$

$$a = -\frac{2u_0}{\delta}$$

$$\rho\left(\cancel{\frac{\partial u}{\partial x}} + \frac{\partial v}{\partial y}\right) + \cancel{u\frac{\partial \rho}{\partial x}} + \cancel{v\frac{\partial \rho}{\partial y}} = 0$$

$$\left(\frac{\partial u}{\partial y}\right)^2 = -\frac{k}{\mu}\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial z^2}\right)$$

Robiąc założenie że $\frac{\partial \theta}{\partial x} = \frac{\partial \theta}{\partial z} = 0$

$$\left(\frac{\partial u}{\partial y}\right)^2 = -\frac{k}{\mu}\frac{\partial^2\theta}{\partial y^2}$$

Rurki włoskowate

Wprowadzając wszystkie przybliżenia z wyjątkiem równania termicznego:

$$\frac{\partial p}{\partial y} = \frac{\partial p}{\partial z} = 0 \qquad \frac{dp}{dx} = \mu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\frac{\partial(\rho u)}{\partial x} = 0$$

$$\text{II)}.\quad u\frac{dp}{dx} + kp\frac{\partial u}{\partial x} = (k-1)\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right]\mu + \kappa\,\Delta^2\theta$$

$$u = f(x, r) \qquad \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = (f')^2$$

$$\frac{\partial u}{\partial y} = f'\frac{y}{r}$$

$$\left.\begin{aligned}\frac{\partial^2 u}{\partial y^2} &= f''\frac{y^2}{r^2} + f'\left(\frac{1}{r} - \frac{y^2}{r^3}\right)\\ \frac{\partial^2 u}{\partial z^2} &= f''\frac{z^2}{r^2} + f'\left(\frac{1}{r} - \frac{z^2}{r^3}\right)\end{aligned}\right\}\quad \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = f'' + f'\frac{1}{r} = \frac{1}{r}d(rf')$$

$$\frac{dp}{dx} = \mu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = \mu\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right] \qquad \rho u = f_1(r).$$

$$u\frac{dp}{dx} + kp\frac{\partial u}{\partial x} = (k-1)\mu\left(\frac{\partial u}{\partial r}\right)^2 + \underbrace{\frac{\kappa}{R}\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\left(\frac{p}{\rho}\right)\right]}$$

$$= -\frac{\kappa p}{R}\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{r}{\rho^2}\frac{\partial\rho}{\partial r}\right]$$

~~I). Zakładając $\kappa = 0$~~

$$u = \frac{r^2}{4}\varphi(x) + \psi(x)\log(r) + \chi(x)$$

$$\frac{\partial}{\partial r}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) = 0 \qquad \frac{\partial u}{\partial r} = \frac{r}{2}\varphi(x) + \frac{1}{r}\psi(x)$$

$$\mu\cdot\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = \mu\cdot\varphi(x) = \frac{dp}{dx} \qquad p = \mu\int\varphi(x)\,dx$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = r\varphi(x)$$

$$r\frac{\partial u}{\partial r} = \frac{r^2}{2}\varphi(x) + \psi(x)$$

Stawiając $x=0$:

$$\frac{n^2}{4}[\varphi'(x)]^2 + \log n \;\psi(x)\,\varphi(x) + \chi(x)\,\varphi(x) + k\left[\frac{n^2}{4}\varphi'(x)\int\varphi(x)dx + \log n\,.\,\psi'(x)\int\varphi(x)dx + \chi'(x)\int\varphi(x)dx\right] =$$

$$= (k-1)\left[\frac{n^2}{4}[\varphi(x)]^2 + \psi(x)\,\varphi(x) + \frac{1}{n^2}[\psi(x)]^2\right]$$

Musi być spełnione dla dowolnych n, x

$$[\varphi(x)]^2 + k\,\varphi'(x)\int\varphi(x)dx = (k-1)[\varphi(x)]^2$$

$$\psi(x)\,\varphi(x) + k\psi'(x)\int\varphi(x)dx = 0$$

$$\chi(x)\,\varphi(x) + k\,\chi'(x)\int\varphi(x)\,dx = (k-1)\,\varphi(x)\,\psi(x)$$

$$0 = \frac{k-1}{n^2}[\psi(x)]^2 \qquad \psi(x) = 0$$

$$\chi\,\varphi = -k\,\chi'\int\varphi$$

$$\frac{\chi}{\chi'}\varphi = -k\int\varphi$$

$$\varphi - \frac{\chi\chi''}{\chi'^2}\varphi + \frac{\chi}{\chi'}\varphi' = -k\varphi$$

$$\varphi\left[1+k - \frac{\chi\chi''}{\chi'^2}\right] = -\frac{\chi}{\chi'}\varphi'$$

$$(1+k)\frac{\chi'}{\chi} - \frac{\chi''}{\chi'} = a\,\frac{4-k}{2-k}$$

$$(1+k)\log\chi - \log\chi' = a\,\frac{4-k}{2-k}\,x +$$

$$\frac{\chi'}{\chi^{1+k}} = c\,e^{a\frac{k-4}{k-2}x}$$

$$\chi^{-k} = -ck\int e^{a\frac{k-4}{k-2}x}dx$$

$$\int\varphi = \frac{(k-2)}{k}\frac{\varphi^2}{\varphi'}$$

$$k\varphi = \frac{k-2}{k}\left[2\varphi - \frac{\varphi^2\varphi''}{\varphi'^2}\right]$$

$$k-4 = (k-2)\,\frac{\varphi\varphi''}{\varphi'^2}$$

$$\frac{k-4}{k-2}\,\frac{\varphi'}{\varphi} = \frac{\varphi''}{\varphi'}$$

$$a\,\frac{k-4}{k-2}\,\varphi = \varphi'$$

$$\frac{\varphi'}{\varphi} = a\,\frac{k-4}{k-2}$$

$$\varphi = b\,e^{a\frac{k-4}{k-2}x}$$

Ścisły warunek $u=0$ na ścianie byłby spełniony tożsamościowo gdyby:
u = skończone dla $z=0$

$$\frac{\delta^2}{4}\varphi(x)+\chi(x)=0$$

$$\chi(x)=-\frac{\delta^2}{4}\varphi(x) \qquad \psi=0$$

To wymagałoby:

$$\varphi^2+k\varphi'\int\varphi=0$$

Co nie zgadza się z równaniem dla φ zatem wogóle ~~niemożliwy~~ stały stan do spełnienia jeśli $\kappa=0$ i jeśli zachowujemy tamte przybliżenia

---

Spróbować przy $\kappa\geq 0$:

$$u=\frac{r^2-\delta^2}{4}\varphi(x)$$

$\rho u=f(r)$:

$$\frac{\partial u}{\partial r}=\frac{r}{2}\varphi(x)$$

$$\frac{r^2}{4}\rho\varphi(x)-\frac{\delta^2}{4}\rho\varphi(x)=\rho u=f_1(r)$$

$\frac{\partial}{\partial x}=0$:

$$\frac{r^2-\delta^2}{4}\frac{\partial}{\partial x}(\rho\varphi)=0 \qquad \frac{\partial\rho}{\partial x}\varphi+\rho\frac{\partial\varphi}{\partial x}=0$$

$$\rho\varphi(x)=f(r)$$

$$\frac{1}{\rho}=\frac{\varphi(x)}{f(r)} \qquad \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\frac{1}{f(r)}\right]=F(r)$$

$$\frac{dp}{dx}=\mu\varphi \qquad p=\mu\int\varphi\,dx$$

$$\frac{r^2-\delta^2}{4}\mu\varphi^2(x)+k\mu\frac{r^2-\delta^2}{4}\varphi'\int\varphi=\mu(k-1)\frac{r^2}{4}\varphi^2(x)+\frac{\kappa\varphi(x)}{R}F(r)$$

Musi być spełnione ~~tożsamościowo~~ dla każdego r i x

$$(r^2-\delta^2)\left[\varphi^2_{(x)} + k\,\varphi'_{(x)}\int\varphi\,dx\right] = (k-1)\,r^2\varphi^2_{(x)} + \frac{4K}{\mu R}\varphi_{(x)}\,F_{(r)}$$

$$r^2\left[(2-k)\,\varphi^2_{(x)} + k\,\varphi'_{(x)}\int\varphi\,dx\right] - \delta^2\left[\varphi^2_{(x)} + k\,\varphi'_{(x)}\int\varphi\,dx\right] = \frac{4K}{R\mu}\varphi_{(x)}\,F_{(r)}$$

$$2r\left[\qquad\qquad\qquad\right] = \frac{4K}{R\mu}\varphi_{(x)}\,F'_{(r)}$$

$$2\left[\qquad\qquad\qquad\right] = \frac{4K}{R\mu}\varphi_{(x)}\,F''_{(r)}$$

$$\frac{F''}{F'} = \frac{1}{r}$$

$$\log F' = \log r + \ldots$$

$$F' = a\,r = \frac{dF}{dr}$$

$$F = a\,r^2 + b$$

$$(2-k)\,\varphi^2 + k\,\varphi'\int\varphi = \frac{2aK}{R\mu}\varphi \quad \Big\| \quad -\delta^2\left[\varphi^2 + k\,\varphi'\int\varphi\right] = \frac{4bK}{R\mu}\varphi$$

$$2(2-k)\,\varphi\varphi' + k\,\varphi''\int + k\,\varphi'\varphi = \frac{2aK}{R\mu}\varphi'$$

$$[2-k]\left[\varphi^2\varphi'' - 2\varphi\varphi'^2\right] - k\,\varphi'^2\varphi = \frac{2aK}{R\mu}\left[\varphi\varphi'' - \varphi'^2\right]$$

$$[2-k]\left[\frac{\varphi''}{\varphi'} - 2\frac{\varphi'}{\varphi}\right] - k\frac{\varphi'}{\varphi} = \frac{2aK}{R\mu}\left[\frac{\varphi''}{\varphi\varphi'} - \frac{\varphi'}{\varphi^2}\right]$$

$$(1-k^2)\,\varphi = \left[\frac{2aK}{R\mu} + \frac{4bK}{\delta^2 R\mu}\right]$$

$f = \text{const}$

Wtedy, jednak: $\theta = \dfrac{p}{R\rho} = \dfrac{p\,u}{R\rho\,u} = \mu_{(x)}\dfrac{\frac{r^2-\delta^2}{4}\varphi_{(x)}}{R}$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right) = \frac{p\varphi}{R}F(r) = \frac{p\varphi}{R}\left(\frac{ar^2}{2}+b\right)$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right) = \frac{p\varphi}{R}\left[\frac{ar^3}{2}+br\right]$$

$$r\frac{\partial\theta}{\partial r} = \frac{p\varphi}{R}\left[\frac{ar^4}{8}+\frac{br^2}{2}\right]+\cancel{f(x)}$$

$$\theta = \frac{p\varphi}{R}\left[\frac{ar^4}{32}+\frac{br^2}{4}\right]+\cancel{f_1(x)\log r}+f_2(x)$$

$$p = p_1+(p_2-p_1)\frac{x}{l} \quad \Big\| \quad \frac{dp}{dx} = \frac{p_2-p_1}{l} = \mu\varphi$$

$$\varphi = \frac{p_2-p_1}{l\mu} = \frac{2a\kappa}{(2-k)R\mu} = \frac{-4b\kappa}{\delta^2 R\mu} \qquad a = -\frac{2b}{(2-k)\delta^2}$$

$$a = \frac{2-k}{2}\frac{(p_2-p_1)R}{l\kappa} \qquad b = -\frac{(2-k)}{2}\delta^2 a$$

$$\theta = \theta_0 + \frac{p_2-p_1}{Rl\mu}\left[p_1+(p_2-p_1)\frac{x}{l}\right]\left[\frac{r^4}{32}-\frac{(2-k)\delta^2 r^2}{8}\right]\frac{2-k}{2}\frac{p_2-p_1}{l\kappa}R$$

$$\left[-\frac{\delta^4}{32}+\frac{(2-k)\delta^4}{8}\right]$$

$$\theta-\theta_0 = \left(\frac{p_2-p_1}{l}\right)^2\frac{2-k}{16}\frac{1}{\kappa\mu}\left[p_1+(p_2-p_1)\frac{x}{l}\right]\underbrace{\left[\frac{r^4-\delta^4}{4}-(2-k)\delta^2(r^2-\delta^2)\right]}$$

$$\underbrace{(r^2-\delta^2)\left[\frac{r^2}{4}+\frac{\delta^2}{4}-(2-k)\delta^2\right]}$$

$$\frac{1}{4}(r^2-\delta^2)\left[r^2+(4k-7)\delta^2\right]$$

$$\rho = \frac{p}{R\theta}$$

$$\rho u = \frac{p\,\frac{r^2-\delta^2}{4}\,\varphi}{R\left[\theta_0 + \frac{\mu\varphi}{R}\cdot\frac{2-k}{64}\,\frac{p_2-p_1}{l\,k}\,R\left(r^4-\delta^4-4(2-k)\delta^2 r^2+4(2-k)\delta^4\right)\right]}$$

$$M = 2\pi\int_0^\delta r\rho u\,dr = \frac{\pi p\varphi}{2R}\int\frac{(r^3 - r\delta^2)\,dr}{\theta_0 + \dots}$$

Nie można spełnić warunku $\rho u = f(r)$

To samo dwuwymiarowe:

$$\frac{\partial p}{\partial y} = 0 \qquad v =$$

$$\begin{cases} \frac{\partial p}{\partial x} = \mu\frac{\partial^2 u}{\partial y^2} \\ \frac{\partial(\rho u)}{\partial x} = 0 \\ u\frac{\partial p}{\partial x} + kp\frac{\partial u}{\partial x} = (k-1)\mu\left(\frac{\partial u}{\partial y}\right)^2 + \kappa\frac{\partial^2\theta}{\partial y^2}\left(+\frac{\partial^2\theta}{\partial x^2}\right) \end{cases}$$

$$\frac{\partial^3 u}{\partial y^3} = 0$$

$$u = \frac{y^2}{2}\varphi(x) + y\,\psi(x) + \chi(x)$$

$$\rho u = f(y)$$

$$\frac{\partial p}{\partial x} = \mu\varphi(x)$$

$$p = \mu\int\varphi(x)\,dx + p_0$$

$$u = \frac{y^2}{2}\frac{1}{\mu}\frac{dp}{dx} + \psi(x)\,y + \chi(x)$$

$$\frac{\partial u}{\partial y} = \frac{y}{\mu}\frac{dp}{dx} + \psi(x)$$

$$\theta = \frac{p}{R\rho} \qquad \frac{\partial^2\theta}{\partial y^2} = \frac{p}{R}\frac{\partial^2}{\partial y^2}\left(\frac{1}{\rho}\right)$$

$$\left.\begin{matrix} y=0 \\ y=\delta \end{matrix}\right\} u = 0 \qquad \chi = 0$$

$$\psi = -\frac{\delta}{2}\varphi(x)$$

$$u = \frac{y(y-\delta)}{2}\frac{1}{\mu}\frac{dp}{dx}$$

$$\frac{\partial u}{\partial y} = \frac{2y-\delta}{2\mu}\frac{dp}{dx}$$

$$\rho u = \frac{y(y-\delta)}{2\mu}\frac{dp}{dx}\rho = f(y)$$

$$\rho = \frac{F_1(y)}{\frac{dp}{dx}} \qquad \frac{1}{\rho} = \frac{\frac{dp}{dx}}{F_1(y)}$$

$$\frac{\partial^2\theta}{\partial y^2} = \frac{p\frac{dp}{dx}}{R}\frac{\partial^2}{\partial y^2}\left[\frac{1}{F_1(y)}\right]$$

$$\frac{y^2-y\delta}{2}\left(\frac{dp}{dx}\right)^2 + k\,p\,\frac{d^2p}{dx^2}\,\frac{y^2-y\delta}{2} = (k-1)\left(\frac{2y-\delta}{2}\right)^2\left(\frac{dp}{dx}\right)^2 + p\,\frac{dp}{dx}\,\frac{\kappa}{R}\,F_{(y)}$$

$$\cancel{(y-\delta)\left(\frac{dp}{dx}\right)^2}$$

$$\left(\frac{dp}{dx}\right)^2\left[\frac{y^2-y\delta}{2} - (k-1)\left(\frac{2y-\delta}{2}\right)^2\right] + k\,p\,\frac{d^2p}{dx^2}\,\frac{y^2-y\delta}{2} = p\,\frac{dp}{dx}\,\frac{\kappa}{R}\,F_{(y)}$$

$$\cancel{\left(\frac{dp}{dx}\right)^2\left[y-\frac{\delta}{2} - (k-1)(2y-\delta)\right] + k\,p\,\frac{d^2p}{dx^2}\left(y-\frac{\delta}{2}\right) = p\,\frac{dp}{dx}\,\frac{\kappa}{R}\,F'_{(y)}}$$

$$y^2\left[\left(\frac{3}{2}-k\right)\left(\frac{dp}{dx}\right)^2 + \frac{k}{2}\,p\,\frac{d^2p}{dx^2}\right] + y\delta\left[\left(k-\frac{3}{2}\right)\left(\frac{dp}{dx}\right)^2 - \frac{k}{2}\,p\,\frac{d^2p}{dx^2}\right]$$

$$-\frac{(k-1)\delta^2}{4}\left(\frac{dp}{dx}\right)^2 = p\,\frac{dp}{dx}\,\frac{\kappa}{R}\,F_{(y)}$$

$$(y^2-y\delta)\left[\left(\frac{3}{2}-k\right)\left(\frac{dp}{dx}\right)^2 + \frac{k}{2}\,p\,\frac{d^2p}{dx^2}\right]$$

$$(2y-\delta)\left[\qquad\qquad\right] = p\,\frac{dp}{dx}\,\frac{\kappa}{R}\,F'_{(y)}$$

$$2\left[\qquad\qquad\right] = \cdot\ -\ F''_{(y)}$$

$$y-\frac{\delta}{2} \qquad = \frac{F'}{F''}$$

$$\frac{F''}{F'} = \frac{1}{y-\frac{\delta}{2}}$$

$$\lg F' = \lg\left(y-\frac{\delta}{2}\right) + \cdots$$

$$F' = a\left(y-\frac{\delta}{2}\right)$$

$$F = a\left(\frac{y^2}{2} - \frac{y\delta}{2}\right) + b = \frac{a}{2}\,y\,(y-\delta) + b$$

$$(y^2-y\delta)\left[\qquad\qquad\right]-\frac{(k-1)\delta^2}{4}\left(\frac{dp}{dx}\right)^2=p\frac{dp}{dx}\frac{k}{R}\left[\frac{a}{2}(y^2-y\delta)+b\right]$$

$$\left.\begin{aligned}\left(\frac{3}{2}-k\right)\left(\frac{dp}{dx}\right)^2+\frac{k}{2}p\frac{d^2p}{dx^2}&=\frac{a}{2}\frac{k}{R}p\frac{dp}{dx}\\ \frac{(k-1)\delta^2}{4}\left(\frac{dp}{dx}\right)^2&=-b\frac{k}{R}p\frac{dp}{dx}\end{aligned}\right\}$$

$$\frac{1}{p}\frac{dp}{dx}=-\frac{4bk}{(k-1)\delta^2R}$$

$$\boxed{p=A\,e^{-\frac{4bk}{(k-1)\delta^2R}x}}$$

$$\cancel{\left(\frac{3}{2}-k\right)}\left[\frac{4bk}{(k-1)\delta^2Rp}\right]\left[\underbrace{\frac{3}{2}-k+\frac{k}{2}}_{\frac{3-k}{2}}\right]=-\frac{a}{2}\frac{k}{pR}$$

$$\boxed{a=-\frac{4(3-k)b}{(k-1)\delta^2}}$$

$$\frac{\partial^2\theta}{\partial y^2}=\frac{p\frac{dp}{dx}}{R}F=\frac{p\frac{dp}{dx}}{R}\left[\frac{a}{2}(y^2-y\delta)+b\right]$$

$$\frac{\partial\theta}{\partial y}=\frac{p\frac{dp}{dx}}{R}\left[\frac{a}{2}\left(\frac{y^3}{3}-\frac{y^2\delta}{2}\right)+by\right]+\vartheta_{(x)}$$

$$\theta=\frac{p\frac{dp}{dx}}{R}\left[\frac{a}{2}\left(\frac{y^4}{12}-\frac{y^3\delta}{6}\right)+b\frac{y^2}{2}\right]+y\,\vartheta_{(x)}+\eta_{(x)}$$

$y=0$: $\theta_0=\eta_{(x)}$

$y=\delta$: $\cancel{\theta_0}=\frac{p\frac{dp}{dx}}{R}\left[-\frac{a}{2}\frac{\delta^3 y}{12}+b\frac{\delta y}{2}\right]+y\,\vartheta_{(x)}+\cancel{\theta_0}$

$$\theta=\theta_0+\frac{p\frac{dp}{dx}}{R}\left[\frac{a}{2}\left(\frac{y^4+\delta^3y}{12}-\frac{y^3\delta}{6}\right)+b\frac{y^2-y\delta}{2}\right]=\frac{pu}{R\rho u}$$

$R\theta = \dfrac{p}{\rho}$ $\qquad R\theta = \dfrac{p(x)}{f(y)} \cdot \dfrac{y(y-\delta)}{2\mu}\dfrac{dp}{dx} = F(y)\, p(x)\dfrac{dp}{dx} = \dfrac{c}{2}F(y)$

dla $y=0$

Niemożliwe do spełnienia, gdyż jeśli

$p\dfrac{dp}{dx} = \text{const}$ $\qquad p^2 = cx + a$ $\qquad p\dfrac{dp}{dx} = \dfrac{c}{2}$

$p\dfrac{dp}{dx} = \dfrac{c}{2}$ $\qquad p\dfrac{d^2p}{dx^2} + \left(\dfrac{dp}{dx}\right)^2 = 0$

$$\mu\rho = f = \frac{y(y-\delta)}{2\mu}\frac{1}{F(y)}$$

$$\frac{\partial^2\theta}{\partial y^2} = \frac{c}{2R}F''$$

$$u = \frac{y(y-\delta)}{2}\frac{1}{\mu}\frac{dp}{dx}$$

$$\frac{y^2-y\delta}{2\mu}\left(\frac{dp}{dx}\right)^2 + k\,\frac{y^2-y\delta}{2\mu}\,p\frac{d^2p}{dx^2} = (k-1)\mu\left(\frac{2y-\delta}{2\mu}\right)^2\left(\frac{dp}{dx}\right)^2 + \frac{kc}{2R}F''_{(y)}$$

$p = \sqrt{cx+a}$ $\qquad p\dfrac{d^2p}{dx^2} = -\dfrac{c^2}{4(cx+a)}$

$\dfrac{dp}{dx} = \dfrac{c}{2\sqrt{cx+a}}$ $\qquad \left(\dfrac{dp}{dx}\right)^2 = \dfrac{c^2}{4(cx+a)}$

$$\frac{d^2p}{dx^2} = \frac{-c^2}{4\sqrt{(cx+a)^3}}$$

$$\left[(1-k)(y^2-y\delta) - (k-1)\frac{(2y-\delta)^2}{2}\right] \frac{c^2}{4(cx+a)}$$

$$(1-k)\; y^2 - y\delta - 2y^2 + 2y\delta$$

Więc nie spełnia równania II

$(p = f(x)$ etc$)$

Założenie musi być opuszczone ze sobą!

Adiabatycznie

$$\frac{dp}{dx} = \mu \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r}\right) \qquad \frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^k \qquad \rho = f_1(x)$$

$$\frac{\partial(\rho u)}{\partial x} = 0 \qquad p^{\frac{1}{k}} \sim \rho$$

$$\rho u = f_c(r)$$

p niezależne od r zatem : $\frac{\partial}{\partial r}\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right] = 0$

$$u = \frac{r^2}{4}\varphi(x) + \cancel{\psi(x)\log r} + \chi(x)$$

$$\frac{dp}{dx} = \mu\varphi(x)$$

$$u = \frac{r^2-\delta^2}{4}\varphi(x)$$

$$u = \frac{r^2-\delta^2}{4\mu}\frac{dp}{dx}$$

$$\frac{1}{p_0}\frac{dp}{dx} = \frac{k}{\rho_0^k}\rho^{k-1}\frac{d\rho}{dx}$$

$$\frac{\partial}{\partial x}\left[\frac{r^2-\delta^2}{4\mu}\frac{k p_0}{\rho_0^k}\rho^k\frac{d\rho}{dx}\right] = 0$$

$$\frac{r^2-\delta^2}{4\mu}\frac{k p_0}{\rho_0^k}\frac{\rho^{k+1}}{k+1} = x f(r) + \psi(r)$$

$$= (r^2-\delta^2)(Ax+B)$$

$$\cancel{\rho^{k+1} = ax+b}$$

$$p^{\frac{k+1}{k}} = ax+b$$

$$\frac{p_2^{\frac{k+1}{k}} - p_1^{\frac{k+1}{k}}}{l} = a$$

$$p^{\frac{k+1}{k}} - p_1^{\frac{k+1}{k}} = \frac{x}{l}\left(p_2^{\frac{k+1}{k}} - p_1^{\frac{k+1}{k}}\right)$$

itd.

Podobnie jak przy izotermicznym wstawić tylko że $\frac{1+k}{k}$ [illegible]

Jesterminiane z uwzględnieniem $\frac{\partial^2 u}{\partial x^2}$

$$\frac{dp}{dx} = \mu \frac{\partial^2 u}{\partial y^2} + \frac{4}{3}\mu \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial(\rho u)}{\partial x} = 0 \quad = \quad \frac{\partial(p u)}{\partial x} = 0$$

$$p u = \varphi(y)$$

$$u = \frac{\varphi(y)}{f(x)} \qquad p = f(x)$$

$$f'(x) = \mu \frac{\varphi''(y)}{f(x)} + \frac{4}{3}\mu\, \varphi(y) \frac{d^2}{dx^2}\left(\frac{1}{f(x)}\right)$$

$$f'(x) f(x) = \mu \varphi''(y) + \frac{4}{3}\mu\, \varphi(y) f(x) \frac{d^2}{dx^2}\left[\frac{1}{f(x)}\right]$$

$\frac{\partial}{\partial y}$:

$$0 = \mu \varphi'''(y) + \frac{4}{3}\mu \varphi'(y)$$

$$0 = \mu \varphi''''(y) + \frac{4}{3}\mu \varphi''(y)$$

$$\cancel{\frac{\varphi''''}{\varphi'''} = \frac{\varphi''}{\varphi'}}$$

Możliwe tylko jeśli: ~~[illegible]~~

I).

$$f(x) f'(x) = a$$

$$\underbrace{f(x) f'(x)}_{\frac{1}{2}\frac{d}{dx} f^2}$$

$$f^2 = 2ax + b$$

$$f = \sqrt{2ax+b}$$

$$f' = \frac{a}{\sqrt{}}$$

$$f'' = -\frac{a^2}{\sqrt{()^3}}$$

$$f(x) \frac{d^2}{dx^2}\left[\frac{1}{f(x)}\right] = c \qquad \cancel{\varphi'' = c}$$

$$d - \frac{f'}{f^2}$$

$$\underbrace{-\frac{f''}{f} + 2\frac{f'^2}{f^2}}_{} =$$

$$+\frac{a^2}{(2ax+b)^2} + 2\frac{a^2}{()^2} = c$$

Niemożliwe oprócz jeśli $a=0$

t.zn $p = $ const

Druga możliwość: $\varphi''(y) = a \qquad \varphi(y) = b$ niemożliwe bo ...

Zatem to wogóle nie może być przystosowane do warunków początkowych

(Jedyna możliw.: $p = $ const lub $pu = $ const

Izotermiczne z uwzględn. zmienności p, jeśli $v, u = 0$

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2} + \frac{4}{3}\mu \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial p}{\partial y} =$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \qquad \rho u = \frac{\partial \psi}{\partial y} \qquad \rho v = -\frac{\partial \psi}{\partial x} \qquad \frac{p}{\rho} = R\theta$$

$$\frac{v}{u} = -\frac{\frac{\partial \psi}{\partial x}}{\frac{\partial \psi}{\partial y}}$$

$$u = \frac{1}{\rho}\frac{\partial \psi}{\partial y} = \frac{R\theta}{p}\frac{\partial \psi}{\partial y}$$

$$v = -\frac{1}{\rho}\frac{\partial \psi}{\partial x} = -\frac{R\theta}{p}\frac{\partial \psi}{\partial x}$$

$$\frac{\partial u}{\partial x} = \frac{1}{\rho}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{1}{\rho^2}\frac{\partial \rho}{\partial x}\frac{\partial \psi}{\partial y}$$

$$\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial^2 \psi}{\partial x \partial y} + \frac{1}{\rho^2}\frac{\partial \rho}{\partial y}\frac{\partial \psi}{\partial x}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{1}{\rho}\left(u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y}\right)$$

$$= -\frac{1}{\rho^2}\left(\frac{\partial \psi}{\partial y}\frac{\partial \rho}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \rho}{\partial y}\right)$$

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -\frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial y}\frac{\partial \psi}{\partial y} - \frac{\partial \rho}{\partial x}\frac{\partial \psi}{\partial x}\right) + \frac{1}{\rho}\left(\frac{\partial^2 \psi}{\partial y^2} - \frac{\partial^2 \psi}{\partial x^2}\right)$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = +\frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial y}\frac{\partial \psi}{\partial y} + \frac{\partial \rho}{\partial x}\frac{\partial \psi}{\partial x}\right) - \frac{1}{\rho}\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right)$$

$$\Phi = \frac{4}{3}\mu\left[\underbrace{\left(\frac{\partial u}{\partial x}\right)^2 - \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} + \left(\frac{\partial v}{\partial y}\right)^2}\right] + \mu\left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right]^2$$

$$= \frac{1}{\rho^4}\left[\left(\frac{\partial \psi}{\partial y}\frac{\partial \rho}{\partial x}\right)^2 + \left(\frac{\partial \psi}{\partial x}\right)^2\left(\frac{\partial \rho}{\partial y}\right)^2\right] + \frac{1}{\rho^2}\left(\frac{\partial^2 \psi}{\partial x \partial y}\right)^2 - \frac{1}{\rho^3}\frac{\partial^2 \psi}{\partial x \partial y}\left[\frac{\partial \rho}{\partial x}\frac{\partial \psi}{\partial y} + \frac{\partial \rho}{\partial y}\frac{\partial \psi}{\partial x}\right]$$

$$- \frac{1}{\rho^4}\frac{\partial \rho}{\partial x}\frac{\partial \rho}{\partial y}\frac{\partial \psi}{\partial x}\frac{\partial \psi}{\partial y}$$

$$\cancel{-\frac{1}{\rho^2}\left(\frac{\partial^2 \psi}{\partial x \partial y}\right)^2 + \frac{1}{\rho^3}\frac{\partial^2 \psi}{\partial x \partial y}\left[\frac{\partial \rho}{\partial x}\frac{\partial \psi}{\partial y} + \frac{\partial \rho}{\partial y}\frac{\partial \psi}{\partial x}\right]}$$

III).

$$k\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=\frac{4\mu}{3}\left[\left(\frac{\partial u}{\partial x}\right)^2-\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}+\left(\frac{\partial v}{\partial y}\right)^2\right]+\mu\left[\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right]^2$$

$$-\frac{\mu}{3}\left[u\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+v\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]-\mu\left[u\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+v\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)\right]$$

$$=\frac{4}{3}\mu\left[\left(\frac{\partial u}{\partial x}\right)^2-u\frac{\partial^2 u}{\partial x^2}+\left(\frac{\partial v}{\partial y}\right)^2-v\left(\frac{\partial^2 v}{\partial y^2}\right)\right]-\frac{1}{3}\mu\left[4\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}+u\frac{\partial^2 v}{\partial x\partial y}+v\frac{\partial^2 u}{\partial x\partial y}\right]$$

$$u^2\left[\frac{1}{u^2}\left(\frac{\partial u}{\partial x}\right)^2-\frac{1}{u}\frac{\partial^2 u}{\partial x^2}\right]+v^2\left[\ \cdots\ \right]$$

$$+\mu\left[\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2-u\frac{\partial^2 u}{\partial y^2}-v\frac{\partial^2 v}{\partial x^2}\right]$$

$$=-u^2\frac{\partial}{\partial x}\left(\frac{1}{u}\frac{\partial u}{\partial x}\right)$$

$$=-\left[u^2\frac{\partial^2}{\partial x^2}(\lg u)+v^2\frac{\partial^2}{\partial y^2}(\lg v)\right]\frac{4}{3}-\left[u^2\frac{\partial^2}{\partial y^2}(\lg u)+v^2\frac{\partial^2}{\partial x^2}(\lg v)\right]$$

$=$

$$\left[a_1x^3+a_2xy^2)+a_3\right]\left[(8a_1+2a_2)x+\frac{2\mu}{3}b_2y\right]+\left[-\frac{b_2}{3}x^3+b_2y^2x+b_3\right]\left[\left(\frac{8}{3}b_2+6b_1\right)x+\frac{2\mu}{3}a_2y\right]$$

$$+k\left[\frac{p_0}{\mu}+(4a_1+a_2)x^2+\frac{a_2}{3}y^2+\frac{2}{3}b_2xy\right]\left[3a_1x^2+a_2y^2+2b_2xy\right]=$$

		stałych	równań $\frac{\partial^2}{\partial x\partial y}$	równań
Jeśli u, v, p rzędu	1	3 + 3 + p_0	$\frac{\partial^3 v}{\partial x^3}+\frac{\partial^3 v}{\partial x\partial y^2}=\frac{\partial^3 u}{\partial x^2\partial y}+\frac{\partial^3 u}{\partial y^3}$:	0
	2	6 + 6		0
	3	10 + 10	$x^{n-3}\ y^{n-3}$	1
	4	15 + 15		3
	5	21 + 21		6
	6	28		

Wstawiając do równania dla Φ:

Stopień: $2(n-1)$	równań dla spółczyn.	pozostaje zatem dowolnych:
0	0	6
2	6	po + 6
4	15	5
6	28	2
8	45	−3

$u = e^{a+\varphi}$ $\quad v = e^{b+\psi}$ $\quad p = e^{c+\chi}$

$\frac{\partial u}{\partial x} = \frac{\partial \varphi}{\partial x} e^{\varphi} =$ [illegible] $\qquad \frac{\partial v}{\partial y} = \frac{\partial \psi}{\partial y} e^{\psi}$

$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 \varphi}{\partial x^2} e^{\varphi} + \left(\frac{\partial \varphi}{\partial x}\right)^2 e^{\varphi} = \left[\frac{\partial^2 \varphi}{\partial x^2} + \left(\frac{\partial \varphi}{\partial x}\right)^2\right] e^{\varphi} \qquad \frac{\partial^2 v}{\partial x \partial y} = \frac{\partial^2 \psi}{\partial x \partial y} e^{\psi} + \frac{\partial \psi}{\partial x}\frac{\partial \psi}{\partial y} e^{\psi}$

$e^{\chi} \frac{\partial \chi}{\partial x} = \left\{ \frac{4\mu}{3}\left[\frac{\partial^2 \varphi}{\partial x^2} + \left(\frac{\partial \varphi}{\partial x}\right)^2\right] + \mu\left[\frac{\partial^2 \varphi}{\partial y^2} + \left(\frac{\partial \varphi}{\partial y}\right)^2\right]\right\} e^{\varphi} + \frac{\mu}{3}\left[\frac{\partial^2 \psi}{\partial x \partial y} + \frac{\partial \psi}{\partial x}\frac{\partial \psi}{\partial y}\right] e^{\psi}$

$u = x(2a x - y^2)\varphi \qquad \cancel{v = x(2a x - y^2)\psi}$

$\frac{\partial u}{\partial x} =$

$u = x(a_1 x^2 +$ [illegible] $+ a_2 y^2) + a_3 \quad \Big| \quad v = x(b_1 x^2 +$ [illegible] $+ b_2 y^2) + b_3$

$\frac{\partial u}{\partial x} = 3 a_1 x^2 + a_2 y^2 \;\Big|\; \frac{\partial u}{\partial y} = 2 a_2 x y \;\Big|\; \frac{\partial v}{\partial y} = 2 b_2 x y \qquad \frac{\partial v}{\partial x} = 3 b_1 x^2 + b_2 y^2$

$\frac{\partial^2 u}{\partial x^2} = 6 a_1 x \qquad \frac{\partial^2 u}{\partial y^2} = 2 a_2 x \;\Big|\; \frac{\partial^2 v}{\partial y^2} = 2 b_2 x \qquad \frac{\partial^2 v}{\partial x^2} = 6 b_1 x$

$\frac{\partial^2 u}{\partial x \partial y} = 2 a_2 y \qquad \frac{\partial^2 v}{\partial x \partial y} = 2 b_2 y$

$$\frac{\partial p}{\partial x} = \frac{4\mu}{3}\, 6 a_1 x + \frac{\mu}{3}\, 2 b_2 y + \mu\, 2 a_2 x = \mu(8 a_1 + 2 a_2) x + \frac{2\mu}{3} b_2 y$$

$$\frac{\partial p}{\partial y} = \frac{4\mu}{3}\, 2 b_2 x + \frac{\mu}{3}\, 2 a_2 y + \mu\, 6 b_1 x = \mu\left(\frac{8}{3} b_2 + 6 b_1\right) x + \frac{2\mu}{3} a_2 y$$

$$p = p_0 + \mu(4 a_1 + a_2) x^2 + \frac{\mu}{3} a_2 y^2 + \frac{2\mu}{3} b_2 x y$$

$$\frac{2\mu}{3} b_2 = \frac{8}{3} b_2 + 6 b_1$$

$$\underline{b_2 = -3 b_1}$$

$$\frac{\partial^2}{\partial x \partial y} \lg u = \frac{\partial}{\partial y}\left(\frac{1}{u}\frac{\partial u}{\partial x}\right) = -\frac{1}{u^2}\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + \frac{1}{u}\frac{\partial^2 u}{\partial x \partial y}$$

$$u = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 xy + a_5 y^2$$

$$v = b_0 + b_1 x + b_2 y + b_3 x^2 + b_4 xy + b_5 y^2$$

$$\frac{\partial u}{\partial x} = a_1 + 2a_3 x + a_4 y$$

$$\frac{\partial^2 u}{\partial x^2} = 2a_3 \qquad \frac{\partial^2 u}{\partial x \partial y} = a_4 \qquad \frac{\partial^2 u}{\partial y^2} = 2a_5$$

$$\frac{\partial^2 v}{\partial x^2} = 2b_3 \qquad \frac{\partial^2 v}{\partial x \partial y} = b_4 \qquad \frac{\partial^2 v}{\partial y^2} = 2b_5$$

$$\frac{\partial p}{\partial x} = \frac{1}{3}\mu\,(2a_3 + b_4) + \mu\,(2a_3 + 2a_5)$$

$$\frac{\partial v}{\partial y} = b_2 + b_4 x + 2b_5 y$$

$$\frac{\partial v}{\partial x} = b_1 + 2b_3 x + b_4 y$$

$$\frac{\partial u}{\partial y} = a_2 + a_4 x + 2a_5 y$$

$$\frac{\partial p}{\partial x} = \mu\left(\frac{8}{3} a_3 + \frac{b_4}{3} + 2a_5\right)$$

$$\frac{\partial p}{\partial y} = \mu\left(\frac{8}{3} b_3 + \frac{a_4}{3} + 2b_5\right)$$

$$p = p_0 + \mu\overbrace{\left(\frac{8}{3} a_3 + \frac{1}{3} b_4 + 2a_5\right)}^{A} x + \mu\overbrace{\left(\frac{8}{3} b_3 + \frac{1}{3} a_4 + 2b_5\right)}^{B} y$$

$$\mu\left(\frac{8}{3} a_3 + \frac{1}{3} b_4 + 2a_5\right)\left(a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 xy + a_5 y^2\right) +$$

$$\left(\frac{8}{3} b_3 + \frac{1}{3} a_4 + 2b_5\right)\left(b_0 + b_1 x + b_2 y + b_3 x^2 + b_4 xy + b_5 y^2\right) +$$

$$k\left[p_0 + \mu\left(\frac{8}{3} a_3 + \frac{1}{3} b_4 + 2a_5\right)x + \mu\left(\qquad \right)y\right]\left[a_1 + b_1 + (2a_3 + b_4)x + (2b_5 + a_4)y\right]$$

$$= (k-1)\left\{\left[\frac{4}{3}\mu\left\{(a_1 + 2a_3 x + a_4 y)^2 + (b_2 + b_4 x + 2b_5 y)^2 - (a_1 + 2a_3 x + a_4 y)(b_2 + b_4 x + 2b_5 y)\right\}\right.\right.$$

$$\left.\left. + \mu\left[a_2 + b_1 + (a_4 + 2b_3)x + (2a_5 + b_4)y\right]^2\right\}\right.$$

$$\left(\frac{8}{3} a_3 + \frac{1}{3} b_4 + 2a_5\right)a_0 + \left(\frac{8}{3} b_3 + \frac{1}{3} a_4 + 2b_5\right)b_0 + k(a_1 + b_1)\frac{p_0}{\mu} =$$

$$= (k-1)\left[\frac{4}{3}\left(a_1^2 + b_2^2 - a_1 b_2\right) + (a_2 + b_1)^2\right]$$

$$A a_5 + \mathcal{B} b_5 + k\mathcal{B}(2b_5 + a_4) = (k-1)\left[\frac{4}{3}(a_4^2 + 4b_5^2 - 2a_4 b_5) + (2a_5 + b_4)^2\right]$$

Spróbować $a_0 = b_0 = a_2 = b_2 = a_5 = b_5 = 0$

t. zn. że $u = v = 0$ na osi X

$$\left(\frac{8}{3}a_3 + \frac{1}{3}b_4\right)(a_1 x + a_3 x^2 + a_4 xy) + \left(\frac{8}{3}b_3 + \frac{1}{3}a_4\right)(b_1 x + b_3 x^2 + b_4 xy) +$$

$$+ k\left[\frac{p_0}{\mu} + \left(\frac{8}{3}a_3 + \frac{1}{3}b_4\right)x + \left(\frac{8}{3}b_3 + \frac{1}{3}a_4\right)y\right]\left[a_1 + b_1 + (2a_3 + b_4)x + a_4 y\right]$$

$$= (k-1)\left[\frac{4}{3}\left\{(a_1 + 2a_3 x + a_4 y)^2 + (b_4 x)^2 - (a_1 + 2a_3 x + a_4 y)b_4 x\right\} + \left[b_1 + (a_4 + 2b_3)x + b_4 y\right]^2\right]$$

6 równania na spółczynniki $y^2 \quad x^2 \quad xy \quad x \quad y \quad 0$

$u = x(a_1 + a_3 x + a_4 y)$

$v = x(b_1 + b_3 x + b_4 y)$

Jeżeli równocześnie mają a_1, b_1 być $= 0$ na drugiej prostej to chyba tak że $a_1 : b_1 = a_3 : b_3 = a_4 : b_4$

$$\left(\frac{8}{3}a_3 + \frac{1}{3}b_4\right)a_4 x + \left(\frac{8}{3}b_3 + \frac{1}{3}a_4\right)b_4 x + k\left\{\left(\frac{8}{3}b_3 + \frac{1}{3}a_4\right)\left[a_1 + b_1 + (2a_3 + b_4)x + a_4 y\right] +\right.$$

$$\left. + a_4\left[\frac{p_0}{\mu} + (\quad)x + \left(\frac{8}{3}b_3 + \frac{1}{3}a_4\right)y\right]\right\} =$$

$$= (k-1)\left[\frac{8}{3}a_4(a_1 + 2a_3 x + a_4 y) - a_4 b_4 x + 2\left[b_1 + (a_4 + 2b_3)x + b_4^2 y\right]\right]$$

$$k\left\{\left(\frac{8}{3}b_3 + \frac{1}{3}a_4\right)a_4 + a_4\left(\frac{8}{3}b_3 + \frac{1}{3}a_4\right)\right\} = (k-1)\left(\frac{8}{3}a_4^2 + 2b_4^2\right)$$

$$k\, a_4\left(\frac{8}{3}b_3 + \frac{1}{3}a_4\right) =$$

Czy możliwe są rozwiązania formy

$$u = a + e^{\alpha_1 x + \alpha_2 y} \qquad \frac{\partial u}{\partial x} = \alpha_1 e^{\alpha_1 x + \alpha_2 y} \qquad \frac{\partial u}{\partial y} = \alpha_2 e^{\alpha_1 x + \alpha_2 y}$$

$$v = b + e^{\beta_1 x + \beta_2 y} \qquad \frac{\partial v}{\partial x} = \beta_1 e^{\beta_1 x + \beta_2 y} \qquad \frac{\partial v}{\partial y} = \beta_2 e^{\beta_1 x + \beta_2 y}$$

Znaleźć takie u v którzy jako równanie $u=0$, $v=0$ dały np. $x=0$ $y=$

$$p = R \rho \theta$$

$$\rho \frac{\partial \theta}{\partial x} + \theta \frac{\partial \rho}{\partial x} = \frac{\mu}{3R} \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \frac{\mu}{R}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\rho\left[\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x}\right] = -\frac{\partial p}{\partial x} + \frac{4\mu}{3} \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 \quad \Big| \; u \qquad \frac{\partial^2 \rho}{\partial t^2} = -\frac{\partial^2(\rho u)}{\partial x \partial t}$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} = -\frac{\partial p}{\partial x} + \frac{4\mu}{3} \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2(\rho u^2)}{\partial x^2} + \frac{\partial^2 p}{\partial x^2} - \frac{4\mu}{3} \frac{\partial^3 u}{\partial x^3}$$

$$\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2}{\partial x^2}\left[p + \rho u^2 - \frac{4\mu}{3} \frac{\partial u}{\partial x}\right]$$

$$= \frac{\partial^2}{\partial x^2}\left[\rho(R\theta + u^2) - \frac{4\mu}{3} \frac{\partial u}{\partial x}\right]$$

Zaniedbując tarcie i u^2

$$\frac{c}{A} \rho\left(\frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x}\right) + p \frac{\partial u}{\partial x} = \Phi + \kappa \nabla^2 \theta$$

$$\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + k p \frac{\partial u}{\partial x} = (k-1)\Phi + (k-1)\kappa \frac{\partial^2 \theta}{\partial x^2}$$

$$\cancel{\frac{\partial^2 p}{\partial t^2} = R\frac{\partial^2}{\partial t^2}(\rho\theta) = R\left[\rho\frac{\partial^2\theta}{\partial t^2} + 2\frac{\partial\rho}{\partial t}\frac{\partial\theta}{\partial t} + \theta\frac{\partial^2\rho}{\partial t^2}\right]}$$

Zaniedbując tarcie i wyższe potęgi:

$$\frac{\partial^2\rho}{\partial t^2} = \frac{\partial^2}{\partial x^2}\left[R\rho\theta\right] \qquad R\rho\theta$$

$$\frac{R}{k-1}\,\rho\left(\frac{\partial\theta}{\partial t} + \cancel{u\frac{\partial\theta}{\partial x}}\right) + p\frac{\partial u}{\partial x} = \kappa\frac{\partial^2\theta}{\partial x^2}$$

$$\frac{R}{k-1}\,\rho_0\frac{\partial\theta}{\partial t} - R\theta_0\frac{\partial\rho}{\partial t} = \kappa\frac{\partial^2\theta}{\partial x^2}$$

$$\frac{\partial\rho}{\partial t} + \rho_0\frac{\partial u}{\partial x} + \cancel{u\frac{\partial\rho}{\partial x}} = 0$$

Przybliżone obliczenie:

zapominając $u = a\sin(\alpha x - \beta t)$

$$\rho = -\rho_0\frac{\alpha}{\beta}\sin(\alpha x - \beta t)$$

W jednowymiarowem zadaniu: wpływ κ

$$\begin{cases} \dfrac{dp}{dx} = \dfrac{4}{3}\mu\dfrac{d^2u}{dx^2} \qquad \rho u = \rho_0 u_0 \\[2mm] u\dfrac{dp}{dx} + k\,p\dfrac{du}{dx} = (k-1)\dfrac{4}{3}\mu\left(\dfrac{du}{dx}\right)^2 + (k-1)\,\kappa\dfrac{d^2\theta}{dx^2} \end{cases}$$

$$p = p_0 + \frac{4}{3}\mu\frac{du}{dx}$$

$$\theta = \frac{pu}{R\rho_0 u_0}$$

$$\frac{d\theta}{dx} = \frac{1}{R\rho_0 u_0}\left[p_0\frac{du}{dx} + \frac{4}{3}\mu\left(\frac{du}{dx}\right)^2 + \frac{4}{3}\mu u\frac{d^2u}{dx^2}\right]$$

$$\frac{4}{3}\mu u\frac{d^2u}{dx^2} + k p_0\frac{du}{dx} + \cancel{\frac{4}{3}\mu k\left(\frac{du}{dx}\right)^2} = \cancel{\frac{4}{3}\mu k\left(\frac{du}{dx}\right)^2} - \frac{4}{3}\mu\left(\frac{du}{dx}\right)^2 + (k-1)\,\kappa\frac{d^2\theta}{dx^2}$$

$$\frac{4}{3}\mu\left[u\frac{d^2u}{dx^2} + \left(\frac{du}{dx}\right)^2\right] + k p_0\frac{du}{dx} = (k-1)\,\kappa\frac{d^2\theta}{dx^2}$$

$$\frac{4}{3}\mu u\frac{du}{dx} + k p_0 u = (k-1)\,\kappa\frac{d\theta}{dx} + (k-1)\,a$$

$$= (k-1)\,a + \frac{(k-1)\,\kappa}{R\rho_0 u_0}\left[p_0\frac{du}{dx} + \frac{4}{3}\mu\frac{d}{dx}\left(u\frac{du}{dx}\right)\right]$$

$$\cancel{\frac{(k-1)\kappa}{R\rho_0 u_0}\left[\frac{4}{3}\mu\frac{d}{dx}\left(u\frac{du}{dx}\right)+p_0\frac{du}{dx}\right]-\frac{4}{3}\mu u\frac{du}{dx}=kp_0u-a(k-1)}$$

Z przybliżeniem podstawiając jako pierwsze przybliżenie:

$$R\frac{d\theta}{dx}=-\frac{(k-1)p_0}{\rho_0 u_0}\frac{du}{dx}$$

$$\frac{\frac{4}{3}\mu u-\frac{(k-1)\kappa}{R\rho_0 u_0}p_0}{kp_0u-a(k-1)}du=dx$$

$$\int\frac{\frac{4}{3}\mu u-\frac{(k-1)\kappa}{u_0\theta_0}}{u-\alpha}du=kp_0x-b$$

$$=\frac{4}{3}\mu+\frac{\frac{4}{3}\mu\alpha-\frac{(k-1)\kappa}{u_0\theta_0}}{u-\alpha}$$

$$\frac{4}{3}\mu(u-\alpha)+\left[\frac{4}{3}\mu\alpha-\frac{(k-1)\kappa}{u_0\theta_0}\right]\log(u-\alpha)=kp_0x-b$$

$$\frac{\partial^2 \rho}{\partial t^2} = R\left[\rho_0 \frac{\partial^2 \theta}{\partial x^2} + \cancel{2\frac{\partial \rho}{\partial x}\frac{\partial \theta}{\partial x}} + \theta_0 \frac{\partial^2 \rho}{\partial x^2}\right]$$

$$\rho_0 \frac{\partial \theta}{\partial t} - (k-1)\theta_0 \frac{\partial \rho}{\partial t} = \frac{(k-1)\,\varkappa}{R} \frac{\partial^2 \theta}{\partial x^2}$$

$$\rho_0 \frac{\partial^3 \theta}{\partial t \partial x^2} = (k-1)\,\theta_0 \frac{\partial^3 \rho}{\partial t \partial x^2} + (k-1)\frac{\varkappa}{R}\frac{\partial^3 \theta}{\partial x^3}$$

$$\frac{\partial^3 \rho}{\partial t^3} = R\theta_0 \frac{\partial^3 \rho}{\partial t\, \partial x^2} + \varkappa(k-1)\frac{\partial^3 \theta}{\partial x^3}$$

$\downarrow$

w przybliżeniu

$p \sim \rho^k \sim \rho^\theta$

$\theta \sim \rho^{k-1}$

$\lg \theta = c + (k-1)\lg \rho$

$\frac{1}{\theta_0}\frac{\partial^3 \theta}{\partial x^3} = (k-1)\frac{\partial^3 \rho}{\partial x^3}\frac{1}{\rho_0}$

$$\frac{\partial^3 \rho}{\partial t^3} = R\theta_0 k \frac{\partial^3 \rho}{\partial t \partial x^2} + \varkappa\frac{(k-1)^2\theta_0}{\rho_0}\frac{\partial^3 \rho}{\partial x^3}$$

$$\rho = a \sin(\alpha x - \beta t)$$

$$+\beta^3 = R\theta_0 k\, \alpha^2 \beta - \frac{\varkappa(k-1)^2\theta_0}{\rho_0}\alpha^3$$

$$\frac{\beta}{\alpha} = a$$

$$a^3 = R\theta_0 k\, a - \frac{\varkappa(k-1)^2\theta_0}{\rho_0}$$

$a = \sqrt{kR\theta_0}$ jako I przybliżenie

$$\left[a^2 - R\theta_0 k\right] = -\frac{\varkappa(k-1)^2\theta_0}{\rho_0\sqrt{kR\theta_0}}$$

$$a = \sqrt{kR\theta_0 - \frac{\varkappa(k-1)^2\theta_0}{\rho_0 a_0}}$$

$$a = \sqrt{a_0^2 - \frac{\varkappa(k-1)^2\theta_0}{\rho_0 a_0}}$$

$$= a_0\left[1 - \frac{1}{2}\frac{\varkappa(k-1)^2\theta_0}{\rho_0 a_0^3}\right]$$

$$\frac{1}{2}\frac{(0{\cdot}4)^2\; 300 \cdot 0{\cdot}000\ldots}{0{\cdot}0013 \cdot (33000)^3} \quad !$$

$$-\frac{2}{3}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)^2+2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2$$

$$-u\frac{\partial}{\partial x}\left[-\frac{2}{3}\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+2\mu\frac{\partial u}{\partial x}\right]-v\frac{\partial}{\partial y}\left[-\frac{2}{3}\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+2\mu\frac{\partial v}{\partial y}\right]$$

$$-u\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right]\qquad -v\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right]$$

$$k\,p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=\frac{2}{3}\left[u\frac{\partial}{\partial x}\mu\,\mathrm{div}+v\frac{\partial}{\partial y}\mu\,\mathrm{div}-\mu[\mathrm{div}]^2\right]$$

$$-2\left[u\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+v\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right)-\mu\left(\frac{\partial u}{\partial x}\right)^2-\mu\left(\frac{\partial v}{\partial y}\right)^2\right]$$

$$-\left[u\frac{\partial}{\partial y}\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)+v\frac{\partial}{\partial x}\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)-\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right]$$

$$=\frac{2}{3}\left[u^2\left\{\frac{1}{u}\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]-\frac{1}{u^2}\frac{\partial u}{\partial x}\cdot\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right\}+v^2\left\{\frac{1}{v}\frac{\partial}{\partial y}\left[\mu(\quad)\right]-\frac{1}{v^2}\frac{\partial v}{\partial y}(\quad)\right\}\right.$$

$+ \ldots$

$$=\frac{2}{3}\left[u^2\frac{\partial}{\partial x}\frac{\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)}{u}+v^2\frac{\partial}{\partial y}\frac{\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)}{v}\right]$$

$$-2\left[u^2\frac{\partial}{\partial x}\left(\frac{\mu\frac{\partial u}{\partial x}}{u}\right)+v^2\frac{\partial}{\partial y}\left(\frac{\mu\frac{\partial v}{\partial y}}{v}\right)\right]-\left[u^2\frac{\partial}{\partial y}\frac{\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)}{u}+v^2\frac{\partial}{\partial x}\frac{\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)}{v}\right]$$

$$(k-1)\,p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$$

Przyblizone rozwiązanie podstawiając: $p = \sqrt{cx+a} + \varphi$

$$u = \frac{y(y-\delta)}{2\mu} \frac{dp}{dx} + \psi$$

$$\begin{cases} \frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2} \\ \frac{\partial p}{\partial y} = \frac{\mu}{3} \frac{\partial^2 u}{\partial x \partial y} \end{cases} \qquad \frac{\partial(\rho u)}{\partial x} = 0$$

$$u \frac{\partial p}{\partial x} + k p \frac{\partial u}{\partial x} = (k-1)\mu\left(\frac{\partial u}{\partial y}\right)^2 + \kappa(k-1)\frac{\partial^2 \theta}{\partial y^2}$$

$$u = \frac{y(y-\delta)}{4\mu} \cdot \frac{-c}{\sqrt{cx+a}} + \psi \quad \Bigg\| \quad \frac{\partial u}{\partial y} = \frac{2y-\delta}{4\mu} \frac{c}{\sqrt{(cx+a)}} + \frac{\partial \psi}{\partial y} \quad \Bigg| \quad \frac{\partial^2 u}{\partial y^2} = \frac{c}{2\mu\sqrt{}} + \frac{\partial^2 \psi}{\partial y^2}$$

$$\frac{\partial u}{\partial x} = \frac{y(y-\delta)\, c^2}{8\mu\sqrt{(cx+a)}^3} + \frac{\partial \psi}{\partial x} \qquad \frac{\partial^2 u}{\partial x \partial y} = \frac{-(2y-\delta)\,c^2}{8\mu\sqrt{()}^3} + \frac{\partial^2 \psi}{\partial x \partial y}$$

$$\left.\begin{aligned} \frac{c}{2\sqrt{cx+a}} + \frac{\partial \varphi}{\partial x} &= \mu \frac{c}{2\mu\sqrt{}} + \mu \frac{\partial^2 \psi}{\partial y^2} \\ \frac{\partial \varphi}{\partial y} &= \frac{\mu}{3}\left[\frac{-(2y-\delta)\,c^2}{8\mu\sqrt{(cx+a)}^3} + \frac{\partial^2 \psi}{\partial x \partial y}\right] \end{aligned}\right\} \text{Niemożliwe!}$$

$$\kappa(k-1)\frac{\partial^2 \theta}{\partial y^2} = \frac{y(y-\delta)}{4\mu} \frac{-c}{\sqrt{}} \frac{-c}{2\sqrt{}} + k\sqrt{} \frac{y(y-\delta)\, c^2}{8\mu\sqrt{()}^3}$$

$$-(k-1)\mu \frac{(2y-\delta)^2}{16\mu^2} \frac{c^2}{(cx+a)}$$

$$\kappa \frac{\partial^2 \theta}{\partial y^2} = \frac{-2y(y-\delta) - (2y-\delta)^2}{16\mu} \frac{c^2}{(a-cx)} = \frac{-\delta^2 + 4y\delta - 4y^2 + 2y\delta - 2y^2}{}$$

$$= -\underbrace{\frac{(\delta^2 - 6y\delta + 6y^2)}{16\mu}}_{y=0 \quad \delta^2} \frac{c^2}{a-cx}$$

N.p. $y = \frac{\delta}{2}$ $\quad \delta^2 - 3\delta^2 + \frac{6}{4} = -\frac{\delta^2}{2}$ $\quad\Bigg|\quad$ $y^2 - y\delta + \frac{\delta^2}{6} = 0$

$$y = \frac{\delta}{2} \pm \sqrt{\frac{\delta^2}{4} - \frac{\delta^2}{6}} = \frac{\delta}{2} \pm \frac{\delta}{\sqrt{12}}$$

$y = \frac{\delta}{2}$ $\quad \frac{d\theta}{dy} = 0$ $\quad \frac{1}{2} - \frac{3}{4} + \frac{1}{4}$

$$k \frac{d\theta}{\partial y} = -\frac{(\delta^2 y - 3\delta y^2 + 2y^3)}{16\mu} \frac{c^2}{a - cx} \cancel{+} \ {}_{0}$$

$$k\,\theta = -\frac{\left(\delta^2 y^2 - 2\delta y^3 + y^4\right)}{32\mu} \frac{c^2}{a-cx} + b$$

$$\theta = \theta_0 - \frac{y^2 (y-\delta)^2}{32\mu k} \frac{c^2}{a-cx}$$

Maximum $y = \frac{\delta}{2}$ $\qquad \theta = \theta_0 - \frac{\delta^4}{128\mu k} \frac{c^2}{a-cx}$

$$= \theta_0 - \frac{\delta^4}{128\mu k} \frac{c^2}{p^2}$$

$\frac{15}{16}\cdot\frac{8}{7}$

$\frac{1}{3}$

Podobnie przy symetryi walcowatej:

$$u = \frac{r^2 - \delta^2}{8\mu} \frac{-c}{\sqrt{a - cx}} \qquad \Big\| \quad p = \sqrt{a - cx}$$

$$\frac{dp}{dx} = \mu \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r}\right)$$

10^{2}

$$\frac{10^8 \cdot 4 \cdot \quad \cdot 10^{2}}{16 \cdot 128 \cdot 10^{8} \cdot \quad 10^{2} \cdot 42 \cdot 10^{6}}$$

$\frac{1}{8}$

$$u\frac{dp}{dx} + kp\frac{\partial u}{\partial x} = (k-1)\mu\left(\frac{\partial u}{\partial r}\right)^2 + (k-1)\,\kappa \cdot \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right)$$

$$\frac{dp}{dx} = \frac{-c}{2\sqrt{\ }} \qquad \frac{\partial u}{\partial x} = \frac{r^2-\delta^2}{8\mu} \frac{-\frac{c^2}{2}}{\sqrt{(\ )^3}} \qquad \frac{\partial u}{\partial r} = \frac{r}{4\mu} \frac{-c}{\sqrt{\ }}$$

$$\frac{c^2}{2} \frac{r^2-\delta^2}{8\mu} \frac{1}{(\ )} - k \frac{r^2-\delta^2}{8\mu} \frac{\frac{c^2}{2}}{(\ )} - (k-1)\mu \frac{r^2}{16\mu^2} \frac{c^2}{(\ )} = (k-1)\,\kappa \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial\theta}{\partial r}\right)$$

$\cancel{\kappa\frac{\partial\theta}{\partial r} = A r \frac{r^3}{4} + \frac{c}{r}}$

$\cancel{\theta = -\frac{r^4}{4} + m = \frac{\delta^4 - 2^4}{4}}$

$\cancel{\theta = \quad \frac{\delta^4}{4} + -}$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right) = \frac{1}{k\mu}\,\frac{-(r^2-\delta^2)c^2 - x^2c^2}{16(a-cx)} = \frac{c^2}{16k\mu}\,\frac{\delta^2-2r^2}{a-cx}$$

$$\frac{\partial}{\partial r}(\ \ ) = \frac{c^2}{16k\mu}\,\frac{\delta^2 r - 2r^3}{a-cx}$$

$$r\frac{\partial\theta}{\partial r} = \frac{c^2}{16k\mu}\,\frac{\frac{\delta^2r^2}{2} - \frac{r^4}{2}}{a-cx} + b$$

$$\frac{\partial\theta}{\partial r} = \frac{c^2}{32k\mu}\,\frac{r\delta^2 - r^3}{a-cx} + \frac{b}{r}$$

$$\begin{cases} \theta = \dfrac{c^2}{32k\mu}\,\dfrac{\frac{r^2\delta^2}{2} - \frac{r^4}{4}}{a-cx} + \cancel{b\lg r} + m \\[2ex] \theta_0 = \quad \cdots \quad \dfrac{\frac{\delta^2}{4}}{a-cx} \quad + n \end{cases}$$

$$\theta = \theta_0 - \frac{c^2}{128\,k\mu}\,\frac{(\delta^2-r^2)^2}{a-cx} = \theta_0 - \frac{(\delta^2-r^2)^2}{128.k\mu}\left[\frac{p_1^2-p_2^2}{lp}\right]^2 = \theta_0 - \frac{u^2}{2k}$$

$$\left.\begin{array}{l} p_1^2 = a \\ p_2^2 = a - cl \end{array}\right\} \quad \frac{p_1^2-p_2^2}{l} = c \qquad r = \frac{\delta}{2}$$

Minimum: $\theta = \theta_0 - \frac{\delta^4}{k\mu}\,\frac{9}{16.128}\left[\frac{p_1^2-p_2^2}{lp}\right]^2$

$$\frac{1}{240}\ \frac{10^{-2}\cdot 10^{-2}\cdot\frac{1}{9}\cdot 10^{12}}{10^{-8}\cdot 42\cdot 10^{6}} \qquad 100$$

N.p. $\delta = 0{\cdot}01$

$p_1 = 10^6 \ \| \ p_2 = \frac{3}{4}\cdot 10^6$

$l = 10$

$k = 0{\cdot}00006$

$\mu = 0{\cdot}00017$

$$\frac{1}{240}\left[\frac{1}{2}\,\frac{10^6}{10}\right]^2\cdot\frac{10^{-8}}{1{\cdot}7\cdot 6\cdot 10^{-9}\cdot 42\cdot 10^{6}} \qquad \frac{100}{9\cdot 2\cdot 42}$$

$$= \frac{10^5}{240\cdot 4\cdot 10\cdot 42} = \frac{10^3}{96\cdot 42}$$

$$= \frac{10}{40} = \underline{\frac{1}{4}\ {}^\circ C}$$

Tym więcej robiąc n.p. $\left.\begin{array}{l}\delta = 0{\cdot}02 \\ l = 5\,\text{cm} \\ p_2 = \frac{1}{2}p_1\end{array}\right\}$ będziemy mieli

$$\Delta\theta = \frac{1}{4}\cdot 16\cdot 4\cdot 3 = 48^\circ\ !!$$

W szczególnym przypadku gdy zboczenia od stanu izotermicznego tylko małe:

$$\theta = \theta_0 + \varphi$$

$$p = R\rho(\theta_0+\varphi) \qquad \rho = \frac{p}{R(\theta_0+\varphi)}$$

$$\frac{\partial}{\partial x}\left(\frac{pu}{\theta_0+\varphi}\right) + \frac{\partial}{\partial y}\left(\frac{pv}{\theta_0+\varphi}\right) = 0 \qquad pu\cdot\left(1-\frac{\varphi}{\theta_0}\right)$$

$$\cancel{\frac{\partial}{\partial x}(pu) + \frac{\partial}{\partial y}(pv) + \frac{1}{\theta_0}\left[\frac{\partial(\varphi u)}{\partial x} + \frac{\partial(\varphi v)}{\partial y}\right] = 0}$$

$$\frac{\partial}{\partial x}\left[pu\cdot\left(1-\frac{\varphi}{\theta_0}\right)\right]$$

$$\text{II).}\quad \frac{\partial}{\partial x}(pu) + \frac{\partial}{\partial y}(pv) - \frac{p\left[u\frac{\partial\varphi}{\partial x} + v\frac{\partial\varphi}{\partial y}\right]}{\theta_0-\varphi} = 0$$

$$\cancel{\text{III).}\quad u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + k\,p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = (k-1)\,\Phi + (k-1)\kappa\,\Delta^2\varphi}$$

Błąd w ogóle w równaniach jest zatem tylko rzędu $\dfrac{pu\frac{\partial\varphi}{\partial x}}{\theta_0}$

$$\theta = \theta_0 + \varphi$$

$$p = R\rho(\theta_0+\varphi) = \pi + \psi$$

Przy tem ~~izotermicznie~~ $\frac{\partial\theta}{\partial z} = 0$ dla powierzchni zatem (ściany ~~rurki~~ nie udzielają ciepła [więc to jest faktycznie przybliżone równanie zamiast $\ldots = \Phi + \kappa\Delta^2\theta$ ale nie $= \Phi$ (bo wtedy musiałoby $\frac{\partial u}{\partial z} = 0$ być na powierzchni)!]

Należy zatem też ~~[illegible]~~ spodziewać się że temperatura nie zostanie zmniejszona, po upływie, mimo że ściany nie są adiabatyczne.

Byłoby rozwiązaniem ścisłem gdyby $\kappa = \infty$.

II przybliżenie:

$$\frac{\partial}{\partial x}(p u) + \frac{\partial}{\partial y}(p v) - \underbrace{R\rho\left(u\frac{\partial \varphi}{\partial x} + v\frac{\partial \varphi}{\partial y}\right)}_{= R\rho\left(u\frac{\partial \vartheta}{\partial x} + v\frac{\partial \vartheta}{\partial y}\right)} = 0$$

~~Jeżeli przez~~ czy możliwe $\theta = \text{const}$ || jeżeli $k = 0$

$$\left.\begin{aligned} &\frac{\partial}{\partial x}(p u) + \frac{\partial}{\partial y}(p v) = 0 \quad = u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \\ &p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = -\frac{\mu}{3}\left[u\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + v\frac{\partial}{\partial y}\left(\quad\right)\right] \\ &\qquad - \mu\left[u\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)\right] \end{aligned}\right\} I + II$$

z drugiej strony

$$p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = \Phi \qquad \text{zatem chyba musiałoby być:}$$

$$\frac{1}{3}\left[u\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + v\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right] + u\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) +$$

$$-\frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2 + 2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2$$

Najprzód dowód że ogólnie niemożliwe przybliżenie jednorodne ...

Podobieństwo dynamiczne

$$\frac{\partial p}{\partial x} = \frac{\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$$

$$u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + k\,p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = (k-1)\mu\left[\left(\frac{\partial u}{\partial x}\right)^2 + \cdots\right] + \kappa\left[\frac{\partial^2 \theta}{\partial x^2} + \cdots\right]$$

Mając rozwiązanie $p = f(xyz)$

$\theta = \varphi(xyz)$ w danym naczyniu (o ściankach nieruchomych)

$\left.\begin{matrix} u \\ v \\ w \end{matrix}\right\} = \psi(xyz)$

I). Otrzymam inne rozwiązanie dla naczynia o rozmiarach n razy większych podstawiając wszędzie nx, ny, nz .. na miejsce xyz

oraz $p = \frac{1}{n} f$

II). $\left.\begin{matrix} u \\ v \\ w \\ p \end{matrix}\right\}$ powiększyć n razy k n^2 razy rozmiary niezmienione

---

III). rozmiary naczynia n razy większe

podstawić $nx\ y\ z$

k, μ ... n razy

IV). rozmiary naczynia n razy $\left\{\begin{matrix} x\ y\ z \\ u\ v\ w \end{matrix}\right.$. p niezmien.

k (n^2) razy

Ogólnie: więc podstawienia: $\begin{matrix} nx \\ ny \\ nz \end{matrix}$ zamiast $\begin{matrix} x \\ y \\ z \end{matrix}$

$\begin{matrix} mu \\ mv \\ mw \end{matrix}$ — — — $\begin{matrix} u \\ v \\ w \end{matrix}$

$\alpha\mu$. — μ

$\beta . k$ — k

$. b p$ . . — p

$n\theta$ . . θ

$$\frac{\mu}{R\theta}\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right]+\frac{\partial p}{\partial x}=\frac{\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \quad \mathrm{I}$$

$$\mathrm{II}$$

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}+k\,p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=(k-1)\mu\,\Phi+\kappa\nabla^2\theta \quad \mathrm{III}$$

I). $\frac{b}{r}\frac{m^2}{n} \equiv \frac{b}{n} \equiv \alpha\frac{m}{n^2}$

III). $m\frac{b}{n} \equiv \alpha\frac{m^2}{n^2} \equiv \beta\frac{r}{n^2}$

~~I). $b\left(\frac{m^2}{r}+1\right) \equiv \alpha\frac{m}{n}$~~

~~III). $bm \equiv \alpha\frac{m^2}{n}+\beta\frac{r}{n}$~~

~~I). Niezmieniając $\alpha = 1$ $\beta = 1$~~

~~$b\left(\frac{m^2}{r}+1\right) \equiv \frac{m}{n}$~~

~~$bm \equiv \frac{m^2}{n}+\frac{r}{n}$~~

~~1). niezmieniając $r = 1$ $b(m^2+1) \equiv \frac{m}{n}$~~

~~$b \equiv \frac{m}{n}+\frac{1}{nm}$~~ $\Big\}$ ~~$b \equiv \frac{1}{nm}$~~

~~$(m^2+1)\left(m+\frac{1}{m}\right) \equiv m$~~ ~~$(m^2+1)^2 \equiv m$~~

~~Niemożliwe~~

~~$\frac{m}{n}+\frac{r}{nm}\left(\frac{m^2}{r}+1\right) \equiv \frac{m^2}{n}$~~ Rozwiązanie $r = -m^2$

~~$(m^2+r) \equiv m^2$~~

~~Niemożliwe!~~

Zaniedbujące ~~[illegible]~~ mając kinet:

~~II. $b = \alpha \frac{m}{n}$~~

~~III). $b \frac{m}{n} = \alpha \frac{m^2}{n^2} + \beta \frac{r}{n^2}$~~

~~$\alpha \frac{m^2}{n^2} = \alpha \frac{m^2}{n^2} + \beta \frac{r}{n^2}$~~ ~~Niemożliwe!~~

$$\text{II}.\quad \frac{b}{r} m^2 = b = \alpha \frac{m}{n}$$
$$\text{III}).\quad mb = \alpha \cancel{\frac{m^2}{n}} = \beta \frac{r}{n}$$

3 równania między 6 wielkościami -

3 dowolne

I).

$m^2 = r$

$$\alpha \frac{m^2}{n} = \beta \frac{r}{n}$$

$\alpha = \beta$

$$\alpha m^2 = \beta r$$
$$\alpha \cancel{r} = \beta \cancel{r}$$

zatem pozostają tylko 2 dowolne

$$b = \alpha \frac{m}{n} = b \frac{m^2}{r}$$
$$mb = \alpha \frac{r}{n}$$

$m^2 = r$

$b = \alpha \frac{m}{n}$

już i tak spełnione

pozostają zatem 2 równania:

$\alpha = \beta$

$$b = \alpha \frac{m}{n} = b \frac{m^2}{r} \quad \left\{ \begin{array}{l} b = \alpha \frac{m}{n} \\ r = m^2 \end{array} \right\}$$

$b = \alpha \frac{m}{n}$ $\quad r = m^2$ 5 wielkości 2 kłopoty 3 dowolne

[illegible]: ochłodzenie i temperatura obiektu

1). Niezmieniając $\begin{cases} \kappa\ \mu : & \alpha = 1 \\ \theta : & r = 1 \end{cases}$

$m = 1 \qquad b = \frac{1}{n}$

Uzupełnić co do ilości ciepła przenoszonego

Tj. Rozmiary n razy powiększyć, ochłodzenie $\frac{1}{n}$ zmniejszyć [illegible] niezmieniona
Wszystko prawie stosuje się do Słońca; masa powiększ. niezmien.

2). Niezmieniając $\kappa\mu$: $\alpha = 1$

rozmiarów : $n = 1$

$b = m \qquad r = m^2$

to samo także dla zadania z czasem t zmiennym!

Powiększając prędkości m razy: należy [illegible] n razy, temp. m^2 razy powięk.

Meteor lecący przez atmosferę: jeśli z prędkością m razy większą leci przez atm. n razy gęstszą ogrzeje się n razy [tylko, nie m^2!] wszystkie, a temp. m^2 wyższa.

Uwzględniając zależność $\kappa\mu$ od temp. i kładąc je prop. θ:

I). $\frac{b}{r} m^2 = b = r \frac{m}{n}$ $\qquad r = m^2 \qquad b = \frac{m^3}{n}$

II). $m\, b = \frac{r^2}{n}$ $\qquad \frac{m^4}{n} = \frac{m^4}{n}$ same przez się spełnione

Pozostaje warunek: $r = m^2 \qquad b = \frac{m^3}{n}$

1). Pozostaje także same

2). $\underline{n=1}$ $\qquad r = m^2 \quad b = m^3$

III). $\alpha = 1$ $\quad m = \frac{1}{n}$

$b = \frac{1}{n^2}$

$r = \frac{1}{n^2}$

Więc porównując zmiany w rzędzie i zmniejszając przy tym $\frac{1}{n}$ należy uwzględnić i tempo zmniejszenia $\frac{1}{n^2}$ (porówn. Helmholtz)

IV. $\alpha = 1$ $\quad m = n$

$b = 1 \qquad r = n^2$

[O jeden stopień większa dokładność przy tych analizach niż przy innych]

$$\cancel{r^2\varphi''\varphi + 2r\varphi\varphi' - r^2\varphi'^2 + k(2\varphi + r\varphi')(p_0 + 2\varphi + r\varphi') = 0}$$

$$r^2\varphi''\varphi + 3r\varphi\varphi' + \varphi^2 + r\varphi\varphi' + r^2\varphi'^2 + k\left[(2\varphi + r\varphi')(p_0 + 2\varphi + r\varphi') - \varphi^2 - r\varphi\varphi' - r^2\varphi'^2\right] = 0$$

$$r^2(\varphi''\varphi + \varphi'^2) + \varphi^2 + 4r\varphi\varphi'$$

$$\left.\underbrace{r^2 d(\varphi\varphi') + 2r\varphi\varphi'}_{d(r^2\varphi\varphi')} + d(r\varphi^2)\right\} = \cancel{d[r\varphi(\varphi + r\varphi')]} = \frac{1}{2} d\left[2r\varphi^2 + 2r^2\varphi\varphi'\right] = \frac{1}{2} d^2(r^2\varphi^2)$$

$$\cancel{3r\varphi(\varphi + \varphi)}$$

$$r^3\varphi''\varphi + 4r^2\varphi\varphi' + r\varphi^2 + r^3\varphi'^2$$

$$\underbrace{r^3 d(\varphi\varphi') + 3r^2\varphi\varphi'}_{d(r^3\varphi\varphi')} + \frac{1}{2} d(r^2\varphi^2)$$

$$d(r^3\varphi\varphi') + \frac{1}{2} d(r^2\varphi^2) + k\left[p_0\, d(r^2\varphi) + \underbrace{3\varphi^2 r + 3\varphi\varphi' r^2}_{\frac{3}{2} d(r^2\varphi^2)}\right] = 0$$

$$r^3\varphi\varphi' + \frac{r^2\varphi^2}{2} + k p_0 r^2\varphi + \frac{3}{2} k r^2\varphi^2 = a$$

$$r^3\varphi\varphi' + \frac{r^2\varphi^2}{2}(1 + 3k) + k p_0 r^2\varphi = a$$

Wypływ kołowy dwuwymiarowy:

$u = z\frac{x}{r} = \varphi x \quad \Big\| \quad v = z\frac{y}{r} = \varphi y$

$\frac{\partial u}{\partial x} = \varphi + \varphi'\frac{x^2}{r}$ $\qquad$ $\frac{\partial u}{\partial y} = \varphi'\frac{xy}{r}$

$\frac{\partial^2 u}{\partial x^2} = \varphi'\frac{3x}{r} - \varphi'\frac{x^3}{r^3} + \varphi''\frac{x^3}{r^2}$ $\qquad$ $\frac{\partial^2 u}{\partial y^2} = \varphi'\frac{x}{r} - \varphi'\frac{xy^2}{r^3} + \varphi''\frac{xy^2}{r^2}$

$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} = \varphi'\frac{3x}{r} + \varphi''\cdot x$

$\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) = 2\varphi + \varphi'\cdot r$

$\frac{\partial}{\partial x} = 3\varphi'\cdot\frac{x}{r} + \varphi''\cdot x =$

$\frac{dp}{dr}\cdot\frac{x}{r} = \frac{4\mu}{3}\left(\varphi'' x + 3\varphi'\cdot\frac{x}{r}\right)$

I). $\frac{dp}{dr} = \frac{4\mu}{3}\left(\varphi'' r + 3\varphi'\right)$

$\Phi = -\frac{2}{3}(2\varphi + r\varphi')^2 + 4\varphi^2 + 2\varphi'^2\frac{x^4+y^4}{r^2}$

$+ 4\varphi\varphi' r + 4\varphi'^2\frac{x^2y^2}{r^2}$

$= -\frac{2}{3}(2\varphi + r\varphi')^2 + 4\varphi^2 + 4\varphi\varphi' r + 2r^2\varphi'^2$

III). $\cancel{\varphi x \cdot \frac{x}{r}}\ \varphi r\,\frac{4\mu}{3}(\varphi'' r + 3\varphi') + k\,p\,(2\varphi + r\varphi') =$

$= \frac{4}{3}(k-1)\varphi^2 + \frac{4}{3}\,r\varphi\varphi' + \frac{4}{3}\,r^2\varphi'^2 = \frac{4}{3}(k-1)\left[\varphi^2 + r\varphi\varphi' + r^2\varphi'^2\right]$

I). $\frac{dp}{dr} = \frac{4\mu}{3}\frac{d}{dr}\left[\varphi' r + 2\varphi\right]$

$p = \frac{4\mu}{3}\left[\varphi' r + 2\varphi\right] + p_0\frac{4\mu}{3}$

$$r^3\varphi\varphi' + \frac{r^2\varphi^2}{2}(1+3k) + k p_0 r^2\varphi = a$$

$$r^2\varphi = z \qquad z\left(r\frac{dz}{dr} - z\right) + \frac{z^2}{2}(1+3k) + k p_0 r z = a$$

$$r\,d\varphi + \varphi\,dr = dz$$

$$r\varphi' = \frac{dz}{dr} - \frac{z}{r} \qquad \cancel{z\frac{dz}{dr} + \frac{z^2}{2}\left(\frac{3k-1}{2}\right)}$$

$$z\frac{dz}{dr} + \frac{z^2}{r}\,\frac{3k-1}{2} + k p_0 z = \frac{a}{r}$$

$$\left.\begin{aligned} r^2\varphi &= z \\ \varphi &= \frac{z}{r^2} \\ \varphi' &= \frac{1}{r^2}\frac{dz}{dr} - \frac{2z}{r^3}\end{aligned}\right\} \quad rz\left(\frac{1}{r^2}\frac{dz}{dr} - \frac{2z}{r^3}\right) + \frac{z^2}{r^2}\,\frac{1+3k}{2} + k p_0 z = a$$

$$z\frac{dz}{dr} - \frac{2z^2}{r^2} + \frac{z^2}{r^2}\left(\frac{1+3k}{2}\right) + k p_0 z = \alpha$$

$$z\frac{dz}{dr} + \frac{z^2}{r^2}\,\frac{3(k-1)}{2} + k p_0 z = a$$

$$r\varphi^2 = z$$

$$\varphi\varphi' = \frac{1}{2r}\frac{dz}{dr} - \frac{z}{2r^2}$$

$$\frac{1}{2}\left(r^2\frac{dz}{dr} - rz\right) + rz\left(\frac{1+3k}{2}\right) + k p_0 r\sqrt{rz} = a$$

$$\cancel{\sqrt{rz} = y^2}$$

$$\cancel{z + r\frac{dz}{dr} = 2y\frac{dy}{dr}}$$

$$\cancel{y\frac{dy}{dr} - y^2 + y^2\frac{1+3k}{2} + k p_0 r\, y = a}$$

$$\cancel{y\frac{dy}{dr} + \frac{y^2}{r}\,\frac{3k-1}{2} + k p_0 y = \frac{a}{r}}$$

$$\cancel{\varphi\frac{d\varphi}{dr}} \quad z\frac{dz}{dr} + \frac{z^2}{r}\,\frac{3k+1}{2} + k p_0\frac{z}{r} = \frac{a}{r^3}$$

$$z\frac{dz}{d\varphi}\frac{d\varphi}{dr}+z\varphi\frac{3k-1}{2}+k p_0 z=\frac{a\varphi}{z}$$

$$\cancel{\varphi\frac{dz}{dz}\quad \varphi'=\frac{1}{z}\varphi}$$

$$\frac{d\varphi}{dr}=\frac{\varphi^2}{\varphi\frac{dz}{d\varphi}-z}$$

$$r+\varphi\frac{dr}{d\varphi}=\frac{dz}{d\varphi}$$

$$\varphi^2\frac{dr}{d\varphi}=\varphi\frac{dz}{d\varphi}-z$$

$$z\varphi^2\frac{dz}{d\varphi}+\left(z\varphi\frac{3k-1}{2}+k p_0 z-\frac{a\varphi}{z}\right)\left(\varphi\frac{dz}{d\varphi}-z\right)=0$$

$$r^2\varphi=z\qquad\qquad \frac{dr}{d\varphi}=\frac{\varphi\frac{dz}{d\varphi}-z}{2\varphi\sqrt{\varphi z}}$$

$$\varphi r^2+2r\frac{dr}{d\varphi}\varphi^2=\frac{dz}{d\varphi}\varphi$$

$$z\frac{dz}{d\varphi}2\varphi\sqrt{\varphi z}+\left(\varphi\frac{dz}{d\varphi}-z\right)\left[z\varphi\frac{3(k-1)}{2}+k p_0 z-a\right]=0$$

$$z=u.v$$

$$uv\left(u\frac{dv}{dr}+v\frac{du}{dr}\right)+\frac{u^2v^2}{r^2}\frac{3(k-1)}{2}+k p_0 uv=\frac{a}{r}$$

$$u^2\left[v\frac{dv}{dr}+\frac{v^2}{r^2}\frac{3(k-1)}{2}\right]+u\left[v^2\frac{du}{dr}+k p_0 v\right]=\frac{a}{r}\qquad \frac{du}{dr}=-\frac{k p_0}{v}$$

$$+u^2k^2p_0^2\left[-\frac{1}{\left(\frac{du}{dr}\right)^3}\frac{d^2u}{dr^2}\right]+\frac{u^2\,3(k-1)}{2r^2}\frac{k^2p_0^2}{\left(\frac{du}{dr}\right)^2}=\frac{a}{r}\qquad v=-\frac{k p_0}{\frac{du}{dr}}$$

$$\frac{1}{v}\frac{dv}{dr}+\frac{3(k-1)}{2}\frac{1}{r}=0\qquad \lg v=-\frac{3(k-1)}{2}\lg r$$

$$\cancel{v=Ae^{\frac{3(k-1)}{r}}}$$

$$v=Ar^{-\frac{3k-1}{2}}$$

$$\cancel{u\left(\frac{du}{dr}+k p_0\frac{1}{v}\right)=\frac{a}{v^2}}$$

$$u\frac{du}{dr}+k p_0 u A r^{\frac{3k-1}{2}}=\frac{a}{A^2}r^{\frac{1-3k}{2}-1}=\frac{a}{A^2}r^{-3k}$$

$$\lg\frac{du}{dr}r$$

$\varphi = z\, r \qquad \varphi = z + r\,\frac{dz}{dr}$ $\qquad 60 - \qquad 100$

50

$$z r^4 \left(z + r\,\frac{dz}{dr}\right) + \frac{z^2 r^4}{} \; \frac{3k+1}{2} + k p_0\, z r^3 = 0 \qquad 3k$$

$$r^5 z \frac{dz}{dr} + z^2 r^4\, \frac{3k+1}{2} + z r^3 k p_0 = 0$$

III).

$R\theta$ ...

$$\cancel{R\frac{\partial\theta}{\partial x} + \frac{\partial\theta}{\partial y} = \frac{1}{\rho}\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right) \ldots \left(\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\right)}$$

$$\iiint \Phi\, dv = \mu \iint \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial}{\partial y}\right)^2 + \ldots \qquad + \frac{1}{3}(\mathrm{div})^2 +$$

$$\mu \iint \left\{ u\left[m\frac{\partial v}{\partial x} + n\frac{\partial w}{\partial x} \pm l\frac{\partial u}{\partial x} - l\frac{\partial v}{\partial y} - l\frac{\partial w}{\partial z}\right] + v\left[\quad\right] + w\left[\quad\right]\right\} dS$$

$$\iint u\frac{\partial}{\partial x}v_n + v\frac{\partial}{\partial y}v_n + w\frac{\partial}{\partial z}v_n - \;\; - v_n\,\mathrm{div}\, v$$

$$\int \Phi = \iiint \left(\frac{\partial u}{\partial x}\right)^2 + \ldots \; \frac{1}{3}\mathrm{div}^2 \; + \iint \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)v_n - \iint v_n\,\mathrm{div}\; dS$$

$$\iiint \rho\left(u\frac{\partial\theta}{\partial x} + \ldots\right)dv = \iint \rho\,\theta\,(ul + vm + wn)\, dS$$

$$\iiint p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)dv = \iint p(ul + vm + wn) - \underbrace{\iiint u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}}$$

$$= \iint \frac{\partial}{\partial n}\left(\frac{u^2+v^2+w^2}{2}\right)dS + \frac{\mu}{3}\iint v_n\,\mathrm{div}\, dS - \mu\iiint\left(\frac{\partial u}{\partial x}\right)^2 \ldots \frac{\mu}{3}$$

$M_0 \quad M_1$

$c_0 \quad c_1 = \frac{c_0}{\sqrt{M_1}}$

$$\left(\frac{c}{A}+R\right)\iint \rho\,\theta(u\,l+v\,m+w\,n)\,dS =$$

$$=\mu\iint\left[\frac{\partial}{\partial n}\left(\frac{u^2+v^2+w^2}{2}\right)+\frac{1}{3}v_n\,\mathrm{div}+\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}\right)v_n - v_n\,\mathrm{div}\right]dS+$$

$$=\mu\iint\left[\frac{\partial}{\partial n}\left(\frac{u^2+v^2+w^2}{2}\right)+\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}\right)v_n-\frac{2}{3}v_n\,\mathrm{div}\right]dS+$$

$$+-\iint\frac{u^2+v^2+w^2}{2}\,\rho\,(v_n)\,dS$$

$$R\left(1+\frac{1}{k-1}\right)\iint\ldots = \frac{Rk}{k-1}\iint\rho\,\theta\,v_n\,dS = \iint = \frac{k}{k-1}\iint p\,v_n\,dS$$

Jeżeli się to zastosuje do rurki prądu

$$q_1\rho_1 v_1 = q_2\rho_2 v_2$$

$$\frac{kR}{k-1}\left(\rho_2\theta_2 v_2 q_2-\rho_1\theta_1 v_1 q_1\right)=-\frac{\rho_2 v_2^3 q_2-\rho_1 q_1 v_1^3}{2}+\mu\left\{\left[\frac{\partial}{\partial s}\frac{v^2}{2}\Big|_2 q_2-\frac{\partial}{\partial s}\frac{v^2}{2}\Big|_1 q_1\right]\right.$$

$$\left.+\left[v_2\frac{\partial v_2}{\partial s}q_2-v_1\frac{\partial v_1}{\partial s}q_1\right]-\frac{2}{3}\left[v_2\frac{\partial v_2}{\partial s}q_2-v_1\frac{\partial v_1}{\partial s}q_1\right]\right\}$$

$$\{\quad\}=\frac{4}{3}\left[v_2\frac{\partial v_2}{\partial s}q_2-v_1\frac{\partial v_1}{\partial s}q_1\right]+\iint\frac{\partial}{}$$

$$+\iiint\nabla^2\frac{u^2+v^2+w^2}{2}\,d\omega \quad q\,ds$$

$$\frac{kR}{k-1}q_1\rho_1 v_1(\theta_2-\theta_1)=\rho_1 q_1 v_1\frac{v_2^2-v_1^2}{2}+\frac{4\mu}{3}\left[\frac{1}{\rho_2}\frac{\partial v_2}{\partial s}-\frac{1}{\rho_1}\frac{\partial v_1}{\partial s}\right]\rho_1 q_1 v_1$$

$$\frac{kR}{k-1}(\theta_2-\theta_1)=-\frac{v_2^2-v_1^2}{2}+\frac{4\mu}{3}\left[\frac{1}{\rho_2}\frac{\partial v_2}{\partial s}-\frac{1}{\rho_1}\frac{\partial v_1}{\partial s}\right]+\int\nabla^2\left(\frac{u^2+v^2+w^2}{2}\right)\frac{ds}{\rho\,v}$$

$$\iint \frac{\partial}{\partial n}\frac{(u^2+v^2+w^2)}{2}\, dS = \iint S\left(\nabla\frac{(u^2+v^2+w^2)}{2}\cdot N\right) dS =$$

$$\iiint \operatorname{div} \nabla \frac{u^2+v^2+w^2}{2}\, dv = \iiint \nabla^2 \frac{u^2+v^2+w^2}{2}\, dv = \iiint \nabla^2 (v^2)\, q\, ds$$

$u^2+v^2+w^2 = v^2$ $\qquad dv = q\, dS$

$$= \iint \left[u\frac{\partial u}{\partial n} + v\frac{\partial v}{\partial n} + w\frac{\partial w}{\partial n}\right] dS = \iint S v$$

Dla warstwy przylegającej do ściany ... u v w ... względn...

$\delta.\frac{\partial u}{\partial N}$ $\quad \delta.\frac{\partial v}{\partial N}$ $\quad \delta.\frac{\partial w}{\partial N}$ $\qquad \frac{\partial u}{\partial n} = \frac{\partial^2 u}{\partial N^2} \ldots$

$= \frac{\partial u}{\partial N} + \ldots$

Zatem:

$\iint u\frac{\partial u}{\partial n}\, dS \ldots\ \delta \iint\left[\left(\frac{\partial u}{\partial N}\right)^2 + \left(\frac{\partial v}{\partial N}\right)^2 + (\ldots)^2\right] ds \cdot dt$

$v \iint\left[\left(\frac{\partial u}{\partial N}\right)^2 + \ldots\right] ds \cdot q$

$q_1 v_1 \rho_1 = q_2 v_2 \rho_2$

$\delta\, dt\, \delta\left(\frac{\partial u}{\partial N}\ldots\right) \rho = \ldots$ $\qquad = \frac{\ldots}{v\rho}$

Zatem ...

Jeśli wszędzie $\nabla^2(u^2+v^2+w^2) = 0$ wtedy wzdłuż $q_1 \frac{\partial v^2}{\partial s}\Big|_1 = q_2 \frac{\partial v^2}{\partial s}\Big|_2$ zatem

$$\frac{k\, p_1}{k-1}\ \ldots\ = -\frac{v_2^2 - v_1^2}{2}$$

Powolny ruch cieczy: odwracalny

zmieniwszy znak $p_1 - p_2$ i znak $\frac{u}{\frac{v}{w}}$ otrzyma się znów także samo rozwiązanie

Szybki ruch nie odwracalny! Tworzenie się promieni cieczy.

U gazów: także powolny ruch nie odwracalny [asymetria: współczyn. etc

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$\xi = \eta = \zeta = 0$

Idealne gazy <u>Niewirowy i dotyk adiabat. bez tarcia i bez przewodn.</u>

Adiabat

$$-\int \frac{dp}{\rho} = \frac{u^2+v^2+w^2}{2} \qquad \frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^k \qquad \rho = \rho_0 \left(\frac{p}{p_0}\right)^{\frac{1}{k}}$$

$$\int \frac{dp}{\rho_0 \left(\frac{p}{p_0}\right)^{\frac{1}{k}}} = \frac{p_0^{\frac{1}{k}}}{\rho_0} \int \frac{dp}{p^{\frac{1}{k}}} = \frac{1}{-\frac{1}{k}+1} \frac{p_0^{\frac{1}{k}}}{\rho_0} p^{-\frac{1}{k}+1}$$

$$\frac{u^2+v^2+w^2}{2} = -\left(\frac{p_0}{p}\right)^{\frac{1}{k}} \frac{p}{\rho_0} \cdot \frac{k}{k-1} \Bigg| = -\frac{\rho_0}{\rho} \frac{p}{\rho_0} \frac{k}{k-1} \Bigg| = \frac{k}{k-1} R\Theta \Bigg|_2^1$$

Przyjmijmy, że: $\frac{p_2}{\rho_2} = -\frac{k}{k-1} \frac{p}{\rho_0} \left(\frac{p_0}{p}\right)^{\frac{1}{k}} \Bigg|_1^2 = +k \frac{p_2}{\rho_0} \left(\frac{p_0}{p_2}\right)^{\frac{1}{k}}$

$$= -2 \frac{k}{k-1} \left[\frac{p_2}{\rho_0} \left(\frac{p_0}{p_2}\right)^{\frac{1}{k}} - \frac{p_0}{\rho_0} \right]$$

$$\frac{p_2}{\rho_0} \left(\frac{p_0}{p_2}\right)^{\frac{1}{k}} \left(\frac{2}{k-1} + 1\right) = \frac{2}{k-1} \frac{p_0}{\rho_0}$$

$$\frac{p_2}{p_0} \left(\frac{p_0}{p_2}\right)^{\frac{1}{k}} = \frac{2}{k+1}$$

$$\left(\frac{p_0}{p_2}\right)^{\frac{1-k}{k}} = \frac{2}{1+k}$$

$$\frac{p_2}{p_0} = \left(\frac{2}{1+k}\right)^{\frac{k}{1-k}}$$

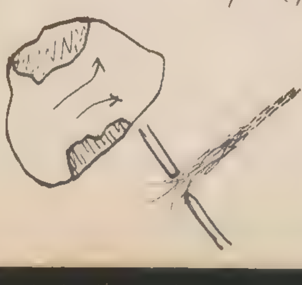
$$\sqrt{k \frac{p_2}{\rho_2}} \cdot \rho_2 = k \sqrt{p_2 \rho_2}$$

$$= \sqrt{k} \sqrt{p_2 \rho_0 \left(\frac{p_2}{p_0}\right)^{\frac{1}{k}}}$$

$$= \sqrt{k} \sqrt{\frac{\rho_0}{p_0^{\frac{1}{k}}}} \; p_2^{\frac{k+1}{2k}}$$

$$= \sqrt{k} \sqrt{\frac{\rho_0}{p_0^{\frac{1}{k}}}} \; p_0^{\frac{k+1}{2k}} \left(\frac{2}{1+k}\right)^{\frac{k}{1-k} \frac{k+1}{2k}}$$

$$= \sqrt{k} \sqrt{\rho_0 p_0} \left(\frac{2}{1+k}\right)^{\frac{k+1}{2(1-k)}}$$



Jeśli rotacyjne : $\nabla^2(u^2+v^2+w^2)=0$

N.p. dwuwymiarowe: $\frac{\partial}{\partial x}\left(u\frac{\partial u}{\partial x}\right)$

$$u\frac{\partial^2 u}{\partial x^2}+v\frac{\partial^2 v}{\partial x^2}+u\frac{\partial^2 u}{\partial y^2}+v\frac{\partial^2 v}{\partial y^2}+\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2=0$$

$$\frac{4}{3}\left(u\frac{\partial^2 u}{\partial x^2}+v\frac{\partial^2 v}{\partial y^2}\right)+u\frac{\partial^2 u}{\partial y^2}+v\frac{\partial^2 v}{\partial x^2}+\frac{1}{3}\left(u\frac{\partial^2 v}{\partial x\partial y}+v\frac{\partial^2 u}{\partial x\partial y}\right)+$$

$$k\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)^2+k\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)p_0=(k-1)\left[\frac{4}{3}\left\{\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right\}+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2-\frac{4}{3}\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}\right]$$

$$\frac{kR}{k-1}(\theta_2-\theta_1)=\frac{\mu}{3}\frac{1}{\rho}\frac{\partial}{\partial s} \qquad \frac{\partial}{\partial x}\frac{dx}{ds}+\frac{\partial}{\partial y}\frac{dy}{ds}-$$

$$=\frac{\mu}{3}\frac{1}{\rho}\frac{u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}(u^2+v^2+w^2)}{\sqrt{u^2+v^2+w^2}}$$

$$\frac{kR}{k-1}(\theta-\theta_0)=\frac{\mu}{3}\frac{1}{\rho}\frac{\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\sqrt{u^2+v^2}}{\sqrt{u^2+v^2}}$$

---

wystarczy jeśli całka nad powierzchnią $=0$

Aby to w każdym punkcie: tylko jeśli dla każdego elementu powierzchni

$\perp$ w całka nad brzegiem $ds\frac{\partial}{\partial n}(u^2+v^2+w^2)=0$

$$ds.\int\nabla(u^2+v^2+w^2)\,\nabla_s v_0 = ds\int s_0.\nabla v_0\,\nabla(u^2+v^2+w^2)=0$$

$$=\int\int(ds.\nabla v_0\,\nabla(u^2+v^2+w^2))$$

$$\int(ds.v)=\int\int(\operatorname{curl} v.n)\,df \qquad \int\int\left[v_0.\operatorname{curl}\nabla v_0\,\nabla(u^2+v^2+w^2)\right]df=0$$

$$\nabla p = \frac{\mu}{3}\nabla \operatorname{div} v + \mu \nabla^2 v \; - \underbrace{\rho (v\nabla) v}_{= -\rho\left(\nabla \frac{v^2}{2} + \nabla v \operatorname{curl} v\right)}$$

~~Fizik. warunki: curl v ⊥ v~~

~~$\int v \operatorname{curl} v = 0$~~

~~$u\xi + v\eta + w\zeta = 0$~~

$$\nabla^2 v = \nabla \operatorname{div} - \operatorname{curl}^2$$

$$\nabla p = \frac{4}{3}\mu \nabla \operatorname{div} - \operatorname{curl}^2 v$$

$$\operatorname{curl}^2 v = \nabla\left(p - \frac{4}{3}\mu \operatorname{div}\right)$$

$\operatorname{curl}^3 v = 0$ $\qquad$ $\nabla^2\left(p - \frac{4}{3}\mu \operatorname{div} v\right) = 0$

$$v = \operatorname{curl}\int \frac{\operatorname{curl} v}{2}\, dv + \nabla \int \frac{\operatorname{div} v}{2}\, dv$$

$$= \cancel{\operatorname{curl}\int \frac{\nabla p - \frac{4}{3}\mu \operatorname{div} v}{2}}\, dv + \cancel{\nabla \int \frac{\operatorname{div} v}{2}}\, dv$$

~~Uwaga: ρ stałym ... $\operatorname{div} v = \frac{1}{\rho}$~~ $\qquad$ $\operatorname{div}(\rho v) = 0$

to znaczy, że $\operatorname{curl} \nabla v_0$ ... $\perp v_0$

$$\begin{vmatrix} i & j & k \\ \frac{u}{v} & \frac{v}{v} & \frac{w}{v} \\ \frac{\partial v}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix}$$

$\frac{u}{V}$

$\int S\,(\ldots)\, df = \ldots\, q,\ v,\ \frac{dv_1}{dt} \ldots$

$\int \nabla^2(\ldots)\, dv$

$\int \nabla(\ldots)$ gas

$$\frac{d}{ds}\left(q \frac{d(\ldots)}{ds}\right) = q\, \nabla^2(\ldots)$$

Przy adiabatycznym obliczeniu efuzji

$$\frac{u^2+v^2+w^2}{2} = \frac{k}{k-1}\left[\frac{p_0}{\rho_0} - \frac{p}{\rho_0}\left(\frac{p_0}{p}\right)^{\frac{1}{k}}\right]$$

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^k$$

$$= \frac{k}{k-1}\left[\frac{p_0}{\rho_0} - \frac{p_0^{\frac{1}{k}}}{\rho_0}\underbrace{p^{1-\frac{1}{k}}}_{\left[p_0\left(\frac{\rho}{\rho_0}\right)^k\right]^{\frac{k-1}{k}}}\right]$$

$$= \frac{k}{k-1}\left[\frac{p_0}{\rho_0} - \frac{p_0}{\rho_0}\left(\frac{\rho}{\rho_0}\right)^{k-1}\right] = \frac{k}{k-1}\frac{p_0}{\rho_0}\left[1-\left(\frac{\rho}{\rho_0}\right)^{k-1}\right]$$

oprócz tego: $\xi = \eta = \zeta = 0$

$$u = \frac{\partial\varphi}{\partial x} \qquad \frac{\partial(\rho u)}{\partial x} + \dots = 0$$

$$u\frac{\partial\rho}{\partial x} + v\frac{\partial\rho}{\partial y} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

$$\begin{cases} \dfrac{\partial\varphi}{\partial x}\dfrac{\partial(\log\rho)}{\partial x} + \dfrac{\partial\varphi}{\partial y}\dfrac{\partial(\log\rho)}{\partial y} + \left(\dfrac{\partial^2\varphi}{\partial x^2} + \dfrac{\partial^2\varphi}{\partial y^2}\right) = 0 \\ \left(\dfrac{\partial\varphi}{\partial x}\right)^2 + \left(\dfrac{\partial\varphi}{\partial y}\right)^2 = f(\rho) = \dfrac{k}{k-1}\dfrac{p_0}{\rho_0}\left[1-\left(\dfrac{\rho}{\rho_0}\right)^{k-1}\right] \end{cases}$$

$$\log\left\{1-\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2\right]\underbrace{\frac{k-1}{k}\frac{\rho_0}{p_0}}_{a}\right\} = (k-1)\left[\log\rho - \log\rho_0\right]$$

$$(k-1)\frac{\partial\log\rho}{\partial x} = \frac{\partial}{\partial x}\log\left\{1-a\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2\right]\right\}$$

$$= \frac{-2a\left(\frac{\partial\varphi}{\partial x}\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial\varphi}{\partial y}\frac{\partial^2\varphi}{\partial x\partial y}\right)}{1-a\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2\right]}$$

$$2a\left[\left(\frac{\partial\varphi}{\partial x}\right)^2\frac{\partial^2\varphi}{\partial x^2} + 2\frac{\partial\varphi}{\partial y}\frac{\partial\varphi}{\partial x}\frac{\partial^2\varphi}{\partial x\partial y} + \left(\frac{\partial\varphi}{\partial y}\right)^2\frac{\partial^2\varphi}{\partial y^2}\right] - (k-1)\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}\right)\left[1-a\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2\right]\right] =$$

Ogólniej dla strugi izotermicznej : $\frac{p_0}{\rho_0} = a$

$$\frac{u^2+v^2+w^2}{2} = -\frac{p_0}{\rho_0} \lg \frac{\rho}{\rho_0} \qquad \frac{\partial \lg \rho}{\partial x} = \frac{a}{2} \frac{\partial}{\partial x}\left[\left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2\right]$$

$$a\left[\left(\frac{\partial \varphi}{\partial x}\right)^2 \frac{\partial^2 \varphi}{\partial x^2} + 2\frac{\partial \varphi}{\partial x}\frac{\partial \varphi}{\partial y}\frac{\partial^2 \varphi}{\partial x \partial y} + \left(\frac{\partial \varphi}{\partial y}\right)^2 \frac{\partial^2 \varphi}{\partial y^2}\right] + \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0$$ i dalej?

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^k \qquad \frac{\theta}{\theta_0} = \left(\frac{\rho}{\rho_0}\right)^{k-1} = \left(\frac{p}{p_0}\right)^{\frac{k-1}{k}}$$

$$\frac{u^2+v^2+w^2}{2} = \frac{k}{k-1}\left[\frac{p_0}{\rho_0} - \frac{p_0}{\rho_0}\frac{\theta}{\theta_0}\right] = \frac{k}{k-1} R\theta_0\left[1-\frac{\theta}{\theta_0}\right] = \frac{k}{k-1} R(\theta_0-\theta)$$

$u^2+v^2+w^2$ będzie miało maximum na osi symetrji

zatem $\frac{\partial}{\partial n}(u^2+v^2+w^2) =$ niesk. małe jak δ^1 (jak

ale przyrostki będzie δ^2 tego samego rzędu co ϱ

→ Jeszcze zadanie jednosymetrowe :

1. izotermicznie : $u\, e^{-\frac{\rho_0}{p_0}\frac{u^2}{2}} = \text{const}$

$\cancel{a\left(\frac{\partial \varphi}{\partial x}\right)^2 \frac{\partial^2 \varphi}{\partial x^2}}$ $\rho u = \text{const}$

~~tylko dla prędkości~~

adiabatycznie: Równanie to ~~[illegible]~~ może być spełnione równocześnie

u - -

przy prędkości niezmiennej . tylko : ~~[illegible]~~

$$\begin{cases} \frac{\partial \rho}{\partial t} + \frac{\partial(\rho \frac{\partial \varphi}{\partial x})}{\partial x} = 0 \\ \frac{\partial \varphi}{\partial t} + \frac{1}{2}\left(\frac{\partial \varphi}{\partial x}\right)^2 = \frac{k}{k-1}\frac{p_0}{\rho_0}\left[1-\left(\frac{\rho}{\rho_0}\right)^{k-1}\right] \end{cases}$$

$$\begin{cases} \frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 & \Big| \; u \\ \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -k\frac{p_0}{\rho_0}\left(\frac{\rho}{\rho_0}\right)^{k-2}\frac{\partial \rho}{\partial x} & \Big| \; \rho \end{cases} \Bigg\} \quad \frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} = -k\frac{p_0}{\rho_0}\left(\frac{\rho}{\rho_0}\right)^{k-2}\rho\frac{\partial \rho}{\partial x}$$

$\alpha = \frac{1}{\kappa}$

$$\Delta^2 \theta = 0 + \alpha \left[u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} + \kappa p \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - (\kappa - 1) \Phi \right]$$

~~$\theta = \theta_0 + \alpha \varphi + \frac{\alpha^2}{2} \psi + \dots$~~

~~$\Delta^2 \theta = \alpha \Delta^2 \varphi + \frac{\alpha^2}{2} \Delta \psi + \dots$~~

~~Dla dostatecznie małych α~~

~~$\Delta^2 \varphi = u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} + \dots$~~

~~Przy tym zarazem:~~

~~$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$~~

$$\theta = \theta_0 + \alpha \theta' + \frac{\alpha^2}{2} \theta'' + \dots$$

$$u = u_0 + \alpha u' + \frac{\alpha^2}{2} u'' + \dots$$

$$p = p_0 + \alpha p' + \frac{\alpha^2}{2} p'' + \dots$$

Wtedy w pierwszym przybliżeniu:

$$\frac{\partial p_0}{\partial x} = \frac{\mu}{3} \frac{\partial}{\partial x} \left(\frac{\partial u_0}{\partial x} + \dots \right) + \dots$$

$$\frac{\partial (\rho_0 u_0)}{\partial x} + \frac{\partial (\rho_0 v_0)}{\partial y} = 0$$

$$\frac{p_0}{\rho_0} = R \theta_0$$

$$\Delta^2 \theta' = \left[u_0 \frac{\partial p_0}{\partial x} + \dots - (\kappa - 1) \Phi_0 \right]$$

Za pomocą tego drugie przybliżenie:

$$\frac{p_0 + \alpha p'}{\rho_0 + \alpha \rho'} = R (\theta_0 + \alpha \theta')$$

$$p_0 + \alpha p' = R (\theta_0 + \alpha \theta') (\rho_0 + \alpha \rho')$$

$$p' = R [\rho_0 \theta' + \rho' \theta_0] = p_0 \left[\frac{\theta'}{\theta_0} + \frac{\rho'}{\rho_0} \right]$$

$$\boxed{\frac{p'}{p_0} = \text{[illegible]} = \frac{\theta'}{\theta_0} + \frac{\rho'}{\rho_0}}$$

$$\rho_0 \left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} \right) + u_0 \frac{\partial \rho'}{\partial x} + v_0 \frac{\partial \rho'}{\partial y} = 0$$

$$\frac{\partial p'}{\partial x} = \frac{\mu}{3} \frac{\partial^2 u'}{\partial x^2} + \dots$$

$$p' = p_0 \frac{\theta'}{\theta_0} + R \theta_0 \cdot \rho'$$

$$\frac{\partial p'}{\partial x} = \frac{\frac{\partial p_0}{\partial x} \cdot \theta' + p_0 \frac{\partial \theta'}{\partial x}}{\theta_0} + R \theta_0 \frac{\partial \rho'}{\partial x}$$

$$R \theta_0 \frac{\partial \rho'}{\partial x} = \frac{\partial p'}{\partial x} - \frac{\partial p_0}{\partial x} \frac{\theta'}{\theta_0} - \frac{p_0}{\theta_0} \frac{\partial \theta'}{\partial x}$$

$$\frac{R \theta_0 \rho_0}{p_0} \left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} \right) + u_0 \frac{\partial p'}{\partial x} + v_0 \frac{\partial p'}{\partial y} - \frac{\theta'}{\theta_0} \left(u_0 \frac{\partial p_0}{\partial x} + v_0 \frac{\partial p_0}{\partial y} \right) - \frac{p_0}{\theta_0} \left(u_0 \frac{\partial \theta'}{\partial x} + v_0 \frac{\partial \theta'}{\partial y} \right) = 0$$

T_{i} : $\quad \dfrac{\partial p'}{\partial x} = \dfrac{\mu}{3} \dfrac{\partial}{\partial x}\left(\dfrac{\partial u'}{\partial x} + \dfrac{\partial v'}{\partial y}\right) + \sim$

$$\frac{\partial p'}{\partial y} = --$$

$$p_0\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right) + u_0 \frac{\partial p'}{\partial x} + v_0 \frac{\partial p'}{\partial y} = \cancel{\frac{u_0}{p_0}} \frac{1}{\theta_0}\left(u_0 \frac{\partial}{\partial x} + v_0 \frac{\partial}{\partial y}\right)(p_0 \theta')$$

Jeśli chcemy utrzymać $\dfrac{\partial p'}{\partial y} = 0$ [takie możliwe tylko jeśli $\dfrac{\partial^2 u'}{\partial x \partial y}$ b. małe w porównaniu do $\dfrac{\partial^2 u'}{\partial y^2}$]

$\therefore v_0 = 0$

$$\begin{cases} \dfrac{\partial p'}{\partial x} = \mu \dfrac{\partial^2 u'}{\partial y^2} \\[2ex] p_0 \dfrac{\partial u'}{\partial x} + u_0 \dfrac{\partial p'}{\partial x} = \dfrac{1}{\theta_0}\, u_0 \dfrac{\partial (p_0 \theta')}{\partial x} \end{cases}$$

$$\frac{\partial^2 p'}{\partial x^2} = \mu \frac{\partial^3 u'}{\partial x \partial y^2}$$

$\Big| : \dfrac{\partial}{\partial y^2}$

$$p_0 \frac{\partial^3 u'}{\partial x \partial y^2} + \frac{\partial^2 u_0}{\partial y^2} \frac{\partial p'}{\partial x} = \frac{1}{\theta_0} \frac{\partial^2}{\partial y^2}\left(u_0 \frac{\partial}{\partial x} \right.$$

$$p_0 \frac{d^2 p'}{dx^2} + \mu \frac{\partial^2 u_0}{\partial y^2} \frac{dp'}{dx} = \frac{\mu}{\theta_0} \frac{\partial^2}{\partial y^2}\left(u_0 \frac{\partial}{\partial x}(p_0 \theta')\right)$$

$$\begin{cases} \frac{p}{R\theta}\, u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial x^2} \\ u \frac{\partial p}{\partial x} + k p \frac{\partial u}{\partial x} = (k-1)\mu\, \Phi + {}^{(k-1)}/_{k}\, \nabla^2\theta \end{cases}$$

Podobieństwo dla różnych gazów ?

gęstość zależy od $R = \frac{1}{\rho_0}$ zmieniając zatem R na ρR etc jak dany;

$$\left.\begin{aligned} \frac{b\, m^2}{\rho r n} &\equiv \frac{b}{n} \equiv \alpha \frac{m}{n^2} \\ \frac{m\, b}{n} &\equiv \alpha \frac{m^2}{n^2} \equiv \beta \frac{r}{n^2} \end{aligned}\right\}$$

$$\left.\begin{aligned} \frac{m^2}{\rho r} &= 1 \\ b &= \alpha \frac{m}{n} = \beta \frac{r}{m n} \end{aligned}\right\}$$ 3 równań na 7 wielkości 4 dowolnych

~~N. p. biorąc dowolnie ρ, α, β, k~~

$$\frac{m^2}{r} \equiv \rho \equiv \frac{m^2}{r} \equiv \beta/\alpha \qquad \underline{\rho \equiv \beta/\alpha}$$

$$b \equiv \alpha \frac{m}{n}$$

(z tego wynika!)

(Podobne mogą być tylko ruchy takich gazów)

przy których $\frac{K}{\mu R}$ ~~tak~~ takie same więc przy których

$\rho_0\, K$ takie same

μ H_2	O_2	N_2	CO_2	CH_4	C_2H_4	N_2O
$\frac{7}{0{\cdot}5} = 14$	$\frac{16.1}{1}$	$\frac{14.1}{1}$	$\frac{22.\ 0{\cdot}71}{0{\cdot}82} = 19$	$\frac{8.\ 1{\cdot}60}{0{\cdot}63} = 21$	$\frac{14.\ 689}{0{\cdot}56} = 22$	$\frac{22.\ 0{\cdot}73}{0{\cdot}82} = 20$

więc wogóle niewielkie różnice możemy się przy każdym ze współczynników zadowolić

Wtedy redukuje się to na:

$$\frac{m^2}{r} = \rho$$ obierając dowolnie 4 wielkości

$$b = \alpha \frac{m}{n}$$ Np. ρ, α, $r=1$,

$$m^2 = \rho$$
$$bn = \alpha m = \alpha\sqrt{\rho}$$
$$\left.\begin{array}{l} m = \sqrt{\rho} \\ bn = \alpha\sqrt{\rho} \end{array}\right.$$

I). robiąc rozmiary niezmienione $n=1$ powstają

$$\left\{\begin{array}{l} m = \sqrt{\rho} = \sqrt{\frac{1}{\rho_0}} \\ b = \alpha\sqrt{\rho} = \frac{\alpha}{\sqrt{\rho_0}} \end{array}\right.$$

Więc prędkości wzrost $\sim \frac{1}{\sqrt{\rho_0}}$

siły ciśnienia $\sim \frac{\alpha}{\sqrt{\rho_0}}$

~~ilości~~ przestrzegany $\sim$ $\rho^{u} = \frac{p^{u}}{R\theta} \| \equiv \frac{bm}{\rho r} = \frac{3}{\rho} = \underline{\underline{\alpha}}$

~~zatem~~ zatem siły

II). ciśnienia niezmienione $b=1$

$$m^2 = \rho$$
$$n = \alpha m = \alpha\sqrt{\rho}$$

$$m = \frac{1}{\sqrt{\rho_0}}$$
$$n = \frac{\alpha}{\sqrt{\rho_0}}$$

$$\rho = \frac{m^2}{r}$$
$$b = \alpha\frac{m}{n}$$

Np. $n=1$
$r=1$

$$m = \sqrt{\rho}$$
$$b = \alpha\sqrt{\rho}$$

~~podobieństwo~~ $\frac{mb}{\rho r} = \alpha$

czas dla podobieństwa: $\tau \sim \frac{1}{m}$

$n=1$
$b=1$

$$m = \frac{1}{\alpha}$$
$$r = \frac{1}{\alpha^2\rho}$$

$$\frac{\partial u}{\partial t} = 0$$

sprowadza się na $\rho u \frac{\partial u}{\partial x}$

$$\frac{m^2}{\rho r} = 1$$
$$m = \sqrt{\rho r}$$
$(r=1)$ $m = \sqrt{\rho}$

~~podobieństwo~~ $\frac{mb}{\rho r}$

czas dla podobieństwa: $\tau \sim$

$h=1$
$n=1$
$m = \sqrt{\alpha}$
$b = \beta\sqrt{\alpha}$

$h=1$
$b=1$
$\alpha = m$
$n = 3\sqrt{\alpha}$

Przy tarciu adiabatycznie:

$$\frac{b\,m^2}{\rho\, r\, n} \equiv \frac{b}{n}$$

Tylko takie będą podobne, które mają równe k

$$\frac{m\,b}{n} \equiv k\,\frac{b}{n} \qquad k=1$$

Izotermicznie z tarciem: $r=1$

$$\frac{b\,m^2}{\rho\, n} \equiv \frac{b}{n} \equiv \alpha\,\frac{m}{n^2}$$

$$\left.\begin{aligned} m^2 &= \rho \\ b n &= \alpha m = \alpha\sqrt{\rho} \end{aligned}\right\}$$

To jest tak samo jak z wzorem [illegible] — $\Phi\, \alpha\, \Delta\theta$ jeśli k = równe!

$$m = \sqrt{\rho} = \frac{1}{\sqrt{\rho_0}}$$

$$b n = \frac{\alpha}{\sqrt{\rho_0}}$$

Zatem jeśli ramiony [illegible]:

ciężar przepływający $\sim \alpha$

$$\frac{\text{ciężar przepływ.}}{\text{różn. ciśnienia}} \sim \frac{\alpha}{\alpha}\sqrt{\rho_0} = \sqrt{\rho_0}$$

gęstość $\frac{1}{R\Theta} = \frac{\alpha}{\sqrt{\rho_0}}\,\rho_0 = \alpha\sqrt{\rho_0}$.

$$\frac{\text{objętość}}{\text{różn. ciśnien}} \sim \frac{\sqrt{\rho_0}}{\alpha\sqrt{\rho_0}} = \frac{1}{\alpha}$$

To będzie się stosować do dowolnie szerokich rurek jeśli $k = \kappa$ i jeśli powiemy [illegible] przy odpowiednich ciśnieniach.

W ogólnej metodzie transpozycyjnej:
Objętości jest mierzona pod względem $\frac{1}{2}(p_1+p_2)$ ~ $\frac{x}{\mu}$
stan ciśnienia

Stawiając $u = u_0 + \mu u'$ etc.

$$\left.\begin{aligned} &u_0 \frac{\partial u_0}{\partial x} + \cdots = -\frac{\partial p_0}{\partial x} \\ &\frac{\partial(\rho_0 u_0)}{\partial x} + \cdots = 0 \\ &u_0 \frac{\partial p_0}{\partial x} + \cdots + k p_0\left(\frac{\partial u_0}{\partial x} + \cdots\right) = \kappa \Delta^2 \theta_0 \end{aligned}\right\} \qquad p_0 = R\theta_0\rho_0$$

$$\left\{\begin{aligned} &u_0 \frac{\partial u'}{\partial x} + \cdots + u' \frac{\partial u_0}{\partial x} + \cdots = -\frac{\partial p'}{\partial x} + \frac{\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y}\right) + \cdots \\ &\rho_0 \frac{\partial u'}{\partial x} + u_0 \frac{\partial \rho'}{\partial x} + \cdots = 0 \\ &u_0 \frac{\partial p'}{\partial x} + \cdots + u' \frac{\partial p_0}{\partial x} + \cdots + k p_0 \left(\frac{\partial u'}{\partial x} + \cdots\right) + k p' \left(\frac{\partial u_0}{\partial x} + \cdots\right) = (k-1)\Phi_0 + \kappa(k-1)\Delta^2\theta' \end{aligned}\right.$$

$$\cancel{p = R\theta\rho} \qquad p' = R\left[\theta_0 \rho' + \theta' \rho_0\right]$$

Bez tarcia i przewodn.

$$\cancel{\frac{\partial p}{\partial x}}$$

$$u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} + w \frac{\partial p}{\partial z} + k p \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$

$$u \frac{\partial \lg p}{\partial x} + v \frac{\partial \lg p}{\partial y} + w \frac{\partial \lg p}{\partial z} + k \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$

$$u \frac{\partial \lg \rho}{\partial x} + v \frac{\partial \lg \rho}{\partial y} + \qquad + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \cdots\right) = 0$$

$$-\rho \left[u \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial x}\right) + v \left(u \frac{\partial u}{\partial y} + v \frac{\partial v}{\partial y} + \cdot\right) + w \left(\quad \right) \right] + k p \left(\frac{\partial u}{\partial x} + \quad \right) = 0$$

$$\left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}\right) \frac{u^2+v^2+w^2}{2} = \frac{k p}{\rho}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

$$u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z} + k\,p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=0 \qquad \text{III)}.$$

$$\text{IV)}.$$

Jeśli się założy $\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^k$:

$$\therefore \quad k\,u\,\frac{\partial \rho}{\partial x}\cdot\rho^{k-1} + \dots \quad + k\,\rho^k\,\frac{\partial u}{\partial x} \quad = 0$$

$$\text{II)}. \quad \frac{\partial(\rho u)}{\partial x} + \frac{\partial \rho v}{\partial y} + \dots = 0$$

stąd się widzi, że zamiast III można także wziąć IV

III otrzyma się wstawiając IV do II

[illegible] Ale z tego nie wynika że z III i II zawsze wypływa IV?

$$\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\log p + k\left(\frac{\partial u}{\partial x} + \quad\right) = 0$$

$$\left(u\frac{\partial}{\partial x} + \dots \quad\right)\log \rho + \left(\frac{\partial u}{\partial x} + \dots\right) = 0 \quad \Big| \cdot -k$$

$$\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\log\left(\frac{p}{\rho^k}\right) = 0$$

To znaczy że każda cząsteczka gazu nie zmienia podczas swego biegu $\log\left(\frac{p}{\rho^k}\right)$ zatem proces adiabat.

Wynika stąd dla ruchu wirowego z równania $\int\frac{dp}{\rho} = \frac{k}{k-1} \cdot \quad = \frac{u^2+v^2+w^2}{2}$

to samo co dla ruchu ogólnego według $\iiint$ -- :

$$\frac{k\,R\,(\theta_2-\theta_1)}{k-1} + \left(\frac{u^2+v^2+w^2}{2}\right)_2 - \left(\frac{u^2+v^2+w^2}{2}\right)_1 = 0$$

~~N.p. przyjmując dla powietrza $u^2v^2w^2$~~

Zatem $\frac{k}{k-1}R\theta + \frac{u^2+v^2+w^2}{2} = \text{const} = \frac{k}{k-1}R\theta_0$, jeśli w początku w spoczynku

maksymalna wartość dla prędkości gdy $\theta=0$ $\quad \frac{u^2+v^2+w^2}{2} = \frac{k}{k-1}R\theta_0 = \frac{k}{k-1}\frac{p_0}{\rho_0}$

$\sqrt{u^2+v^2+w^2} = \sqrt{\frac{2}{k-1}}\,\sqrt{k\frac{p_0}{\rho_0}}$. lub też wygodniejsze $a^2 = k\frac{p}{\rho} = kR\theta$

Więc maksymalna możliwa prędkość $= \sqrt{\frac{2}{k-1}}$ razy prędkość głosu (wyjścia)

$\sqrt{\frac{2}{0{\cdot}4}} = \sqrt{5} = 2{\cdot}4$ wtedy musiałoby już nastąpić oziębienie do ($0°$ abs.)

Tymczasem wedle St Venant & Wantzel przy efuzji

$$\left.\begin{aligned} &\sqrt{} = a \\ &\frac{k}{k-1}\cancel{R}\theta + \frac{k\cancel{R}\theta}{2} = \frac{k}{k-1}\cancel{R}\theta_0 \\ &(2+k-1)\theta = 2\theta_0 \end{aligned}\right\} \qquad \begin{aligned} &\theta = \frac{2\theta_0}{k+1} \\ &\theta_0 - \theta = \theta_0\frac{k-1}{k+1} \end{aligned}$$

Np. $\theta_0 = 300$ $\qquad \theta_0 - \theta = \frac{0{\cdot}4}{2{\cdot}4}\cdot 300 = \underline{\underline{50°}}$ $\qquad \begin{aligned} &+27° \\ &-23° \end{aligned}$

nie zna rozproszenia energii

Odwracalność ruchów względnych

Idealna ciecz: $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \dots = -\frac{\partial p}{\partial x}$

Ten sam kształt linij prądu i rozkład prędkości

zmieniając znak u v w: nie zmienia się znak dp

N.p. kula w cieczy poruszającej się

$\boxed{\text{więc wszystkie zmiany prędkości pozornych}}$

u	$-u$
v	$-v$
w	$-w$
p	$a-p$

Powolne tarcie cieczy: $\frac{\partial p}{\partial x} = \mu\left(\frac{\partial^2 u}{\partial x^2} + \dots\right)$

zmieniając znak u v w: zmienia się znak dp $\boxed{\text{więc wszystkie zmiany odwrócone}}$

N.p. wypływ przez rurki [illegible]

Szybkie tarcie cieczy: $u\frac{\partial u}{\partial x} + \dots = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \dots\right)$

Nieodwracalny: $-u, -v, -w$ nie będzie rozwiązaniem jeśli $+u, +v, +w$ jest rozwiązaniem

---

Idealny gaz: odwracalny tak samo jak ciecz idealna [bez rozproszenia energii; chyba drobne drgania]

nie zmienia się znak dp

Powolne tarcie gazu

Nieodwracalne, bo oprócz dp wchodzi także w grę ~~[illegible]~~ bezwzględna wartość p:

$\frac{\partial p}{\partial x} = \mu\frac{\partial^2 u}{\partial y^2}$

$\frac{\partial(\rho u)}{\partial x} = 0$

$\downarrow$

$\rho\frac{\partial u}{\partial x} + u\frac{\partial \rho}{\partial x}$

jeśli są spełnione przez p, u to będą także spełnione przez $-u, -p$ $\boxed{\text{ale ujemne } p \text{ niemożliwe: dodatkowy warunek!}}$

Szybkie tarcie wszystkie stałe nieodwracalne

Entropia gazu $S = c_v \lg \theta + R \lg v + \text{const}$

$$\cancel{R \lg \frac{1}{\varrho}} = c_v \left(\lg \theta + (k-1) \lg v \right)$$

$$= c_v \lg \frac{\theta}{\varrho^{k-1}} + \text{const} = c_v \lg \frac{p}{\varrho^k} + \text{const!}$$

$$\cancel{\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial x^2}} \quad \ldots$$

$$\cancel{\frac{\partial \varrho u}{\partial x} + \qquad = 0}$$

$p = p_1 + p_2$

$u = u_1 + u_2$

$$\cancel{(\varrho_1 + \varrho_2)\left(\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial x}\right) + (u_1 + u_2)\left(\frac{\partial \varrho_1}{\partial x} + \frac{\partial \varrho_2}{\partial x}\right) + \ldots = 0}$$

Dodawanie rozwiązań niemożliwe u gazów oprócz dla bardzo małych / $\frac{\partial \varrho}{\partial x} \neq 0$

$$\varrho_1 \frac{\partial u_2}{\partial x} + \varrho_2 \frac{\partial u_1}{\partial x} + u_1 \frac{\partial \varrho_2}{\partial x} + u_2 \frac{\partial \varrho_1}{\partial x} + \ldots = 0$$

$$\frac{\partial p}{\partial x} = \frac{\mu}{3} \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \ldots\right) + \mu \frac{\partial^2 u}{\partial x^2} + \ldots$$

$p = p_1 + p_2 \qquad u = u_1 + u_2$ —

$$\frac{\partial p_2}{\partial x} = \frac{\mu}{3} \frac{\partial}{\partial x}\left(\frac{\partial u_2}{\partial x} + \ldots\right) + \mu \nabla^2 u_2$$

$$\cancel{\varrho \frac{\partial u}{\partial x} + u \frac{\partial \varrho}{\partial x}} \ldots = 0$$

$$\varrho\left(\frac{\partial u_1}{\partial x} + \ldots\right) = 0$$

$$\varrho\left(\frac{\partial u_2}{\partial x} + \ldots\right) + u \frac{\partial \varrho}{\partial x} = 0$$

$$(u_1 + u_2)\frac{\partial(\varrho_1 + \varrho_2)}{\partial x} \ldots = 0$$

Przybliżone obliczenie dla kul w gazie

Równania: ~~izotermiczne~~: adiabatyczne

$$\begin{cases} \frac{\partial p}{\partial x} = \frac{\mu}{3}\frac{\partial}{\partial x}\operatorname{div} + \mu\Delta^2 u \\ \end{cases} \quad \Bigg\| \quad k p \operatorname{div} + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z} = 0$$

Supponujemy p jako bardzo duże, podczas gdy $\frac{\partial p}{\partial x}$ małe, otrzyma się $\operatorname{div} = 0$

więc ruchy takie jak w cieczy $\quad p = P + p \quad P = \text{const}$

$$\operatorname{div} = \cancel{\ldots} = -\frac{1}{P+p}\left(u\frac{\partial}{\partial x} - \right)$$

$$= -\frac{1}{P}\left(u\frac{\partial p}{\partial x} - \right) + \frac{p}{P^2}\left(u\frac{\partial p}{\partial x} - -\right)$$

$$p = p_0 + \frac{1}{P}p' + \frac{1}{2}\left(\frac{1}{P}\right)^2 p'' + \cdots$$

$$\begin{cases} \frac{\partial p_0}{\partial x} + \frac{1}{P}\frac{\partial p'}{\partial x} + \cdots = \frac{\mu}{3}\frac{\partial}{\partial x}\left(\overset{=0}{\cancel{\operatorname{div}}} + \frac{1}{P}\operatorname{div}' + \right) + \mu\left[\Delta^2 u_0 + \frac{1}{P}\Delta^2 u' + \cdot\right] \\ k\left(P + p_0 + \frac{1}{P}p' + \right)\left(\overset{=0}{\cancel{\operatorname{div}}} + \frac{1}{P}\operatorname{div}' + \right) + \cancel{\ldots}\left(u_0 + \frac{1}{P}u'\right)\left(\frac{\partial p_0}{\partial x} + \frac{1}{P}\frac{\partial p'}{\partial x}\right) + \quad = 0 \end{cases}$$

~~Porównujemy rozwijając~~ wg potęg $\frac{1}{P}$:

$$\frac{\partial \rho u}{\partial x} + \cdot \quad = 0 \qquad \cancel{\rho = \rho_0 + \frac{1}{P}\rho'}$$

$$\begin{cases} \cancel{\frac{\partial p_0}{\partial x} = \mu\Delta u_0} \\ \cancel{u_0\frac{\partial p_0}{\partial x} + \cdots = 0} \end{cases}$$

$$\cancel{\frac{\partial(\rho_0 u_0)}{\partial x} - \cdots = 0} \quad \Big\| \quad \cancel{\frac{\partial}{\partial x}(\rho u + \rho u') + \cdots = 0}$$

$$\frac{p}{\rho} = R\theta \qquad \frac{p_0 + \frac{1}{P}p'}{\rho_0 + \frac{1}{P}\rho'} = \kappa\,\theta_0 + \frac{1}{P}\theta'$$

$$p' = \kappa\theta_0\rho_0 + \ldots$$

$$\begin{cases} \cancel{\frac{\partial p'}{\partial x} = \frac{\mu}{3}\frac{\partial \operatorname{div}'}{\partial x} + \mu\Delta^2 u'} \\ \cancel{p_0 \operatorname{div}' + u'\frac{\partial p_0}{\partial x}} + u_0\frac{\partial p'}{\partial x} + \cdots = 0 \end{cases}$$

$\cancel{k}\ (\mathrm{div}' + \dots)\left(1 + \frac{p_0}{p} + \frac{1}{p^2}p' + \dots\right) + \left(u_0 + \frac{1}{p}u'\right)\left(\frac{\partial p_0}{\partial x} + \frac{1}{p}\frac{\partial p'}{\partial x}\right) + \dots = 0$

$k\,\mathrm{div}' + u_0\frac{\partial p_0}{\partial x} + v_0\frac{\partial p_0}{\partial y} + w_0\frac{\partial p_0}{\partial z} = 0$

$$\left.\begin{aligned}\frac{\partial p'}{\partial x} &= \frac{\mu}{3}\frac{\partial\,\mathrm{div}'}{\partial x} + \mu\Delta^2 u'\\ \frac{\partial p'}{\partial y} &= \frac{\mu}{3}\frac{\partial\,\mathrm{div}'}{\partial y} + \mu\Delta^2 v'\\ \frac{\partial p'}{\partial z} &= \frac{\mu}{3}\frac{\partial}{\partial z} + \mu\Delta^2 w'\end{aligned}\right\}\quad \begin{aligned}\Delta^2 p' &= \frac{4\mu}{3}\Delta^2\mathrm{div}' \\ p' &= \frac{4\mu}{3}\mathrm{div}' + \varphi\end{aligned}\qquad \Delta^2\varphi = 0$$

$$\mu\frac{\partial\,\mathrm{div}'}{\partial x} + \frac{\partial\varphi}{\partial x} = \mu\Delta^2 u' = \frac{\partial}{\partial x}(\mu\,\mathrm{div}' + \varphi)$$

$$\mu\Delta^2 v' = \frac{\partial}{\partial y}(\quad)$$

$$\mu\Delta^2 w' = \frac{\partial}{\partial z}(\quad)$$

~~z~~ obliczeni $u_0\frac{\partial p_0}{\partial x} + \dots = k\,\mathrm{div}'$

$p = p_0 - \frac{3}{2}\mu c a \frac{x}{r^3}$

$\frac{\partial p}{\partial x} = -\frac{3}{2}\mu c a\left(\frac{1}{r^3} - \frac{3x^2}{r^5}\right)$	$u = -\frac{3}{4}\frac{ca}{\cancel{r}}\left(1-\frac{a^2}{r^2}\right)\frac{x^2}{r^3} + c\left(1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right)$
$\frac{\partial p}{\partial y} = \frac{3}{2}\mu c a\frac{3xy}{r^5}$	$v = -\frac{3}{4}ca\left(1-\frac{a^2}{r^2}\right)\frac{xy}{r^3}$
$\frac{\partial p}{\partial z} = \frac{3}{2}\mu c a\frac{3xz}{r^5}$	$w = -\frac{3}{4}ca\left(1-\frac{a^2}{r^2}\right)\frac{xz}{r^3}$

$u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z} = \cancel{-\frac{27}{8}\mu c^2 a^2 \frac{a^2 x^2}{r^{10}}(x^2+y^2+z^2) + \frac{9}{8}\mu c^2 a^2}$

$$= -\frac{27}{8}\mu c^2a^2\left(1-\frac{a^2}{r^2}\right)\frac{x^2}{r^6} + \frac{9}{8}\mu c^2a^2\left(1-\frac{a^2}{r^2}\right)\frac{x^2}{r^6} - \frac{3}{2}\mu c^2 a\frac{1}{r^3}\left(1-\frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right)$$
$$+ \frac{9}{2}\mu c^2 a\frac{x^2}{r^5}\left(1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right)$$

$-k\,div' = -\frac{3}{2}\mu a c^2 \frac{1}{r^3}\left(1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right)+\frac{9}{2}\mu c^2 a \frac{x^2}{r^5}\left(1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right)$

$$\underbrace{-\frac{18}{8}\mu c^2 a \frac{x^2}{r^5}\left(\frac{a}{r}-\frac{a^3}{r^3}\right)}$$

$$+\frac{9}{2}\mu c^2 a \frac{x^2}{r^5}\left(1-\frac{5}{4}\frac{a}{r}+\frac{1}{4}\frac{a^3}{r^3}\right)$$

symetrycznie dla $\pm x$

Dla $r=a$, $r=\infty$

$div'=0$

Dla $x=0$: $r>a$ $\quad div'<0$

Dla $x=r$: (oś X) $\quad div' = \mu\frac{c^2 a}{r^3}\left[3-\frac{9}{2}\frac{a}{r}+\frac{3}{2}\frac{a^3}{r^3}\right] = 3\mu\frac{c^2 a}{r^3}\left[1-\frac{3}{2}\frac{a}{r}+\frac{1}{2}\frac{a^3}{r^3}\right]$

$\frac{a}{r}=1-\delta \qquad 1-\frac{3}{2}(1-\delta)+\frac{1}{2}(1-\delta)^3 = \cancel{\frac{3}{2}\delta}-\cancel{\frac{3}{2}\delta}+\frac{3}{2}\delta^2-\frac{\delta^3}{2}$

>0

$div'>0$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial^2 w}{\partial x\partial z} = \frac{\partial\, div}{\partial x} = \Delta^2 u + \underbrace{\frac{\partial\zeta}{\partial y}-\frac{\partial\eta}{\partial z}}_{=-\frac{\partial\varphi}{\partial x}}$$

$''\qquad ''$

$$\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+\frac{\partial^2 u}{\partial y^2}\ \Big\|\ \frac{\partial}{\partial z}\left(\frac{\partial w}{\partial x}-\frac{\partial u}{\partial z}\right)+\frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial\, div'}{\partial y} = \Delta^2 v + \underbrace{\cancel{\frac{\partial\xi}{\partial z}}-\frac{\partial\zeta}{\partial x}}_{-\frac{\partial\varphi}{\partial y}\ ?}$$

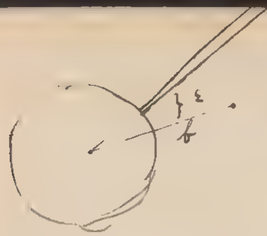
$\xi=0 \qquad \frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}=0$

$$\frac{\partial\, div'}{\partial z} = \Delta^2 w + \underbrace{\frac{\partial\eta}{\partial x}-\cancel{\frac{\partial\xi}{\partial y}}}_{-\frac{\partial\varphi}{\partial z}}$$

$\frac{\partial\zeta}{\partial x}=\frac{\partial\varphi}{\partial y} \qquad \frac{\partial\eta}{\partial x}=-\frac{\partial\varphi}{\partial z}$

$\frac{\partial^2\zeta}{\partial x\partial z}+\frac{\partial^2\eta}{\partial x\partial y}=0$ - . . . .

$\frac{\partial\xi}{\partial z}+\frac{\partial\eta}{\partial y}=f(y.z)$



$$\frac{3\varepsilon}{b} \cdot d\omega \int_a^\infty \frac{dr\, \frac{1}{r^2}\left(1-\frac{5}{4}\frac{a}{r}+\frac{1}{4}\frac{a^3}{r^3}\right)}{\sqrt{b^2+r^2-2br\cos\varepsilon}} \cdot \cos^2\theta$$

$$\frac{\partial}{\partial x}\left(\frac{x^m}{r^n}\right) = m\frac{x^{m-1}}{r^n} - n\frac{x^{m+1}}{r^{n+2}}$$

$$\frac{\partial^2}{\partial x^2}(\quad) = m(m-1)\frac{x^{m-2}}{r^n} - n(2m+1)\frac{x^m}{r^{n+2}} + n(n+2)\frac{x^{m+2}}{r^{n+4}}$$

$$\frac{\partial}{\partial y}\left(\frac{x^m}{r^n}\right) = -n\frac{x^m y}{r^{n+2}}$$

$$\frac{\partial^2}{\partial y^2}(\quad) = -n\frac{x^m}{r^{n+2}} + n(n+2)\frac{x^m y^2}{r^{n+4}}$$

$$\Delta^2\left(\frac{x^m}{r^n}\right) = m(m-1)\frac{x^{m-2}}{r^n} - \frac{x^m}{r^{n+2}}\left[n(2m+1)+2n\right] + \frac{x^m}{r^{n+2}}\, n(n+2)$$

$$= m(m-1)\frac{x^{m-2}}{r^n} + \frac{x^m}{r^{n+2}}\left[\underbrace{n^2+2n-2mn-3n}_{n^2-2mn-n}\right]$$

$m=4 \quad n=\begin{cases}10\\8\end{cases}$

100
−80
−10
10

64
−8
−64

$m=2$

$$\Delta^2\left(\frac{x^2}{r^n}\right) = \frac{2}{r^n} + (n^2-5n)\frac{x^2}{r^{n+2}}$$

$n=3$: $\Delta^2\left(\frac{x^2}{r^3}\right) = \frac{2}{r^3} - 6\frac{x^2}{r^5} = 2\left(\frac{1}{r^3} - \frac{3x^2}{r^5}\right)$

$n=4$ $\Delta^2\left(\frac{x^2}{r^4}\right) = \frac{2}{r^4} - 4\frac{x^2}{r^6} = 2\left(\frac{1}{r^4} - \frac{2x^2}{r^6}\right)$

$n=5$ $\Delta^2\left(\frac{x^2}{r^5}\right) = \frac{2}{r^5}$

$n=6$ $\Delta^2\left(\frac{x^2}{r^6}\right) = \frac{2}{r^6} + 6\frac{x^2}{r^8} = 2\left(\frac{1}{r^6} + 3\frac{x^2}{r^8}\right)$

8

$\frac{x^2}{r^5} = \frac{1}{3}\frac{1}{r^3} - \frac{1}{6}\Delta^2\left(\frac{x^2}{r^3}\right) \Big\| = \frac{2}{r^8} + 24\frac{x^2}{r^{10}}$

$n=10: \Big\| = \frac{2}{r^{10}} + 50\frac{x^2}{r^{12}}$

$$\frac{x^2}{r^5} = \frac{1}{3}\ \frac{1}{r^3} - \frac{1}{6}\ \Delta^2\left(\frac{x^2}{r^3}\right)$$

$$\frac{x^2}{r^6} = \frac{1}{2}\ \frac{1}{r^4} - \frac{1}{4}\ \Delta^2\left(\frac{x^2}{r^4}\right) \qquad \Big|\ -\frac{5}{4}a$$

$$\frac{x^2}{r^8} = -\frac{1}{3}\ \frac{1}{r^6} + \frac{1}{6}\ \Delta^2\left(\frac{x^2}{r^6}\right) \qquad \Big|\ \frac{1}{4}a^3$$

$$\frac{x^2}{r^5} - \frac{5}{4}a\,\frac{x^2}{r^6} + \frac{1}{4}a^3\frac{x^2}{r^8} = \frac{1}{3}\ \frac{1}{r^3} - \frac{5}{8}\ \frac{a}{r^4} - \frac{1}{12}\ \frac{a^3}{r^6}$$

$$- \Delta^2\left\{\frac{1}{6}\ \frac{x^2}{r^3} - \frac{5}{16}\ \frac{ax^2}{r^4} - \frac{1}{24}\ \frac{a^3x^2}{r^6}\right\}$$

$$\frac{Q}{2}\mu c^2 a\left\{\left(\frac{x^2}{r^5}\ \cdots\ -\frac{1}{4}a^3\frac{x^2}{r^8}\right) - \frac{1}{3}\left(\frac{1}{r^3} - \frac{3}{4}\ \frac{a}{r^4} - \frac{1}{4}\ \frac{a^3}{r^6}\right)\right\} = \frac{Q}{2}\mu c^2 a$$

$$\left(-\frac{5}{8} + \frac{1}{4}\right)\frac{a}{r^4} - \Delta^2\left\{\frac{1}{6}\ \frac{x^2}{r^3}\ \cdots\ \right\}$$

$$= -\frac{Q}{2}\mu c^2 a\left[\frac{3}{8}\ \frac{a}{r^4} + \frac{1}{6}\ \Delta^2\left(\frac{x^2}{r^3} - \frac{15}{8}\ \frac{ax^2}{r^4} - \frac{1}{4}\ \frac{a^3x^2}{r^6}\right)\right]$$

$$\Delta^2\left(\frac{1}{r^2}\right) = -\frac{\partial}{\partial x}\left(\frac{2x}{r^4}\right) + \cdots = 2\Sigma\left(-\frac{1}{r^4} + \frac{4x^2}{r^6}\right) = \frac{2}{r^4}$$

$$\cancel{= -\mu c^2 a\ \frac{27}{16}}\ \Delta^2\left\{-\mu c^2 a\left[\frac{27}{32}\ \frac{a}{r^2} + \frac{3}{4}\left(\frac{x^2}{r^3} - \frac{15}{8}\ \frac{ax^2}{r^4} - \frac{1}{4}\ \frac{a^3x^2}{r^6}\right)\right]\right\}$$

Zatem $\operatorname{div}' = \Delta^2\psi$

$$\left.\begin{aligned}
\mu\Delta^2 u' &= \mu\Delta^2\frac{\partial\psi}{\partial x} + \frac{1}{\mu}\frac{\partial\varphi}{\partial x}\\
\mu\Delta^2 v' &= \mu\Delta^2\frac{\partial\psi}{\partial y} + \frac{1}{\mu}\frac{\partial\varphi}{\partial y}\\
\mu\Delta^2 w' &= \mu\Delta^2\frac{\partial\psi}{\partial z} + \frac{1}{\mu}\frac{\partial\varphi}{\partial z}\\
\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} &= \Delta^2\psi
\end{aligned}\right\}$$

Rozwiązanie typu: $u' = \frac{\partial\psi}{\partial x} + U$

$v' = \frac{\partial\psi}{\partial y} + V$

$w' = \frac{\partial\psi}{\partial z} + W$

$$\Delta^2 U = \frac{1}{\mu}\frac{\partial\varphi}{\partial x}$$
$$\Delta^2 V =$$
$$\Delta^2 W =$$
$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0$$

$\Delta^2\varphi = 0$ } zupełnie to samo jak dla nov. wo

$$-k\frac{\partial\psi}{\partial x} = -\mu c^2 a\frac{1}{2}\left[-\frac{27}{8}\frac{a}{r^3}\frac{x}{2} + 3\left(\frac{x}{r^3} - \frac{15ax}{8r^4} - \frac{a^3x}{4r^6}\right) - \frac{3}{2}\left(\frac{3x^3}{r^5} - \frac{15}{2}\frac{ax^3}{r^6} - \frac{3}{2}\frac{a^3x^3}{r^8}\right)\right]$$ 177

$$= -\mu c^2 a\left[\frac{3}{2}\frac{x}{r^3}\left[1 - 3\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right] - \frac{9}{4}\frac{x^3}{r^5}\left[1 - \frac{5}{2}\frac{a}{r} - \frac{1}{2}\frac{a^3}{r^3}\right]\right]$$

$$-k\frac{\partial\psi}{\partial y} = -\mu c^2 a\left[-\frac{27}{16}\frac{ay}{r^4} - \frac{9}{4}\frac{x^2y}{r^5}\left[1 - \frac{5}{2}\frac{a}{r} - \frac{1}{2}\frac{a^3}{r^3}\right]\right]$$

$$k\frac{\partial\psi}{\partial z} = -\mu c^2 a\left[\;—\;-\;\;—\;\;—\;-\;\right]$$

Dla $r = a$:

$$-k\left.\frac{\partial\psi}{\partial x}\right|_{r=a} = -\mu\frac{c^2 a}{2}\left[\frac{3x}{a^3}\left(-\frac{9}{4}\right) - \frac{9}{2}\frac{x^3}{a^5}(-2)\right] = \frac{\mu c^2}{2a}\left[\frac{27}{4}\frac{x}{a} - 9\frac{x^3}{a^3}\right]$$

$$= 9\frac{\mu c^2}{2a}\left[\frac{3}{4}\frac{x}{a} - \frac{x^3}{a^3}\right]$$

$$-k\left.\frac{\partial\psi}{\partial y}\right|_{r=a} = +\mu c^2 a\left[\frac{27}{8}\frac{y}{a^3} + \frac{9}{2}\frac{x^2y}{a^5}(-2)\right] = 9\frac{\mu c^2}{2a}\left[\frac{3}{8}\frac{y}{a} - \frac{x^2y}{a^3}\right]$$

$$-k\left.\frac{\partial\psi}{\partial z}\right|_{r=a} = \cancel{\mu c^2 a}\left[\;—\;—\;—\;—\;\right] = \frac{9\mu c^2}{2a}\left[\frac{3}{8}\frac{z}{a} - \frac{x^2z}{a^3}\right]$$

Próba utworzenia $\Delta^2\psi$:

$$3\frac{1}{r^3}\left[1 - \frac{3a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right] - 9\frac{x^2}{r^5}\left[1 - \frac{3a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right] + \frac{3x^2}{r^3}\left[\frac{3a}{r^3} + \frac{3}{4}\frac{a^3}{r^5}\right] -$$

$$- \frac{27}{2}\cancel{\frac{x^2}{r^5}\left[1 - \frac{5}{2}\frac{a}{r} - \frac{1}{2}\frac{a^3}{r^3}\right]} + \frac{45}{2}\cancel{\frac{x^4}{r^7}\left[1 - \frac{5}{2}\frac{a}{r} - \frac{1}{2}\frac{a^3}{r^3}\right]} - \frac{9}{2}\frac{x^4}{r^5}\left[\frac{5}{2}\frac{a}{r^3} + \frac{3}{2}\frac{a^3}{r^5}\right]$$

$$-\frac{27}{8}\frac{a}{r^4} - \frac{9}{2}\cancel{\frac{x^2}{r^5}\left[1 - \frac{5}{2}\frac{a}{r} - \frac{1}{2}\frac{a^3}{r^3}\right]} + \frac{27}{2}\frac{ay^2}{r^6} + \frac{45}{2}\cancel{\frac{x^2y^2}{r^7}\left[1 - \frac{5}{2}\frac{a}{r} - \frac{1}{2}\frac{a^3}{r^3}\right]} -$$

$$-\frac{9}{2}\frac{x^2y^2}{r^5}\left[\frac{5}{2}\frac{a}{r^3} + \frac{3}{2}\frac{a^3}{r^5}\right]$$

$$-\frac{27}{8}\frac{a}{r^4} - \frac{9}{2}\cancel{\frac{x^2}{r^5}\left[1 - \frac{5}{2}\frac{a}{r} - \frac{1}{2}\frac{a^3}{r^3}\right]} + \frac{27}{2}\frac{az^2}{r^6} + \frac{45}{2}\cancel{\frac{x^2z^2}{r^7}\left[1 - \frac{5}{2}\frac{a}{r} - \frac{1}{2}\frac{a^3}{r^3}\right]} -$$

$$-\frac{9}{2}\frac{x^2z^2}{r^5}\left[\frac{5}{2}\frac{a}{r^3} + \frac{3}{2}\frac{a^3}{r^5}\right]$$

$$= 3\frac{1}{r^3}\left[1-\frac{3a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right]-\frac{27}{4}\frac{a}{r^4}+\frac{27}{2}\frac{a(y^2+z^2)}{r^6}-\frac{9x^2}{r^5}\left[1-\frac{3a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right]$$

$$+\frac{3x^2}{r^3}\left[\frac{3a}{r^3}+\frac{3}{4}\frac{a^3}{r^5}\right]-\frac{9}{2}\frac{x^2}{r^3}\left[\frac{5}{2}\frac{a}{r^3}+\frac{3}{2}\frac{a^3}{r^5}\right]$$

$$=\frac{3}{r^3}\left[1-\frac{3a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right]-\frac{27}{4}\frac{a}{r^4}+\frac{27}{2}\frac{a}{r^4}\left[1-\frac{x^2}{r^2}\right]-\frac{9x^2}{r^5}\left[1-\frac{3a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right]$$

$$+\frac{3x^2}{r^5}\left[3\frac{a}{r}+\frac{3}{4}\frac{a^3}{r^3}\right]-\frac{9}{2}\frac{x^2}{r^5}\left[\frac{5}{2}\frac{a}{r}+\frac{3}{2}\frac{a^3}{r^3}\right]=$$

$$=\frac{3}{r^3}\left[1-\frac{3a}{4r}-\frac{1}{4}\frac{a^3}{r^3}\right]+\frac{x^2}{r^5}\left[\frac{a}{r}\left(\overset{-9+}{-\frac{27}{2}}+27+9-\frac{45}{4}\right)+\frac{a^3}{r^3}\left[\frac{9}{4}+\frac{9}{4}-\frac{27}{4}\right]\right]$$

$$=\frac{3}{r^3}\left[1-\frac{3a}{4r}-\frac{1}{4}\frac{a^3}{r^3}\right]+\frac{9}{4}\frac{x^2}{r^5}\left[-1+\frac{5}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right]$$ Zgadza się

Teraz należy jeszcze $U\ V\ W$ tak oznaczyć żeby na powierzchni owe wartości były spełnione:

a w nieskończoności:

!! , $w=0$

$$U=-\frac{27\mu c^2}{8a}\frac{x}{a}\left[1-\frac{4}{3}\frac{x^2}{a^2}\right]$$

$$V=-\frac{27\mu c^2}{8a}\frac{y}{a}\left[\frac{1}{2}-\frac{4}{3}\frac{x^2}{a^2}\right]$$

$$W=-\frac{27\mu c^2}{8a}\frac{z}{a}\left[\frac{1}{2}-\frac{4}{3}\frac{x^2}{a^2}\right]$$

Ogólne rozwiązanie:

$$U=\frac{1}{\mu}\sum\left\{\frac{r^2}{2(2n+1)}\frac{dp_n}{dx}+\frac{n\,r^{2n+3}}{(n+1)(2n+1)(2n+3)}\frac{d}{dx}\left(\frac{p_n}{r^{2n+1}}\right)\right\}+\sum\frac{d\varphi_n}{dx}$$

$$V=\qquad\frac{d}{dy}\qquad\qquad\frac{d}{dy}\qquad\qquad\frac{d\varphi_n}{dy}$$

$$W=$$

$p_0 = \text{const.}$ $\quad p_{-1} =$

$p_1 = x$ $\quad p_{-2} =$

$p_2 =$ $\quad p = r^m S_m \quad \frac{1}{r^{m+1}} S_m$

$\frac{x^2}{r^3}$

$\frac{2x}{2r} - \frac{3x^3}{2r^5}$

$p_n =$

$xu + yv + zw \Big|_{r=\infty} = 0$

$S_0 = 1$

$S_1 = \cos$

$S_2 = \frac{1}{2}(3\cos^2 - 1)$

$S_3 = \frac{1}{2}(5\cos^3 - 3\cos)$

$x \qquad \frac{x}{r^3}$

$\frac{1}{2} \; 3 \quad p_0 = 1 \;\Big\|\; p_{-1} = \frac{1}{r}$

$p_1 = x \;\Big\|\; p_{-2} = \frac{x}{r^3}$

$p_2 = \frac{1}{2}(3x^2 - r^2) \;\Big\|\; p_{-3} = \frac{1}{2r^3}\left(3\frac{x^2}{r^2} - 1\right)$

$xu + yv + zw \Big|_{\infty} = \frac{1}{\mu} \sum \frac{n\, r^n}{2(2n+3)} p_n + \frac{1}{r} \sum n \varphi_n \Big|_{\infty} = 0$

$n = -1 \qquad n < 0$

$$\frac{\partial p_{-3}}{\partial x} = -\frac{3x}{2r^5}\left(\frac{3x^2}{r^2} - 1\right) + \frac{3x}{r^5} - \frac{3x^3}{r^7} = \frac{3x}{r^5}\left[\frac{3}{2} - \frac{5}{2}\frac{x^2}{r^2}\right]$$

$$= \frac{9x}{2r^5}\left[1 - \frac{5}{3}\frac{x^2}{r^2}\right]$$

$$\frac{\partial p_{-3}}{\partial y} = -\frac{3y}{2r^5}\left(\frac{3x^2}{r^2} - 1\right) - \frac{3yx^2}{r^7} = \frac{3y}{r^5}\left[\frac{1}{2} - \frac{5}{2}\frac{x^2}{r^2}\right] = \frac{9}{2}\frac{y}{r^5}\left[\frac{1}{3} - \frac{5}{3}\frac{x^2}{r^2}\right]$$

$$\frac{\partial}{\partial x}\left(\frac{p_{-3}}{r^{-5}}\right) = \frac{\partial}{\partial x}\left[\frac{r^2}{2}\left(\frac{3x^2}{r^2} - 1\right)\right] = \frac{1}{2}\frac{\partial}{\partial x}\left[3x^2 - r^2\right] = 3x - x = 2x$$

$$\frac{\partial}{\partial y}\left(\frac{p_{-3}}{r^{-5}}\right) = \frac{1}{2}\frac{\partial}{\partial y}\left[3x^2 - r^2\right] = -y$$

$n = -3$

$$U = -\frac{r^2}{2.5}\,\frac{3x}{r^5}\left[\frac{3}{2} - \frac{5}{2}\frac{x^2}{r^2}\right] + \frac{\cancel{2}\,1}{2.5.\cancel{2}}\,\frac{1}{r^3}\,2x = \frac{1}{10}\frac{x}{r^3}\left[-\frac{5}{2} + \frac{15}{2}\frac{x^2}{r^2}\right]$$

$$= -\frac{1}{4}\frac{x}{r^3}\left[1 - 3\frac{x^2}{r^2}\right]$$

$$V = -\frac{r^2}{2.5}\,\frac{3y}{r^5}\left[\frac{1}{2} - \frac{5}{2}\frac{x^2}{r^2}\right] - \frac{1}{10.}\frac{1}{r^3}\,y \qquad = \frac{1}{10}\frac{y}{r^3}\left[-\frac{5}{2} + \frac{15}{2}\frac{x^2}{r^2}\right]$$

$$= -\frac{1}{4}\frac{y}{r^3}\left[1 - 3\frac{x^2}{r^2}\right]$$

$$U = A\frac{x}{r^3}\left[1 - 3\frac{x^2}{r^2}\right] + B\frac{x}{r^3} + C\frac{x}{r^5}\left[\frac{3}{2} - \frac{5}{2}\frac{x^2}{r^2}\right]$$

$$V = A\frac{y}{r^3}\left[1 - 3\frac{x^2}{r^2}\right] + B\frac{y}{r^3} + C\frac{y}{r^5}\left[\frac{1}{2} - \frac{5}{2}\frac{x^2}{r^2}\right]$$

$$\left[A\frac{1}{a^3} + B\frac{1}{a^3} + \frac{3}{2}C\frac{1}{a^5}\right]\cancel{x} = \frac{1}{a}$$

$$\cancel{\left[-3A\frac{1}{a^5} - \frac{5}{2}C\frac{1}{a^7}\right]x^3 = -\frac{4}{3}\frac{x^3}{a^3}}$$

$$\frac{C}{a^5} = \frac{1}{2a} \qquad C = \frac{a^4}{2}$$

$$A\frac{1}{a^3} + B\frac{1}{a^3} + \frac{1}{2}C\frac{1}{a^5} = \frac{1}{2}\frac{1}{a}$$

$$-3A\frac{1}{a^5} - \frac{5}{2}C\frac{1}{a^7} = -\frac{4}{3}\frac{1}{a^3}$$

$$\frac{3A}{a^5} + \frac{5}{4}\frac{1}{a^3} = \frac{4}{3}\frac{1}{a^3}$$

$$\frac{3A}{a^5} = \left(\frac{4}{3} - \frac{5}{4}\right)\frac{1}{a^3} = \frac{1}{12}\frac{1}{a^3}$$

$$A = \frac{a^2}{36}$$

$$A + B + \frac{1}{2} C \frac{1}{a^2} = \frac{1}{2} a^2$$

$$B = \frac{1}{2} a^2 - \frac{a^2}{36} - \frac{1}{3}\frac{a^2}{2} = \frac{2}{9} a^2$$

$$U = -\frac{27 \mu c^2}{8a}\left[\frac{a^2}{36}\left\{\frac{x}{r^3}\left[1 - 3\frac{x^2}{r^2}\right] + 8\frac{x}{r^3}\right\} + \frac{a^4}{2}\frac{x}{r^5}\left[\frac{3}{2} - \frac{5}{2}\frac{x^2}{r^2}\right]\right]$$

$$U = -\frac{27 \mu c^2}{8a}\left[\left(\frac{1}{4}\frac{x}{r^3}\right) - \frac{a^2}{12}\frac{x^3}{r^5} + \frac{a^4}{4}\frac{x}{r^5}\left[3 - 5\frac{x^2}{r^2}\right]\right]$$

$$V = -\frac{27 \mu c^2}{8a}\left[\left(\frac{y}{4r^3}\right) - \frac{a^2}{12}\frac{y x^2}{r^5} + \frac{a^4}{4}\frac{y}{r^5}\left[1 - 5\frac{x^2}{r^2}\right]\right]$$

$$U = +\frac{27 \mu c^2 a}{32}\left[\left(-\frac{x}{r^3} + \right)\frac{1}{3}\frac{x^3}{r^5} - \frac{a^2 x}{r^5}\left(3 - 5\frac{x^2}{r^2}\right)\right]$$

$$V = \frac{27 \mu c^2 a}{32}\left[\left(-\frac{y}{r^3} + \right)\frac{1}{3}\frac{y x^2}{r^5} - \frac{a^2 y}{r^5}\left(1 - 5\frac{x^2}{r^2}\right)\right]$$

Zkouška: $\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} \sim = \frac{x^2}{r^5} - \frac{5}{3}\frac{x^4}{r^7} - \frac{3a^2}{r^5} + \frac{15 a^2 x^2}{r^7} + \frac{15 a^2 x^2}{r^7} - \frac{35 x^4 a^2}{r^9}$

$$+\frac{1}{r^3} - \frac{3x^2}{r^5} \quad + \frac{1}{3}\frac{x^2}{r^5} - \frac{5}{3}\frac{x^2 y^2}{r^7} - \frac{a^2}{r^5} + \frac{5a^2 y^2}{r^7} + \frac{5a^2 x^2}{r^7} - \frac{35 x^2 y^2 a^2}{r^9}$$

$$+\frac{1}{r^3} \Big\} = 0 \quad + \frac{1}{3}\frac{x^2}{r^5} - \frac{5}{3}\frac{z^2 x^2}{r^7} - \frac{a^2}{r^5} + \frac{5a^2 z^2}{r^7} + \frac{5a^2 x^2}{r^7} - \frac{35 x^2 z^2 a^2}{r^9}$$

$$-\frac{5}{3}\frac{x^2}{r^5} \qquad -\frac{35 x^2 a^2}{r^7}$$

$$= -\frac{5a^2}{r^7}(r^2 - x^2) + \frac{5a^2x^2}{r^7} \qquad + \frac{5a^2x^2}{r^7}$$

$$= \frac{1}{r^7}\left\{ r^4 - \frac{5}{3}(x^4 + y^4 + z^4) - \ldots \right. \qquad = 0$$

$$\Delta^2 U \sim \frac{1}{3}\left(\frac{x}{r^5} - 10\frac{x^3}{r^7}\right) - 3a^2\left(10\frac{x}{r^7}\right) + 5a^2 \quad 6\frac{x}{r^7} \quad \frac{2x}{r^5} - \frac{10}{3}\frac{x^3}{r^7}$$

$$\Delta^2 V \sim \frac{2}{3}\frac{y}{r^5} - \frac{10}{3}\frac{x^2 y}{r^7}$$

$$\Delta W \sim \frac{2}{3}\frac{z}{r^5} - \frac{10}{3}\frac{x^2 z}{r^7}$$

$$\frac{\partial}{\partial x}\nabla^2 U + \frac{\partial}{\partial y}\nabla^2 V + \frac{\partial}{\partial z}(\ldots) = 0$$

vpadá (?) ...

Czasookresowe prądu cieplnego

nie były elektrostatyki:

temp. stała U stała

$\Delta^2\theta$ $\Delta^2 U$

siła $= \frac{\partial U}{\partial x}$

$\theta = \frac{1}{4\pi}\int \Delta^2\theta \, dv$ $U = \frac{1}{4\pi}\int \frac{\Delta^2 U}{r} dv$

$\Delta^2\theta \, \frac{\partial\theta}{\partial x}$? mechanik. $\Delta^2 U . \frac{\partial U}{\partial x}$

dyf. prawo grav.

ścisły związek

gradient grawit.

$$\frac{p_1 - p_2}{l} \quad \frac{a^4 \pi}{8\mu} = F$$

$$p_1 - p_2 = \frac{8\mu}{a^4\pi} F\, l \qquad \frac{40000 . \cancel{8} . 0{\cdot}0015 . \cancel{1}000}{\cancel{3} . \cancel{16}} = 0{\cdot}06 \text{ atm}$$

$$-\frac{5z}{r^7}$$

$$-\frac{5}{r^7} + \frac{35 z^2}{r^9}$$

$$-\frac{7z}{r^9} \qquad -\frac{7}{r^9} + \frac{63 z^2}{r^{11}}$$

$\frac{1}{3}\frac{yx^2}{r^5} - \frac{a^2y}{r^5} + \frac{a^2x^2y}{r^7}$

$\frac{1}{3}\cdot\frac{2xy}{r^5} - \frac{5x^3y}{3r^7} + \frac{5a^2xy}{r^7} + \frac{15a^2xy}{r^7} - \frac{35a^2x^3y}{r^9}$

$\frac{2}{3}\frac{y}{r^5} - \frac{10}{3}\frac{x^2y}{r^7} - \frac{25x^2y}{3r^7} + \frac{35x^4y}{3r^9} + \frac{15a^2y}{r^7} - 2\cdot 5\cdot 7\frac{a^2x^2y}{r^9} - 3\cdot 5\cdot 7\frac{a^2x^2y}{r^9}$

$+9\cdot 35\frac{a^2x^4y}{r^{11}}$

$\frac{1}{3}\frac{x^2}{r^5} - \frac{5x^2y^2}{3r^7} - \frac{a^2}{r^5} + \frac{a^2y^2}{r^7} + \frac{5a^2x^2}{r^7} - \frac{35a^2x^2y^2}{r^9}$

$-\frac{5}{3}\frac{x^2y}{r^7} - \frac{10x^2y}{3r^7} + \frac{5\cdot 7}{3}\frac{x^2y^3}{r^9} + \frac{15a^2y}{r^7} - 35\frac{a^2y^3}{r^9} - \frac{105a^2x^2y}{r^9} + 3\cdot 5\cdot 9\frac{a^2x^2y^3}{r^{11}}$

$\cancel{\frac{1}{3}\frac{yx^2}{r^7}\cdot 2 - \frac{a^2y}{r^7}\cdot 2 + \frac{a^2x^2y\, 42}{r^9}}$

$-\frac{5}{3}\frac{yx^2}{r^7} + \frac{35}{3}\frac{yx^2z^2}{r^9} + \frac{5a^2y}{r^7} - \frac{35a^2yz^2}{r^9} - \frac{35a^2x^2y}{r^9} + 9\cdot 35\frac{a^2x^2yz^2}{r^{11}}$

$+\frac{35}{3}\frac{x^2y}{r^7}$ $\quad -\frac{35a^2yz^2}{r^9}$ $\quad +9\cdot 35\frac{a^2x^2y}{r^9}$

$-\frac{35a^2y}{r^7}$

-105
-210
-35
$350 \quad + 315$

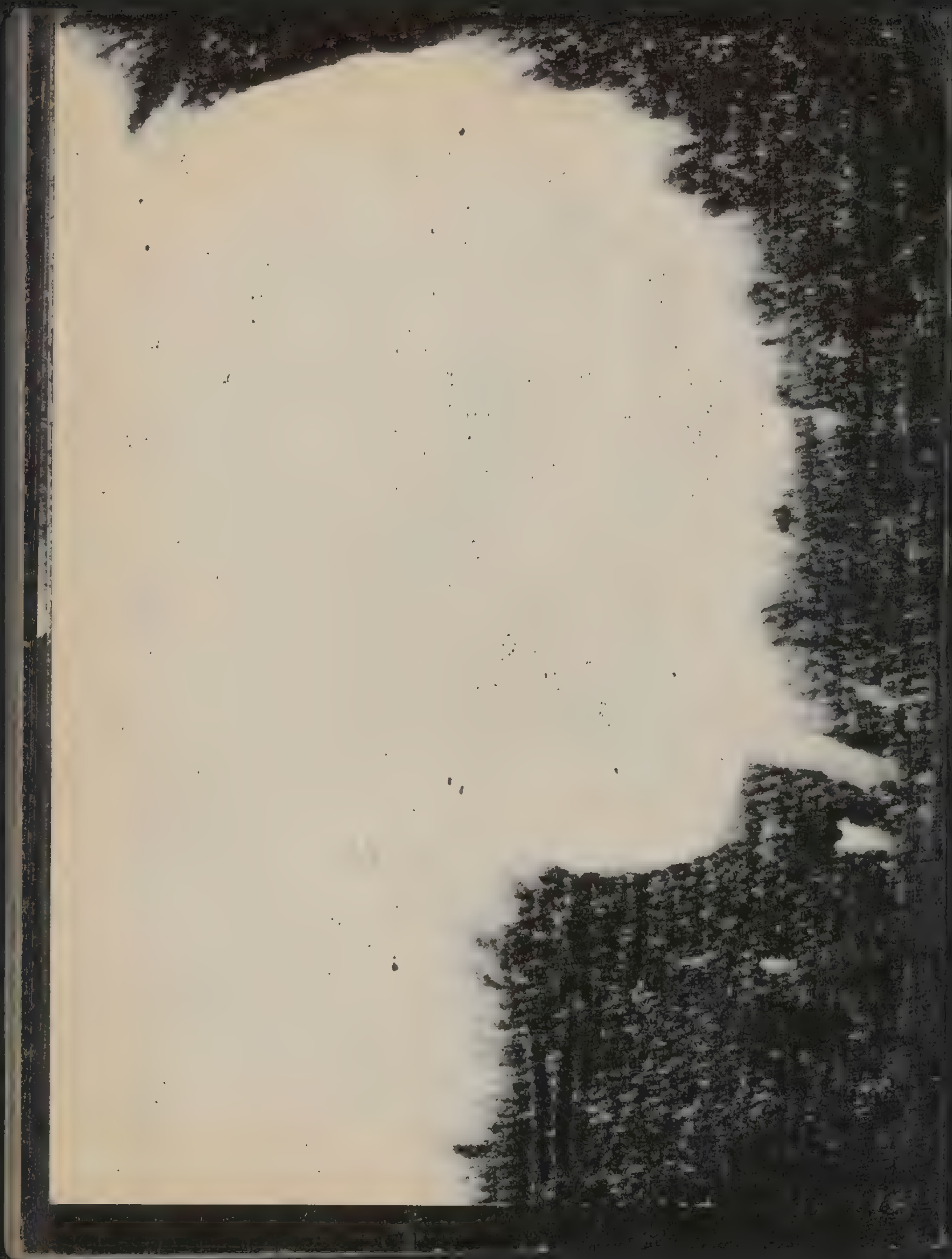
$\frac{2}{3}\frac{y}{r^5} - \frac{10}{3}\frac{x^2y}{r^7} \qquad = 0 \qquad = -35\frac{a^2x^2}{r^9}$

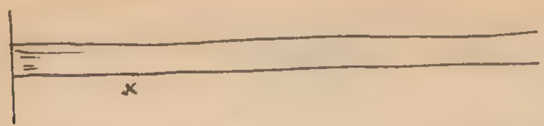
$\frac{2}{r^5} - \frac{10x^2}{r^7} - \frac{10x^2}{r^7} + \frac{70x^4}{r^9} \qquad \frac{10}{3}\frac{1}{r^5} - \frac{10}{3}$

$\frac{2}{3r^5} - \frac{10}{3}\frac{x^2}{r^7} - \frac{10x^2}{3r^7} + \frac{70x^2x^2}{3r^9}$

$\frac{2}{3r^5} - \frac{10}{3}\frac{z^2}{r^7} - \frac{10x^2}{3r^7} + \frac{70x^2z^2}{3r^9}$

$\frac{70x^2}{3r^7}$





$$\frac{\partial}{\partial r}\left[r\frac{\partial u}{\partial r}\right] = r\frac{\partial p}{\partial x} \qquad \frac{\partial^2}{\partial r^2}\left(r\frac{\partial u}{\partial r}\right) = -\frac{\partial p}{\partial x} + \cancel{r\frac{\partial^2 p}{\partial x\partial r}}$$

$$\cancel{r\frac{\partial u}{\partial r} = f(x) + x}$$

$$r\frac{\partial^2}{\partial r^2}\left(r\frac{\partial u}{\partial r}\right) = +\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \qquad \cancel{-\frac{f_1(x)}{r^2} = -r\frac{\partial p}{\partial x}}$$

$$\log\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = +\log(r) + f_1(x)$$

$$r = R \qquad u = 0$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = r f_1(x)$$

$$r = 0 \qquad u = \text{endlich}$$

$$\cancel{u = \frac{r}{\partial r} f(x) + \varphi(x)}$$

$$\psi(x) = 0$$

$$r\frac{\partial u}{\partial r} = \frac{r^2}{2}\varphi(x) + \psi(x)$$

$$\frac{\partial u}{\partial r} = r\varphi(x) + \frac{1}{r}\psi(x)$$

$$u = \frac{r^2}{2}\varphi(x) + \psi(x)\log r + \chi(x)$$

$$0 = \frac{R^2}{2}\varphi(x) + \chi(x)$$

$$\chi(x) = -\frac{R^2}{2}\varphi(x)$$

$$u = \frac{r^2 - R^2}{2}\varphi(x)$$

$$\frac{\partial}{\partial x}(\rho u) = \text{const} \qquad pu = \cancel{x F(x)} + \Phi(x) = p\frac{r^2 - R^2}{2}\varphi(x)$$

$$\cancel{\Phi(x) = 0 \quad \Phi(x) \quad \varphi(x) = x}$$

$$p\varphi(x) = x \qquad \cancel{\frac{\partial p}{\partial x} = 0 \quad x F(x) + \Phi(x) = 0}$$

$$u = \frac{r^2 - R^2}{2}\frac{x}{p}$$

$u = (a^2 - r^2)\frac{k}{\mu}$

$\frac{\partial u}{\partial r} = -2r\frac{k}{\mu}$

$-p\frac{\partial u}{\partial x}\Big| + \mu\left[2\left(\frac{\partial u}{\partial r}\right)^2 = \kappa\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)\right.$

$-8\mu\left(\frac{k}{\mu}\right)^2 r^2 = \frac{\partial}{\partial r}\left(\quad\right)$

$\frac{8}{3}\mu\left(\frac{k}{\mu}\right)^2 r^2 = \frac{\partial\theta}{\partial r} + \frac{c}{r} \Big\| \theta = \frac{8}{9}\mu\left(\frac{k}{\mu}\right)^2 r^3 + c$

$\theta_0 = \frac{8}{9}\mu \quad a^3$

$\theta - \theta_0 = \frac{8}{9}\frac{\mu}{\kappa}\left(\frac{k}{\mu}\right)^2(r^3 - a^3)$

$J_\lambda = \frac{A}{\lambda^5}e^{-\frac{B}{\lambda\theta}}$

$\frac{\partial J}{\partial\lambda} = -\frac{5}{\lambda} + \frac{B}{\lambda^2\theta} = 0$

$\lambda_m\theta = \frac{B}{5}$

$B = 5\lambda_m\theta$

8.863 . ~~288~~ . 5
3 5452 144
3545

$B = 1276.3$

$\frac{B}{\theta = 288} = 4.4316$

$^{10}\log J_\lambda = {}^{10}\log A - 5\log\lambda - \frac{B}{\lambda\theta}\,{}^{10}\log e$ | $\theta = 1440$, 273, 1167

$\frac{B}{\theta} = 8863 \cdot 0.4343$
3439
35452
2659
3545
205
3.8486

$\lambda = 1:\quad \log J = -3.8486$
.1514

58531
23930
0.34601 5

$\lambda = \frac{1}{2}$ $+\ 0.30103\ .5$
1.50515
-7.6972
38080 -10

$\lambda_{max} = 1.735$

0.23930
-1.1965
$-2.2182$5
$-3.4147$5

$\lambda = 2:$ -1.50515
-1.9243
-3.4295

477.2

$\lambda = 3$

$$\begin{array}{r} -1.2829 \\ \cancel{1.1.3.1.7} \\ -\underline{2.3856} \\ -3.6685 \end{array}$$

$\lambda = 4$

$$\begin{array}{r} 0.96215 \\ 3.0103 \\ \hline -3.97245 \end{array}$$

$\lambda_{max} = 1.157$

$$\begin{array}{r} 0.40921 \\ \underline{0.06333} \\ 0.34588 \end{array}$$

$$\begin{array}{r} \underline{0.06333.5} \\ +0.31665 \\ -2.2176 \\ -0.3166 \\ -\cancel{1.9010} \\ -2.6342 \end{array}$$

$\theta = 1440$

$$\begin{array}{r} 1440 \\ \underline{720} \\ 2160 \\ \underline{273} \\ 1887 \end{array}$$

$\frac{\Im}{\theta} = 1.28287$

2.5657

$\lambda = 1$: $\lg \Im = -2.5657$

-5.1314

$\lambda = 0.5$

$$\begin{array}{r} +1.5052 \\ \hline -3.0262 \end{array}$$

$\lambda = 2$ $\lg \Im = -1.28285$

$$\begin{array}{r} 1.50515 \\ \hline -2.7880 \end{array}$$

$\lambda = 3$

$$\begin{array}{r} 0.8552 \\ \underline{2.3856} \\ 3.2408 \end{array}$$

$\lambda = 4$

$$\begin{array}{r} 0.6414 \\ \underline{3.0103} \\ 3.6517 \end{array}$$

$3^4 = 81$

$\theta = 720$

273

$\frac{\Im}{\theta} = 7.7572$

$\lambda_{max} = 3.47$

$\lambda = 3$:

$$\begin{array}{r} -2.3856 \\ -\underline{2.5857} \\ -4.9713 \end{array}$$

$\theta = 1440 \cdot \frac{5}{4}$

7200

1800

273

1527

$\cancel{25657}$

$38486 \cdot \frac{4}{5}$

153944

$\lambda = 1 \quad \cancel{\tfrac{1}{2}} \; J = 3'07888$

$\lambda = 2 : \quad 1'53944$

$1'50515$

$-3'0446$

$\lambda = \frac{1}{2} \quad -6'15776$

$+1'50515$

$-4'65261$

$\lambda_{max} = \quad 1'735 \cdot \frac{4}{5}$

$0'347$

$1'388$

$\cancel{48840}$

$0'14239$

$0'34601$

$2'21825$

$0'71195$

$2'9302$

$\lambda = 3 \quad -1'02629$

$2'3856$

$-3'4119$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \eta\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)u + \frac{1}{3}\eta\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) - \frac{\partial p}{\partial x}$$

$$x = r\cos\varphi \qquad u = q\cos\varphi \qquad q,\ w = f.(r,z)$$

$$y = r\sin\varphi \qquad v = q\sin\varphi$$

$$z \qquad w = w$$

$$\frac{\partial s}{\partial x} = \frac{\partial s}{\partial r}\cos\varphi - \frac{\partial s}{\partial \varphi}\frac{\sin\varphi}{r}$$

$$\frac{\partial s}{\partial y} = \frac{\partial s}{\partial r}\sin\varphi + \frac{\partial s}{\partial \varphi}\frac{\cos\varphi}{r}$$

$$\boxed{\theta} \quad \frac{\partial q}{\partial r} + \frac{q}{r} + \frac{\partial w}{\partial z}$$

$$\frac{\partial \theta}{\partial x} = \left[\frac{\partial^2 q}{\partial r^2} + \frac{1}{r}\frac{\partial q}{\partial r} - \frac{q}{r^2} + \frac{\partial^2 w}{\partial r\,\partial z}\right]\cos\varphi$$

$$\frac{\partial u}{\partial x} = \frac{\partial q}{\partial r}\cos^2\varphi + \frac{q}{r}\sin^2\varphi$$

$$\frac{\partial u}{\partial y} = \left(\frac{\partial q}{\partial r} - \frac{q}{r}\right)\sin\varphi\cos\varphi$$

$$\frac{\partial u}{\partial z} = \frac{\partial q}{\partial z}\cos\varphi$$

$$\frac{\partial v}{\partial x} = \left(\frac{\partial q}{\partial r} - \frac{q}{r}\right)\sin\varphi\cos\varphi$$

$$\frac{\partial v}{\partial y} = \frac{\partial q}{\partial r}\sin^2\varphi + \frac{q}{r}\cos^2\varphi$$

$$\frac{\partial v}{\partial z} = \frac{\partial q}{\partial z}\sin\varphi$$

$$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial r}\cos\varphi \qquad \frac{\partial p}{\partial x} = \frac{\partial p}{\partial r}\cos\varphi$$

$$\frac{\partial w}{\partial y} = \frac{\partial w}{\partial r}\sin\varphi \qquad \frac{\partial p}{\partial y} = \frac{\partial p}{\partial r}\sin\varphi$$

$$\frac{\partial w}{\partial z} = \frac{\partial w}{\partial z} \qquad \frac{\partial p}{\partial z} = \frac{\partial p}{\partial z}$$

$$\frac{\partial^2}{\partial x^2} + \qquad = \frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \varphi^2} + \frac{1}{r}\frac{\partial u}{\partial r}$$

$$\left[\frac{\partial^2 q}{\partial z^2} + \frac{\partial^2 q}{\partial r^2} + \frac{1}{r}\frac{\partial q}{\partial r} - \frac{q}{r^2}\right]\cos\varphi = \nabla^2 u$$

$$\nabla^2 w = \frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial z^2} + \frac{1}{r}\frac{\partial w}{\partial r}$$

$$\frac{\partial(\theta)}{\partial z} = \frac{\partial^2 q}{\partial r\,\partial z} + \frac{1}{r}\frac{\partial q}{\partial z} + \frac{\partial^2 w}{\partial z^2}$$

$$\rho\left[\frac{\partial q}{\partial t}\cancel{\sin\psi}+q\cancel{\cos\psi}\left(\frac{\partial q}{\partial r}\cos^2\psi+\frac{q}{r}\sin^2\psi\right)+q\sin\psi\left(\frac{\partial q}{\partial r}-\frac{q}{r}\right)\sin\psi\cancel{\cos\psi}+w\frac{\partial q}{\partial z}\cancel{\sin\psi}\right]$$

$$=\eta\,\cancel{\sin\psi}\left[\frac{\partial^2 q}{\partial z^2}+\frac{4}{3}\left(\frac{\partial^2 q}{\partial r^2}+\frac{1}{r}\frac{\partial q}{\partial r}-\frac{q}{r^2}\right)+\frac{1}{3}\frac{\partial^2 w}{\partial r\,\partial z}\right]-\frac{\partial p}{\partial r}$$

$$\left[\frac{\partial w}{\partial t}+q\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right]=\frac{\eta}{\rho}\left[\frac{4}{3}\frac{\partial^2 w}{\partial z^2}+\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{3}\left(\frac{\partial^2 q}{\partial r\,\partial z}+\frac{1}{r}\frac{\partial q}{\partial z}\right)\right]-\frac{1}{\rho}\frac{\partial p}{\partial z}$$

$$\left[\frac{\partial q}{\partial t}+q\frac{\partial q}{\partial r}+w\frac{\partial q}{\partial z}\right]=\frac{\eta}{\rho}\left[\frac{\partial^2 q}{\partial z^2}+\frac{4}{3}\left(\frac{\partial^2 q}{\partial r^2}+\frac{1}{r}\frac{\partial q}{\partial r}-\frac{q}{r^2}\right)+\frac{1}{3}\frac{\partial^2 w}{\partial r\,\partial z}\right]-\frac{1}{\rho}\frac{\partial p}{\partial r}$$

Equation of motion with ~~cylindrical~~ radial symmetry (independent of ψ)

$$\cancel{c\rho\frac{\partial\theta}{\partial t}}=-\frac{2}{3}\mu\,\boxed{\theta}^2+\mu\left[2\left\{\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right\}+\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)^2+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right]$$

$$-\mu\ldots$$

$$c\rho\left[\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}\right]$$

$$c\rho\left[\frac{\partial\theta}{\partial t}+q\frac{\partial\theta}{\partial r}+w\frac{\partial\theta}{\partial z}\right]=-\mu\left[\frac{\partial q}{\partial r}+\frac{q}{r}+\frac{\partial w}{\partial z}\right]-\frac{2}{3}\mu\left[\frac{\partial q}{\partial r}+\frac{q}{r}+\frac{\partial w}{\partial z}\right]^2$$

$$+\mu\left[2\left\{\left(\frac{\partial q}{\partial r}\right)^2\cos^4\psi+2\frac{q}{r}\frac{\partial q}{\partial r}\cos^2\psi\sin^2\psi+\left(\frac{q}{r}\right)^2\sin^4\psi\right.\right.$$
$$\sin^4\psi\qquad +2\qquad\qquad +\qquad\qquad \cos^4\psi$$
$$\left.+\left(\frac{\partial w}{\partial z}\right)^2\right\}$$

$$\underbrace{\sin^4\psi+\cos^4\psi+\ldots}_{=1-2\sin^2\psi\cos^2\psi}$$

$$=\left(\frac{\partial q}{\partial r}\right)^2+\left(\frac{q}{r}\right)^2-2\sin^2\psi\cos^2\psi\underbrace{\left[\left(\frac{\partial q}{\partial r}\right)^2+\left(\frac{q}{r}\right)^2-2\frac{\partial q}{\partial r}\frac{q}{r}\right]}_{\left(\frac{\partial q}{\partial r}-\frac{q}{r}\right)^2}$$

$$\left.+\left(\frac{\partial w}{\partial r}+\frac{\partial q}{\partial z}\right)^2\sin^2\psi+\left(\frac{\partial q}{\partial z}+\frac{\partial w}{\partial r}\right)^2\cos^2\psi+4\left(\frac{\partial q}{\partial r}-\frac{q}{r}\right)^2\sin^2\psi\cos^2\psi\right]$$

$$2\left(\frac{\partial w}{\partial z}\right)^2+2\left(\frac{q}{r}\right)^2+2\left(\frac{\partial q}{\partial r}\right)^2\cancel{-4\sin^2\psi\cos^2\psi\left(\frac{\partial q}{\partial r}-\frac{q}{r}\right)^2}+\left(\frac{\partial w}{\partial r}\right)^2+\left(\frac{\partial q}{\partial z}\right)^2+2\frac{\partial q}{\partial z}\frac{\partial w}{\partial r}$$
$$\cancel{+4\sin^2\psi\cos^2\psi\left(\frac{\partial q}{\partial r}-\frac{q}{r}\right)^2}$$

$$\frac{c}{A}\rho\left[\frac{\partial\theta}{\partial t}+q\frac{\partial\theta}{\partial r}+w\frac{\partial\theta}{\partial z}\right]=-p\left[\frac{\partial q}{\partial r}+\frac{q}{r}+\frac{\partial w}{\partial z}\right]+\mu\Bigg[-\frac{2}{3}\left(\frac{\partial q}{\partial r}+\frac{q}{r}+\frac{\partial w}{\partial z}\right)^2+2\left[\left(\frac{\partial w}{\partial z}\right)^2+\left(\frac{q}{r}\right)^2+\left(\frac{\partial q}{\partial r}\right)^2\right]$$

$$+\underbrace{\left(\frac{\partial w}{\partial r}\right)^2+\left(\frac{\partial q}{\partial z}\right)^2+2\frac{\partial q}{\partial z}\frac{\partial w}{\partial r}}_{=\left(\frac{\partial w}{\partial r}+\frac{\partial q}{\partial z}\right)^2}\Bigg]$$

Thermal equation in respect to motion with radial symmetry

I. ex. Stationary flow in tube:

$$q=0 \qquad w=\frac{p_1^2-p_2^2}{8\mu p l}(R^2-r^2) \qquad p^2=p_1^2-\underbrace{\frac{p_1^2-p_2^2}{l}}_{=a}z$$

$$w=\frac{a(R^2-r^2)}{8\mu p}$$

with neglection of $\mu\frac{\partial w}{\partial z}$

$$\frac{\partial w}{\partial r}=-\frac{ar}{4\mu p}$$

$$\frac{c\rho}{A}w\frac{\partial\theta}{\partial z}=-p\frac{\partial w}{\partial z}+\mu\left(\frac{\partial w}{\partial r}\right)^2$$

$$\frac{c\rho}{A}\frac{\partial\theta}{\partial z}\frac{a(R^2-r^2)}{8\mu p}=+p\frac{a(R^2-r^2)}{8\mu p^2}\frac{dp}{dz}+\mu\frac{a^2r^2}{16\mu^2p^2}$$

$$=\frac{1}{8\mu p^2}\left[-\frac{a^2(R^2-r^2)}{2}+\mu\frac{a^2r^2}{2\mu}\right]$$

$$=\frac{1}{8\mu p^2}\cdot\frac{a^2(2r^2-R^2)}{2}$$

$$\frac{c}{A}\frac{\partial\theta}{\partial z}=\frac{a(2r^2-R^2)}{2p\rho(R^2-r^2)}=\cancel{\frac{a(2r^2-R^2)}{2(R^2-r^2)}}$$

$$=\cancel{\frac{a}{2}\frac{1}{\sqrt{p_1^2-az}}\left[\frac{R^2}{R^2-r^2}-2\right]}$$

$$\frac{p}{\rho}=\alpha\theta$$

$$\rho=\frac{p}{\alpha\theta}$$

$$p\rho=\frac{p^2}{\alpha\theta}$$

$$\frac{c}{A}\frac{\alpha}{\theta}\frac{\partial\theta}{\partial z}=\frac{a(2r^2-R^2)}{2p^2(R^2-r^2)}=\frac{a}{2}\frac{1}{p_1^2-az}\left[\frac{R^2}{R^2-r^2}-2\right]$$

$$\frac{c}{\rho R^2}\int 2\rho r w \rho\theta\, dr = \frac{2}{R^2}\int w\rho\theta r\,dr =$$

$$\frac{c}{A\alpha}\log\theta = -\frac{1}{2}\log(p_1^2 - az)\left[\frac{R^2}{R^2-r^2}-2\right] + \text{const}$$

$$\frac{c\alpha}{A}\log\theta_0 = -\frac{1}{2}\log p_1^2 \quad [\quad] + -$$

$$\cancel{\frac{c\alpha}{A}\left[\theta^2-\theta_0^2\right] = \log\frac{p_1^2}{p_1^2-az}\left[\frac{R^2}{R^2-r^2}-2\right]}$$

$$\frac{c}{\alpha A}\log\frac{\theta}{\theta_0} = \log\sqrt{\frac{p_1^2}{p_1^2-az}}\left[\frac{R^2}{R^2-r^2}-2\right]$$

$$\theta = \theta_0\left[\frac{p_1^2}{p_1^2-az}\right]^{\frac{1}{2}\frac{\alpha A}{c}\frac{2r^2-R^2}{R^2-r^2}}$$

$$= \theta_0\left[1-\frac{az}{p_1^2}\right]^{-\frac{1}{2}\frac{\alpha A}{c}\frac{2r^2-R^2}{R^2-r^2}}$$

$z = l$:

$$\theta = \theta_0\left[\frac{p_1}{p_2}\right]^{\frac{2r^2-R^2}{R^2-r^2}\cdot\frac{\alpha A}{c}}$$

$r = 0$:

$$\theta = \theta_0\left(\frac{p_1}{p_2}\right)^{-\frac{\alpha A}{c}}$$

$$\cancel{\int\left(\frac{R^2}{R^2-r^2}-2\right)r\,dr = R^2\log(R^2-r^2)}$$

$r = R$: $\theta = \infty$

With taking into account the conduction of heat:

$$\frac{c\rho}{A}\frac{\partial\theta}{\partial z}\frac{a(R^2-r^2)}{8\mu p} = \frac{1}{8\mu p^2}\frac{a^2(2r^2-R^2)}{2} + \kappa\left[\underbrace{\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}}_{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)}\right]$$

$$\frac{c}{\alpha A}\frac{1}{\theta}\frac{\partial\theta}{\partial z}\frac{a(R^2-r^2)}{8\mu} = \frac{1}{8\mu(p_1^2-az)}\frac{a^2(2r^2-R^2)}{2} + \kappa$$

$$\theta = \cancel{b_0 + b_1 z + (b_2 + b_3 z) r^2}$$

$$\frac{\partial\theta}{\partial z} = b_1 + b_3 r^2 \qquad r\frac{\partial\theta}{\partial r} = 2r^2(b_2 + b_3 z)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(\ ) = 4(b_2 + b_3 z)$$

$$\frac{c}{\alpha A}\,\alpha(R^2 - r^2)(b_1 + b_3 r^2)(p_1^2 - az) = a^2\left(r^2 - \frac{R^2}{2}\right)(b_0 + b_1 z + b_2 r^2 + b_3 z r^2) +$$
$$+ 4\kappa(b_2 + b_3 z)(b_0 + b_1 z + b_2 r^2 + b_3 z r^2)(p_1^2 - az)$$

$$\frac{c}{\alpha A}\,a R^2 b_1 p_1^2 = -a^2\frac{R^2}{2}b_0 + 4\kappa p_1^2 b_0 b_2$$

$$\frac{c}{\alpha A}\,\alpha(R^2 b_3 - b_1)p_1^2 = a^2\left(b_0 - \frac{R^2}{2}b_2\right) + 4\kappa b_2^2 p_1^2$$

$$-\frac{c}{\alpha A}\,a^2 R^2 b_1 = a^2\left(-\frac{R^2}{2}\right)b_1 + 4\kappa\left[(b_0 b_3 + b_1 b_2)p_1^2 - a\, b_0 b_2\right]$$

$$+\frac{c}{\alpha A}\,a^2(b_1 - R^2 b_3) = a^2\left(b_1 - \frac{R^2}{2}b_3\right) + 4\kappa\left[2b_2 b_3 p_1^2 - a\, b_2^2\right]$$

$$) = a^2\left[\ -\frac{R^2}{2}(p_1^2 + a b_0)\right] + 4\kappa p_1^2(b_0 b_3 + b_1 b_2)$$

$$0 = a^2\left[b_1 p_1^2 + b_0 - \frac{R^2}{2}(b_3 p_1^2 + b_2)\right] + 8\kappa b_2 b_3 p_1^4$$

$$1 + \frac{1}{2}\frac{\alpha A}{c}\frac{a}{p_1^2}\underbrace{\frac{2r^2 - R^2}{R^2 - r^2}}\, z -$$

$$= \frac{R^2}{R^2 - r^2} - 2 = \frac{1}{1 - \frac{r^2}{R^2}} - 2 = 1 + \frac{r^2}{R^2} + \frac{r^4}{R^4} \dots - 2 = -1 + \frac{r^2}{R^2} + \frac{r^4}{R^4}$$

$$-\frac{1}{2}\frac{\alpha A}{c}\frac{2r^2 - R^2}{R^2 - r^2} - 1$$

$$\cancel{\frac{\partial\theta}{\partial r} = +\theta_0 \frac{a}{p_1^2}\frac{\alpha A}{2c}\frac{2r^2 - R^2}{R^2 - r^2}\left[1 - \frac{az}{p_1^2}\right]}$$

$$\ln\theta = \ln\theta_0 + \ln\left[1 - \frac{az}{p_1^2}\right] - \frac{2\alpha A}{c}\frac{2r^2 - R^2}{R^2 - r^2}$$

$$\frac{1}{\theta}\frac{\partial\theta}{\partial z} = \cancel{-\frac{a}{p_1^2}\frac{1}{1 - \frac{az}{p_1^2}}} = \frac{-a}{p_1^2 - az}$$

$$= 1 - \frac{1}{\beta}\frac{a^2}{p_1^2}z + \frac{a^2}{\beta\, p_1^2 R^2}2z^2 +$$

$$\cancel{\frac{-\alpha}{p_1^2 - az}}\;\frac{c}{\alpha A}\;\cancel{\frac{\alpha(R^2 - r^2)}{2}} = \alpha$$

$$-\eta\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)=\frac{\partial p}{\partial z}$$

$$\frac{\partial}{\partial z}(\rho w)=0$$

$$\theta = b_0 + b_1 z + b_2 z r^2 + b_3 z^2 r^2 + b_4 z r^4 + b_5 z^2 r^4 + \cdot \quad + b_{11} z^2$$

$$\frac{2c}{\alpha A}\, a(R^2-r^2)(p_1^2-az)\frac{\partial\theta}{\partial z} = a^2(2r^2-R^2)\theta + 16\mu k(p_1^2-az)\frac{\theta}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)$$

$$r\frac{\partial\theta}{\partial r} = 2b_2 z r^2 + 2b_3 z^2 r^2 + 4b_4 z r^4 + 4b_5 z^2 r^4$$

$$\frac{\partial}{\partial r}(\ ) = 4b_2 z r + 4b_3 z^2 r + 16 b_4 z r^3 + 16 b_5 z^2 r^3$$

$$\frac{2ca}{\alpha A} = \beta$$

$$\frac{2ca}{\alpha A}(R^2-r^2)(p_1^2-az)(b_1 + b_2 r^2 + 2b_3 z r^2 + b_4 r^4 + 2b_5 z r^4 + 2b_{11} z) =$$

$$= a^2(2r^2-R^2)(b_0 + b_1 z + b_{11} z^2 + b_2 z r^2 + b_3 z^2 r^2 + b_4 z r^4 + b_5 z^2 r^4) +$$

$$+ 64\mu k(p_1^2-az)(b_0 + b_1 z + b_{11} z^2 + b_2 z r^2 + b_3 z^2 r^2 + b_4 z r^4 + b_5 z^2 r^4)(b_2 z + b_3 z^2 + 4b_4 z r^2 + 4 b_5 z^2 r^2)$$

$$\beta \cancel{R^2} p_1^2 b_1 = -a^2 \cancel{R^2} b_0$$

z)
$$-\beta R^2 a b_1 + 2\beta R^2 p_1^2 b_{11} = -a^2 R^2 b_1 + 64\mu k\, p_1^2 b_0 b_2$$

r^2)
$$-\beta p_1^2 b_1 + \beta R^2 p_1^2 b_2 = 2a^2 b_0$$

zr^2)
$$2\beta R^2 p_1^2 b_3 - \beta R^2 a b_2 + \beta \cancel{R^2} a b_1 = -a^2 R^2 b_2 + 2a^2 b_1$$

$$\mathrm{I\,II}: \quad \beta R^2 p_1^2 b_2 = a^2 b_0 = -p_1^2 \beta b_1$$

$$b_1 = -\frac{a^2 b_0}{\beta p_1^2}$$

$$b_2 = \frac{a^2 b_0}{\beta p_1^2 R^2}$$

$$\mathrm{II}). \quad 2\beta p_1^2 b_{11} = \beta a b_1 - a^2 b_1 + 64\mu k \frac{p_1^2 b_0 b_2}{R^2}$$

$$b_{11} = b_1 \left[\frac{a}{2p_1^2} - \frac{a^2}{2\beta p_1^2}\right]$$

$$= \frac{-a^3}{2p_1^2}\frac{b_0}{\beta p_1^2}\left(1 - \frac{a}{\beta}\right)$$

$$\left[1-2\alpha+\frac{17}{16}\alpha^2\right]^{-1} = 1+2\alpha-\frac{17}{16}\alpha^2+4\alpha^2 \dots$$

$$\frac{1}{1-x} = 1+x+x^2+\dots \qquad = 1+2\alpha+\frac{47}{16}\alpha^2$$

Dieterici-Vergleich:

$$\frac{v_a}{v_i} = e^{h^2} = e^{\frac{\frac{ms^2}{2}}{\frac{ms^2}{3}}} = e^{\frac{\frac{ms^2}{2}}{RT}}$$

$\frac{ms^2}{2} = \int \dots$
$= LK \dots$

$$\psi_f = \frac{2ma}{v_f}$$

$$\cancel{\psi} = \frac{RT}{2a} \lg \frac{v_a}{v_i} = \frac{\cancel{2}m\cancel{a}}{v_f}$$

$$\lg \frac{v_a - b}{v_i - b} = \lg \frac{v_a}{v_i} \cdot \frac{1-\frac{b}{v_a}}{1-\frac{b}{v_i}}$$

$$= \lg \frac{v_a}{v_i} + \lg\left(1-\frac{b}{v_a}\right) - \lg\left(1-\frac{b}{v_i}\right)$$

$$= \lg \frac{v_a}{v_i} - \frac{b}{v_a} + \frac{b}{v_i} - \frac{b^2}{v_a^2} + \frac{b^2}{v_i^2}$$

$$= \lg \frac{v_a}{v_i} - b\left(\frac{1}{v_a}-\frac{1}{v_i}\right) - b^2\left(\frac{1}{v_a^2}-\frac{1}{v_i^2}\right)$$

(17f)

$$a\left(\frac{1}{v_f}-\frac{1}{v_g}\right) = RT\left[\lg\frac{v_g-b}{v_f-b} - \left(\frac{v_g}{v_g-b} - \frac{v_f}{v_f-b}\right)\right]$$

$$1+\frac{b}{v_f}+\frac{b^2}{v_f^2}$$

$$= RT\left[\lg\frac{v_a}{v_i} - b\left(\frac{1}{v_g}-\frac{1}{v_f}\right) - b^2\left(\frac{1}{v_g^2}-\frac{1}{v_f^2}\right)\right.$$

$$\left. + b\left(\frac{1}{v_g}-\frac{1}{v_f}\right) - b^2\left(\quad\right)\right]$$

$$= p(v_g - v_f)$$

Boltzmanns (978)

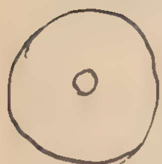
$$p(v_g - v_f) = rT\left[\lg \frac{v_g-b}{v_f-b} + a\left(\frac{1}{v_g} - \frac{1}{v_f}\right)\right]$$

$$p = \frac{rT}{v-b} - \frac{a}{v^2}$$

$$rT\underbrace{\left[\frac{-\cancel{v_g}}{v_g-b} + \frac{v_f}{v_f-b} + \lg\frac{v_g-b}{v_f-b}\right]} = 2a\underbrace{\left(\frac{1}{v_g} - \frac{1}{v_f}\right)}_{=L_i}$$

= identical with Maxwells Equation ?

$$\neq \lg\frac{v_g}{v_f} - b\left(\frac{1}{v_g} - \frac{1}{v_f}\right) - b^2\left(\frac{1}{v_g^2} - \frac{1}{v_f^2}\right)$$



$$4\pi\left[R^2 - \left(\sqrt[3]{R^3-r^3}\right)^2\right]$$

$$= 4\pi R^2\left[1 - \left(1-\left(\frac{r}{R}\right)^3\right)^{\frac{2}{3}}\right]$$

$$= 4\pi R^2 \; \cancel{1-1} + \frac{2}{3}\left(\frac{r}{R}\right)^3$$

$$= \frac{8\pi}{3}\frac{r^3}{R} = \frac{2v}{R}$$

m_2 m_1 m_3 m_4 m_5

$$W = m_1 m_2 \chi_{12} + m_1 m_3 \chi_{13} + m_1 m_4 \chi_{14} + m_1 \chi_{15} \ldots$$
$$+ m_2 m_3 \chi_{23} + m_2 m_4 \chi_{24} + \ldots$$
$$+ m_3 m_4 \chi_{34} + \ldots$$

$$= \frac{1}{2} \sum m_i m_k \chi_{ik}$$

$$= \frac{1}{2} \sum m_i \sum m_k \chi_{ik}$$

$$= \frac{1}{2} \int_0^M \int_0^\infty 4\pi\rho f\, df\, \chi\, dM = 2\pi M \rho \int_0^\infty f \chi\, df$$

[illegible] $= \frac{nB}{2}$ [illegible] $B = \ldots$ [illegible]

[illegible]

[illegible] $W = \frac{1}{2}\sum m U$ [illegible] 2). U wird bei den beobachteten

Stellen kleiner und diese beiden Wirkungen sind gleich groß $= B$

also im Ganzen Arbeit $\delta W = B$

Wenn nun aber der entstandene Hohlraum wieder verschwindet so wird letztere

Arbeit $= \frac{1}{2} B$ wieder gewonnen; also wirkliche innere Verdichtung für 1 Stelle $= \frac{B}{2}$!

$$dN = A e^{-h(U+L)}\, du\, dv\, dw \dots dx\, dy\, dz$$

$$= A e^{-h(U + L_a + L_i)}\, d\omega_a\, d\omega_i\, dv$$

$$N_u = A e^{-hU} dv \int e^{L_a + L_i}\, d\omega_a\, d\omega_i = B e^{-hU} dv$$

$$= N_{ges}\, e^{-hU} dv$$

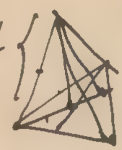
$$\frac{\rho_f}{\rho_g} = e^{-hU} = e^{-h\int}$$

$$p = \frac{mc^2}{3\pi(d-2r)^3}$$

$\cancel{b = \frac{3}{4} k(d-2r)}$

$$4\,\frac{4}{3}\pi r^3 = b \qquad 2r = \sqrt[3]{\frac{3b}{2\pi}} \qquad p = \frac{mc^2}{3\pi\left(\sqrt[3]{v} - \sqrt[3]{\frac{3b}{2\pi}}\right)^3}$$

$\cancel{d^3 = v}$ $\qquad$ $\cancel{d = \sqrt[3]{v}}$

d 

$$h = \sqrt{d^2 - \left(\frac{2}{3}\frac{d}{2}\sqrt{3}\right)^2} = d\sqrt{1 - \frac{1}{3}} = d\sqrt{\frac{2}{3}}$$

$$p = \frac{mc^2}{3\pi v\left(1 - \sqrt[3]{\frac{3b}{2\pi v}}\right)^3} \qquad d\,\frac{1}{2}\sqrt{3}$$

$$\frac{4}{3}\pi r^3 = b_1 \qquad r = \sqrt[3]{\frac{3b_1}{4\pi}} \qquad p = \frac{mc^2}{3\pi\left(\sqrt[3]{v} - \sqrt[3]{\frac{6b_1}{\pi}}\right)^3}$$

$$\cancel{2}\,\frac{d}{2}\cdot\frac{d}{2}\sqrt{3}\cdot d\sqrt{\frac{2}{3}} = \frac{d^3}{\sqrt{2}} = v \qquad d = \sqrt[3]{v\sqrt{2}} \qquad \sqrt{2} = \frac{3b}{2\pi v}$$

$$\frac{b}{v} = \frac{\pi\sqrt{8}}{3}$$

$$p = \frac{mc^2}{3\pi\left(\sqrt[3]{v\sqrt{2}} - \sqrt[3]{\frac{6b_1}{\pi}}\right)^3} = \frac{mc^2}{\left(\sqrt[3]{3\sqrt{2}\pi v} - \sqrt[3]{18 b_1}\right)^3} = \frac{mc^2}{3\pi\sqrt{2}\cdot v\left[1 - \sqrt[3]{\frac{6}{\pi\sqrt{2}}\frac{b_1}{v}}\right]^3}$$

$$\frac{6\, b_1}{\pi\sqrt{2}} = \mathcal{B} \qquad p = \frac{n\, m\, c^2}{3\pi\sqrt{2}\cdot v\left(1-\sqrt[3]{\frac{\mathcal{B}}{v}}\right)^3}$$

$$\left[\frac{\mathcal{B}}{v}\right]^{\frac{1}{3}} = \left[1 \cancel{+} - \frac{v-\mathcal{B}}{v}\right]^{\frac{1}{3}} = 1 - \frac{v-\mathcal{B}}{3v}$$

$$p = \frac{n\, m\, c^2}{3\pi\sqrt{2}\cdot v\left(\frac{v-\mathcal{B}}{3v}\right)^3} = \frac{n\, m\, c^2}{\cancel{3}\pi\sqrt{2}}\,\frac{9\, v^2}{(v-\mathcal{B})^3} = n\, m\, c^2\, \frac{3}{2}\,\frac{\mathcal{B}}{b_1}\,\frac{v^2}{(v-\mathcal{B})^3}$$

$$p v = n\, m\, c^2 \cdot \frac{9}{\pi\sqrt{2}}\,\frac{1}{\left(1-\frac{\mathcal{B}}{v}\right)^3} = \frac{9}{\pi\sqrt{2}}\, \underline{\underline{n m c^2}} \left[1 + 3\,\frac{\mathcal{B}}{v}\right.$$

$$\frac{1+\frac{2}{3}x}{1-\frac{x}{3}} = \left(1+\frac{2}{3}x\right)\left(1-\frac{x}{3}\right)^{-1} =$$

$$= \left(1+\frac{2x}{3}\right)\left(1 + \underset{\frac{2x}{3}}{\frac{x}{3}} + \underset{\frac{2x^2}{9}}{\frac{x^2}{9}} + \underset{\frac{2x^3}{27}}{\frac{x^3}{27}}\right)$$

$$= 1 + x + \frac{x^2}{3} + \frac{x^3}{9}$$

$$\frac{V}{nm} = v \qquad \Big\| \quad \frac{2\pi N\sigma^3}{3pm} = b$$

$$\frac{b}{v} = \frac{2\pi\sigma^3 n}{3V}$$

$$\frac{nm\bar{c}^2}{2} \qquad \bar{c}^2 = 3rT$$

$$W_i'' = 3\rho^2 a V$$

$$rT\left[1+\frac{\sqrt{2}}{3}\,\frac{\sigma}{\lambda}\right] = pv + \underbrace{\frac{1}{3}\,\frac{W_i''}{nm}}_{\frac{a}{v}}$$

$$p + \frac{a}{v^2} = \frac{rT}{v}\left[1+\frac{\sqrt{2}}{3}\,\frac{\sigma}{\lambda}\right]$$

Nach Jäger für Flüssigkeiten und nach Sutherland für feste Körper

$$\lambda = \cancel{}\, d - 6$$

Somit

$$p + \frac{a}{v^2} = \frac{rT}{v}\left[1 + \frac{\sqrt{2}}{3}\,\frac{6}{d-6}\right] \qquad \frac{d}{6} = \sqrt[3]{\frac{v}{v_0}}$$

$$= \frac{rT}{v}\left[1 + \frac{\sqrt{2}}{3}\,\frac{1}{\sqrt[3]{\frac{v}{v_0}} - 1}\right] \qquad \sqrt[3]{\frac{v}{v_0}} = \left[1 + \frac{v-v_0}{v_0}\right]^{\frac{1}{3}} = 1 + \frac{1}{3}\,\frac{v-v_0}{v_0}$$

$$= \frac{rT}{v}\left[1 + \frac{\sqrt{2}}{\cancel{3}}\,\frac{1}{\cancel{\tfrac{1}{3}}\,\frac{v-v_0}{v_0}}\right] = \frac{rT}{v}\left[\frac{v - v_0 + v_0\sqrt{2}}{v - v_0}\right]$$

$$= \frac{rT}{v}\,\frac{v_0\sqrt{2}}{v-v_0} = \frac{rT\sqrt{2}}{v-v_0} \quad \cancel{= rT\sqrt{2}}$$

$$\cancel{pv - pv_0 + \frac{a}{v} - \frac{a v_0}{v^2} = rT\sqrt{2}}$$

$$\cancel{\frac{a}{v} - \frac{a v_0}{v^2} = rT\sqrt{2}}$$

$$\cancel{\frac{dv}{dT}\left[\frac{a}{v^2} - \frac{2 a v_0}{v^3}\right] = -r\sqrt{2}} \qquad \cancel{\alpha = \frac{1}{3}\,\frac{1}{v}\,\frac{dv}{dT}}$$

$$v - v_0 = rT\sqrt{2}\,\frac{v_0^2}{a}$$

genähert:

$$\frac{a}{v_0^2}(v - v_0) = rT\sqrt{2}$$

$$\frac{a}{v_0^2}\,\frac{dv}{dT} = r\sqrt{2} \qquad \alpha = \frac{1}{3}\,\frac{1}{v}\,\frac{dv}{dT} = \frac{v_0}{3}\,\frac{\overbrace{r\sqrt{2}\,T}^{=\frac{a}{v_0^2}(v-v_0)}}{a\,T}$$

$$= \frac{v - v_0}{3 v_0 T} = \frac{r\sqrt{2}}{3}\,\frac{v_0}{a} = \frac{r}{\rho}\,\frac{\sqrt{2}}{3}\,\frac{1}{a}$$

$$dp - \frac{2a}{v^3}\,dv = -\frac{rT\sqrt{2}}{(v-v_0)^2}\,dv \qquad \cancel{k = \frac{1}{v}\frac{dv}{dp}} \quad k = v\frac{dp}{dv}$$

$$v\frac{dp}{dv} = \frac{2a}{v^2} - \frac{rT\sqrt{2}\,v}{(v-v_0)^2} = \frac{2a}{v^2} - \frac{a\,v}{v_0^2(v-v_0)} = a\,\frac{2vv_0^2 - 2v_0^3 - v^3}{v_0^2v^2(v-v_0)}$$

$$= -\frac{a}{v_0(v-v_0)} = \frac{a^2}{r\sqrt{2}\,v_0^3\,T}$$

$$k = \frac{v_0(v-v_0)}{a} = rT\sqrt{2}\,\frac{v_0^3}{a^2}$$

$$\frac{k}{\alpha} = \frac{v-v_0}{r}\,\frac{3}{\sqrt{2}} \neq \frac{2(v-v_0)}{2} = \frac{3}{\sqrt{2}}\,T\sqrt{2}\,\frac{v_0^2}{a} = \frac{3}{\sqrt{2}}\,\frac{T\sqrt{2}}{\rho_0^2}\,\frac{1}{a}$$

$$a = \frac{r}{\rho}\,\frac{\sqrt{2}}{3}\,\frac{1}{\alpha}$$

$$k = \frac{rT\sqrt{2}}{\rho^3}\,\frac{\rho^2}{r^2}\,\frac{9}{2}\alpha^2 = \frac{9}{2}\,\frac{T\sqrt{2}}{\rho r}\,\alpha^2 \qquad k = \frac{2}{9}\,\frac{\rho r}{\sqrt{2}\,T\alpha^2} = \frac{\sqrt{2}}{9}\,\frac{r}{T}\cdot\frac{\rho}{\alpha^2} = \frac{\sqrt{2}}{9}\,\frac{R}{T}\,\frac{\rho}{\mu\alpha^2}$$

$$r = \frac{R}{\mu}$$

$$= \frac{1}{\mu}\,\frac{pv}{T} = \frac{1}{\mu}\,\frac{p}{\rho T} = \frac{1}{\mu}\,\frac{\overset{1000}{\cancel{76\cdot13.6}}\cdot 980}{0.00129\cdot 273} \qquad \mu = \frac{\text{Atomgew.}}{14.5}$$

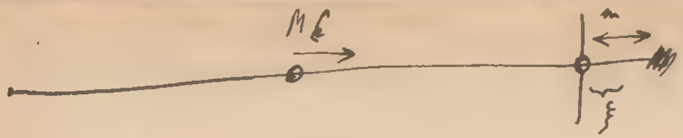
$$\frac{\sqrt{2}}{9}\,\frac{R}{T} = \frac{\sqrt{2}}{9}\,\frac{980.000}{(273)^2\cdot 0.00129}$$

2.4362	0.30103
4.8724	0.15051
0.1106 − 3	5.9912
0.9542	6.1417
2.9372	2.9372
	3.2045

$$C_n = \frac{\varrho}{\mu \alpha^2} \qquad \frac{8{\cdot}8 \;.\; 14{\cdot}5}{63{\cdot}3 \;.\; [168 \,.\, 10^9]^2} = \frac{14{\cdot}5}{\alpha \,.\, \frac{\alpha A}{\nu}} = \frac{14{\cdot}5 \;.\; 10^{15}}{168 \,.\, \cancel{} \,.\, 121 \,.\, \cancel{}}$$

$$\begin{array}{r} 3{\cdot}2045 \\ 1{\cdot}1614 \\ \hline 4{\cdot}3659 \end{array} \qquad \begin{array}{r} 19{\cdot}3659 \\ -\;4{\cdot}3081 \\ \hline 15{\cdot}0578 \end{array} \qquad \begin{array}{r} 2{\cdot}2253 \\ 2{\cdot}0828 \\ \hline 4{\cdot}3081 \end{array}$$

$$1150 \,.\, 10^{12}$$



$\xi = \cancel{M}\ a \sin \alpha t$

$\dot\xi\big|_0 = a\alpha = \cancel{A}v$

$$\overline{MC^2} = \overline{mc^2}$$

$$\frac{\overline{MC^2} + \overline{mc^2}}{2} + \overline{V} = \frac{MC_0^2}{2}$$

$$\overline{V} = \frac{\overline{mc^2}}{2}$$

$$C^2 = \frac{1}{3} C_0^2$$

$M = 2m$

~~$M+m$~~

$$\left.\begin{aligned} MV^2 + mv^2 &= MV_0^2 + mv_0^2 \\ MV + mv &= MV_0 + mv_0 \end{aligned}\right\}$$

$$M(V^2 - V_0^2) = m(v_0^2 - v^2)$$

$$\frac{M(V - V_0) = m(v_0 - v)}{V + V_0 = v + v_0}$$

$$\underline{V - v = -V_0 + v_0} \quad \Big|\, m \,\Big|\, -M$$

$$V = \frac{(M-m)V_0 + 2mv_0}{(M+m)} = \frac{V_0 + 2v_0}{3}$$

$$v = \frac{\cancel{(m-M)v_0 + 2MV_0}}{} = \frac{-v_0 + 4V_0}{3}$$

$$\frac{v_0(m-M) + 2MV_0}{m+M}$$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

$$\cancel{\alpha = \frac{\delta}{3\theta v}} \qquad \beta = \frac{2\alpha}{9}\,\frac{\delta^3}{b^2 R}$$

$$\cancel{p + \frac{a}{v^2} = \frac{R\theta\, v^2}{\delta}} \qquad \frac{9}{2} R\theta \ldots$$

Nach Jäger:

$$\left(p + \frac{a}{v^2}\right) = 9\,\frac{b^2 R\theta}{2(v-b)^3}$$

$$1 - \frac{2a}{v^3}\,\frac{dv}{dp} = -9\,\frac{b^2 R\theta}{2}\,\frac{3}{(v-b)^4}\,\frac{dv}{dp}$$

$$1 = \frac{dv}{dp}\left[\frac{2a}{v^3} - \underbrace{\frac{9\cdot 3\; b^2 R\theta}{2\;(v-b)^4}}\right]$$

$$\left(p+\frac{a}{v^2}\right)\frac{3}{v-b} = \frac{dv}{dp}\,\frac{a}{v^2}\left[\frac{2}{v} - \frac{3}{v-b}\right]$$

$$= \frac{dv}{dp}\,\frac{a\,[2v-2b-3v]}{v^3(v-b)}$$

$$\frac{v^2}{a} = \frac{2(v-b)^3}{9\,b^2 R\theta}$$

$$\frac{dp}{dv} = -\frac{a(v+2b)}{v^3(v-b)}$$

$$\frac{a}{v^2}(v-b)^3 = \frac{9}{2}\,b^2 R\theta$$

$$\left[-\frac{2a}{v^3}(v-b)^3 + 3\frac{a}{v^2}(v-b)^2\right]\frac{dv}{d\theta} = \frac{9}{2} R b^2 = \frac{dv}{d\theta}\; a\left(\frac{v-b}{v}\right)^2\left[\underbrace{-\frac{2(v-b)}{v} + 3}_{\frac{v+2b}{v}}\right]$$

$$\cancel{\left[-2a(v-b)^3 + 3av(v-b)^2\right]\frac{dv}{d\theta}\frac{1}{v^3}}$$

$$\cancel{\left[-2av^3 + 6av^2 b - 6avb^2 + 2ab^3 + 3av^3 - 6av^2 b + 3avb^2\right]\frac{1}{v^3}\frac{dv}{d\theta}}$$

$$= \frac{a}{v^2}\,\frac{(v-b)^3}{\theta}$$

$$\frac{1}{v}\frac{dv}{d\theta} = \frac{v-b}{v+2b}\,\frac{1}{\theta} = \alpha$$

$$\frac{1}{v}\frac{dv}{dp} = \beta = \frac{v^2(v-b)}{a(v+2b)} = \frac{\theta\alpha v^2}{a} = \frac{2\alpha}{9}\,\frac{(v-b)^3}{b^2 R}$$

$\alpha_{H_2} = \frac{1}{5500}$

$v - b = (v + 2b)\,\theta\alpha$

$1 - \frac{b}{v} = \left(1 + 2\frac{b}{v}\right)\theta\alpha \neq 3\theta\alpha$

~~280. 3~~

$\frac{300\,3}{5500} = \frac{9}{55} = \frac{1}{6}$

$b = \frac{5}{6} v$ [wenn α ... 0 / b ...

$R = \frac{pv}{\theta} = \frac{nm\bar{c}^2}{\theta\,3} = \rho\frac{c^2}{3\theta}$

$\cancel{(1855)^2} = \frac{(485)^2 \cdot 0.00129 \cdot 10^4}{3 \cdot 273}$

2·6857
5·3714
0·1106 − 3
5·4820 − 3
−2·9133
2·5687 = $370 \cdot 10^4$

2·4362
0·4771
2·9133

$\rho = \frac{2}{9}\alpha\left(\frac{1}{6}\right)^3 \frac{1}{370 \cdot 10^4} = \frac{2}{9}\,\frac{1}{216}\,\frac{1}{5500}\,\frac{1}{370 \cdot 10^4}$

0·9542
2·3345
3·7404
6·5682
13·5973
3010
13·2963

$= 2 \cdot 10^{-13}$!

$\rho_{H_2} = 0.00004 \cdot \frac{1}{13.6 \cdot 76 \cdot 980}$

$= 4 \cdot 10^{-12}$

also 20 ...

$(C_2H_5)OH$: $\alpha = 0{\cdot}0018$.

$$1 - \frac{b}{v} \neq \cancel{2\theta\alpha \quad 3.000 \cdot 0.0018}$$

$$= \cancel{1} \quad \text{~~[illegible] unmöglich!~~}$$

$$\theta\alpha = \frac{0{\cdot}54}{\cancel{1}}$$

$$\frac{b}{v} = \frac{1-\theta\alpha}{1+2\theta\alpha}$$

$$\frac{b}{v} = \frac{0{\cdot}46}{2{\cdot}08} = 0{\cdot}23$$

$$\rho = \frac{2}{9}\,\alpha\,(0{\cdot}23)^{3}\,\frac{1}{370.10^4} = \frac{\overset{0{\cdot}0004}{\cancel{0{\cdot}0018}}\,.\,(0{\cdot}23)^3}{370.\ 10^4}$$

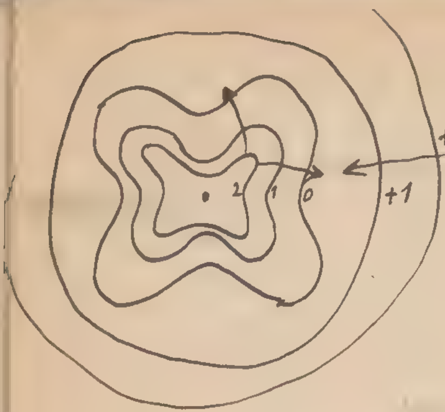
0·3617 − 1 . 3
0·0851 − 2
0·6021 − 4
0·6872 − 6
6·5682
0·1190 − 12

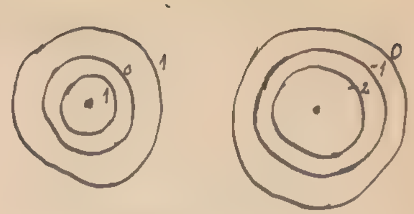
$$= 1{\cdot}3.\ 10^{-12}$$

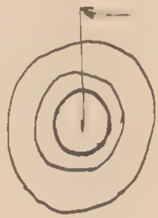
In Wirklichkeit:

$$0{\cdot}00009.\ 10^{-6} = 0{\cdot}9.\ 10^{-10}$$ fast 100 mal so groß!

α sollte mit θ kleiner werden, in Wirklichkeit umgekehrt







$$\varphi = \cancel{F_1} + F_2 + F_3 + \cdots$$

$$= \cancel{(x, y, z)} + \overset{a}{(}x^2 + y^2, z^2) + \cdots$$

$$X = -\frac{\partial \varphi}{\partial x} = -\left[\cancel{\frac{\partial F_1}{\partial x}} + \frac{\partial F_2}{\partial x} + \cdots \right. \qquad F_1 = 0$$

$$= - (A_2 x + B y + \cdots) - (A_3 x^2 + \cdots) -$$

of Ilm ~~…~~ Vxyz
hu stabil

$$\frac{\partial (F_3 + F_5 + \cdots)}{\partial x} < \frac{\partial (F_2 + F_4 + \cdots)}{\partial x}$$

$$-\tfrac{1}{2}(X x + Y y + Z z) = F_2 + \tfrac{3}{2} F_3 + 2 F_4 + \cdots \quad = L$$

φ soll < 0 sein $\quad F_2 + F_3 + F_4 + \cdots < 0$

$$F_2 + \tfrac{3}{2} F_3 + 2F_4 + \cdots > 0 > -\varphi$$

$$\underbrace{3F_3 + 5F_5 + \cdots}_{} < \underbrace{2F_2 + 4F_4 + \cdots}_{>0}$$

Dawniejsza t.g. bez uwzględnienia

Nowsza t.g. przy uwzględnieniu sił etc.

Fundamenta:

Maxw. ~~średniej~~ rozdział en. kin.

Maxw.-Boltzm. rozdział prędkości $\qquad h = \frac{1}{\theta}$

<u>Clausius Virial</u>

$$\bar{\varphi} = \int \varphi e^{-h\varphi} d\varphi \qquad \nabla \bar{\varphi} = \int \nabla\varphi . e^{-h\varphi}\varphi - h \int \varphi \nabla\varphi . e^{-h\varphi} d\varphi \; \underbrace{- \nabla h \int \varphi^2 e^{-h\varphi} d\varphi}_{>0}$$

$$\nabla(h\varphi) > 0$$

$$\nabla\left(\frac{\varphi}{\theta}\right) > 0$$

$$\int \nabla(\varphi e^{-h\varphi}) d\varphi < 0$$

$$\frac{2L}{3d^2(d-b)} - p - \frac{2}{d^3}\underbrace{Amn\rho \lg\frac{R}{a}}_{\rho} = 0$$

$$p + \frac{a}{v^2} = \cancel{p}\frac{R\theta}{d^2(d-b)}$$

$$\cancel{pv+a} = \frac{R\theta \quad \cancel{p_4}}{v\underbrace{\left[1-\frac{b}{d}\right]}_{\frac{1}{3}\frac{v-v_0}{v_0}}}$$

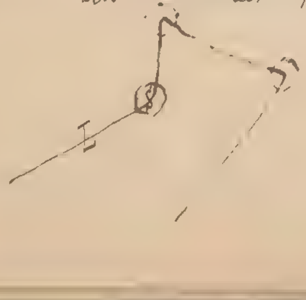
$$= \frac{R\theta \,.\, 3}{v-v_0}$$

Zanim przejdę do właściwego przedmiotu chciałbym z góry ~~zrobić~~ uczynić dwa zastrzeżenia 1). że w obec ogromnej ilości materyału, który się nagromadził w ostatnich latach, muszę zrezygnować z jednakowego traktowania całego przedmiotu i ograniczę się na pewne punkta, które mi się wydają ważniejsze, mianowicie, zupełnie pominę ~~muszę~~ teoryę roztworów, bo ta samo przez się już wyrosła do olbrzymich rozmiarów i, tworzy całość dosyć odrębną która zasługuje na ~~osobne~~ szczególne osobne omówienie, ~~a do tego nie jest w bezpośrednim związku z ... innemi zagadnieniami i~~

2) że nie myślę mówić jako adwokat teoryi kinetycznej bo to mi się wydaje niepotrzebnem. Wprawdzie walka przeciwko tej teoryi – rozpoczęta przez niektórych fizyków – czasami przyjmuje formy bardzo stanowcze, czego dowód następujący krótki referat o książce Ostwalda „Über Gastheorie" (w ogólnie znanem czasopiśmie niemieckim) który tutaj jako Curiosum chciałbym przytoczyć:

Ale przeważna część fizyków przecież przyznaje że zasadniczych błędów (w podstawie tej teoryi) nie wykazano, że ~~ta teorya~~ ona tłomaczy mnóstwo zjawisk inną drogą jeszcze niewyjaśnionych ~~jeszcze~~ i że doprowadziła (z mniejszą dowolności) do nowych wyników, zatem użyteczność jej nie podlega kwestyi. ~~Tego~~ Ogólnie więc bez wyjątków niemożna krytykować, co mi ~~jest~~ wydaje mi się niezupełnie uzasadnionem, bo sądzę że to się więcej przyda niż wyliczenie wszystkich formułek które na podstawie podobnych rozumowań w ostatnich latach zostały wyprowadzone, chociaż że może Panowie odniosą wrażenie iż tytuł mojego referatu musiałby raczej brzmieć: „Braki postępów na polu teoryi kin. materyi!"

Dawniejsza teorya kin. t.j. teorya skonstruowana przez Clausiusa i w znacznie ~~poprawionej~~ doskonalszej formie przez Maxwella była oparta na pojęciu tzw. drogi swobodnej t.j. drogi, ~~[illegible]~~ na której cząsteczka porusza się z jednostajną prędkością, w tym samym kierunku, aż następuje (dwutworne) spotkanie z inną cząsteczką.
(albo [illegible])



~~Ogólnie~~ Średnią miarą cząsteczek na tych kawałkach dróg przebytych ~~[illegible]~~ ważano 29 średnią ~~energią~~ wartości kinet. ogóln.; ~~[illegible]~~

nie uwzględniano 1). ~~zmiany~~ zmienności prędkości w bliskości drugiej cząstki t. j. w obrębie ~~[illegible]~~ działania

2). czasu trwania spotkania

3). drogi przebytej podczas tego czasu (w porównaniu z długością drogi średniej I)

jednym słowem nie uwzględniano rzeczywistości sfery sił wywieranych przez cząstki.

To postępowanie oczywiście ~~[illegible]~~ jest usprawiedliwione ~~[illegible]~~ jeśli gaz jest ~~rozrzedzony~~ stosunkowo rozrzedzony; jak wszystkie gazy przy ciśnieniu atm.

Tym sposobem wyjaśniono prawo Boyl-Marlo-Avogadro: $pv = R\theta$, a jakościowo wyjaśniono zjawiska tarcia wewnętrznego, przewodzenia ciepła i dyffuzyi. Wywód owego prawa jest niezależny od hipotezy co do składu sił działających przy spotkaniu podczas gdy ilościowe objaśnienie tamtych zjawisk wymagałoby ~~[illegible]~~ takiej hipotezy.

A oprócz tego zdołano ~~[illegible]~~ przepowiedzieć istnienie kilku specyalnych zjawisk, ~~[illegible]~~ które (prawa) dopiero a posteriori stwierdzono doświadczalnie ~~[illegible]~~

co w każdym razie korzystnie świadczy o użyteczności tych teoryi.

Do tego należy przedewszystkiem nagła zmiana (Discontinuität) w stanie gazu ~~w bliskości powi~~ otaczającego ciała stałe, która się okazuje ~~[illegible]~~ n. p. przy tarciu wewnętrznem i przy przewodzeniu ciepła (także elektryczności ?!).

Według zwykłej teoryi hydromechaniki cieczy lub gazów lepkich rozkład prędkości ~~musi~~ miałby być ciągły t.j. nigdzie nie mogą graniczyć ze sobą dwie warstwy ~~[illegible]~~ (~~różniące prędkością~~ ~~między któremi istnieje skończona~~ różnica prędkości ~~[illegible]~~ o skończonej prędkościami ~~[illegible]~~ różniącemi się o skończoną wielkość). ~~[illegible]~~

Tak samo przy przewodzeniu ciepła: ~~[illegible]~~ ~~[illegible]~~ ~~dotykających~~ temperatury dwóch ciał ~~[illegible]~~ dotykających się ~~były~~ musiałyby być takie same z obu stron ~~[illegible]~~ powierzchni ~~granicznej~~ styczności. Tymczasem ~~takie ślizganie się gazów wzdłuż powierzchni ciał stałych~~ praktycznie teorya gazów wykazuje że ~~podobne~~ następującej prędkości dla tej prędkości zjawisko ~~[illegible]~~ ~~[illegible]~~ ~~zachodzi~~ w gazach rozrzedzonych (nagła zmiana, skok, zachodzi więc na powierzchni oddzielającej gaz od ciała stałego, co tak dla tarcia wewnętrznego (Warburg) jak i dla przewodzenia ~~temperatury~~ ciepła ( ) zostało dowiedzionem. Różnica prędkości ~~stycznej~~ ciała stałego i warstwy stycznej gazu oznaczona jest przez $\Delta u = -\beta \frac{\partial u}{\partial n}$, a tak samo skok temperatury $\Delta \theta = -\gamma \frac{\partial \theta}{\partial n}$, przy czem β i γ są ~~[illegible]~~ współczynnikami (rozmiaru długości) proporcjonalnemi do ~~[illegible]~~ drogi swobodnej λ i nie wiele się od niej różniącemi.

	Cl.-M.	M.	[illegible]
η	$\sqrt{\theta}$	θ	
κ	$\sqrt{\theta}$	θ	
D	$\theta^{3/2}$	θ^2	

~~Obliczenie~~

~~Wywód wszystkich tych praw jest niemożliwy~~

~~Do wszystkich tych rezultatów~~ jakościowych

Do jakościowego objaśnienia wszystkich tych zjawisk wystarcza przyjęcie ogólnych zasad teorji kinetycznej, bez specyalnej hipotezy co do istoty sił działających między cząsteczkami.

Wiadomość tychże jest niezbędna do ilościowo dokładnych obliczeń, (tylko prawo Boyle-Charles-Avogadro tego nie wymaga).

Odnosi się to nie tylko do bezwzględnej wartości spółczynników tarcia wewnętrznego, przewodnictwa ciepła, dyfuzyi ale mianowicie także do ich zależności od temperatury (~~...~~ [illegible]) ~~[illegible]~~ aż dotąd tylko pod założeniem że siły są odwrotnie proporcyonalne do 5ej potęgi odległości zdołano wyrachować (Maxwell .. Boltzmann

~~[illegible]~~ Wszystkie dotychczasowe obliczenia tych wielkości ~~oparte na~~ (Clausiusa, Meyera etc.) (takie ~~[illegible]~~ gdyby były) ~~[illegible]~~ kulami oparte na hipotezie że cząsteczki wywierają siły sztywnemi są fałszywe, a właściwa droga do obliczeń tego rodzaju wskazana przez Boltzmanna jest niezmiernie mozolna.

~~Co do zmienności tych spółczynników w zależności od temperatury~~

~~Łatwo jednak wykazać że według drugiej teorji~~ Przeciwnie [illegible] że

Spółczynniki ~~k~~, jak ~~[illegible]~~ proste rozumowanie wskazuje, musiałyby być proporcyonalne do θ według pierwszego założenia, a do $\sqrt{\theta}$ według drugiego, podczas gdy ~~one w rzeczywistości~~ wartości znalezione między temi

6 granicami się schodzą. Zatem w rzeczywistości inne musi zachodzić

skombinował dla tego wystąpił pod tym względem

prawo siły. (Sutherland ~~przyjął co do tego~~ hipotezę, że owe cząstki

(pierwsze są to twarde, dające zawsze)

są kulami à la Clausius-Maxwell, które ~~oprócz tego jeszcze wzajemnie~~ jednak

t. zn. że przyjął cząstki typu

przyciągają się siłą $\frac{\mu}{r^4}$, ~~więc sprzeciwiają wszelkim hipotezom~~ Van d. Waalsa.

że dodanie

Pewnie ~~że~~ takich sił musiałoby mieć ten skutek że ~~cząstki~~

cząstki wydawałyby się mniej „twardemi" t. z. że sfera działalności

przy niższej temperaturze, ~~że mniejszej prędkości cząstek już w~~

~~takiej odległości by się~~ wydawałaby się większą aniżeli przy wyższej

temperaturze, i że wskutek tego zmienność owych spółczynników

ale czy to jest .. stosującem do uzasadnienia owego prawa?)

byłaby ~~mniejszą~~ większą aniżeli $\sqrt{\theta}$. ~~Teraz~~ Sutherland ma dwie

(zawartego)

stałe do dyspozycji: ~~wielkość promien~~ średnicę cząstek i stałą (C)

określającą wielkość siły, więc nic dziwnego że może je tak dobrać

dla każdego gazu

że ~~zgadzają się one~~ formułka $\eta = \eta_0 \frac{1+\alpha C}{1+\frac{C}{\theta}}\sqrt{1+\alpha\theta}$ zgadza się

wystarczająco z doświadczeniami. Ale praw tego rodzaju mnóstwo

by można znaleźć.

W prawie Sutherlanda w każdym razie jest zawarta słuszna zasada że 1
siły w większych odległościach mogą być przyciągające w mniejszych silnie
odpychające, ale już samo pojęcie kul sztywnych jest niejako naiwną abstrakcyą
którą używamy tylko do ułatwienia rachunków i łatwiejszego uzmysłowienia,
pojęcie z ~~dwoma~~ pojęciem sił przyciągających
a żeby prawo Sutherlanda łącząc to ~~ze ściśle~~ –
hipotezą
miało odpowiadać rzeczywistości, jest to ~~rzeczą~~ bardzo nieprawdopodobną
przeciwko której w dalszym ciągu jeszcze ważne objekcye czynić będziemy.
(jak w ogóle przeciwko sferom V.d.W.

(ponieważ ~~ściśle~~ przy zbliżaniu się do p. kr. droga swobodna już do tego samego rzędu wielkości się zbliży.

~~Nowsza teorya kinetyczna~~ Aby módz zastosować teoryę kinetyczną do
gazów zgęszczonych, do cieczy i ciał stałych niezbędnem jest uwzględnienie
gdyż te już w bliskim punkcie tego samego rzędu wielkości jest jak.
wielkości sfery działania sił odpychających co jest celem nowszej teoryi kinetycznej.
~~tam~~ używane
Dawniejsze teorye kin. ~~Met~~ Metody ukazują się ~~tu~~ ~~tam~~ niewystarczającemi do pokonania
tutaj
ogromnych trudności ~~z tąd~~ powstających; a tylko dwa ~~drogi znamy~~
~~aż dotąd, któ~~ instrumenta matematyczne znamy aż dotąd, które
na których
nam tutaj usługi oddawać mogą i ~~których~~ racyonalna teorya się
opierać musi t.j. Clausius'a twierdzenie o wiriale i Maxwella i
Boltzmanna prawo o rozdziale energii kinetycznej i prędkości w systemach
mechanicznych

~~Zajmiemy się najpierw drugą częścią twierdzenia, ponieważ pierwsza nie~~
~~podstawia żadnych nowych~~

Clausiusa twierdzenie o silniku odnosi się do systemu złożonego z punktów materyalnych między którymi działają siły XYZ
Oznaczając całk. en. kin. przez L wypisuje zasadę:

$$L \quad + \frac{1}{2}\sum(Xx + Yy + Zz) = 0$$

gdzie L oznacza całkowitą energię kinetyczną systemu punktów materyalnych, X, ~~YZ składowe całkowitej siły działającej na punkt~~ xyz, a suma (nazwane silnikiem) odnosi się do wszystkich cząsteczek w -

Dowodzi tego w bardzo prosty sposób zważając że pod założeniem stan systemu jest stały, jakaś będź suma funkcyi utworzona z prędkości i spółrzędnych musi być niezależna od czasu, zatem także

$$\frac{d}{dt}\sum m\left(x\frac{dx}{dt} + y\frac{dy}{dt} + z\frac{dz}{dt}\right) = 0$$ co przy wykonaniu różniczkowania

daje powyższy wzór.

Zastosowując to twierdzenie do gazu (lub cieczy etc.) poddanego jednakowemu ciśnieniu p w naczyniu o objętości V otrzymuje się, oddzielając p od reszty sił XYZ, (które mają teraz oznaczać wewnętrzne siły) otrzymuje się

$$L = \frac{3}{2}pv - \sum(Xx + Yy + Zz)$$

co służy jako podstawa do obrachowania równania zasadniczego izotermicznego (Zustandsgleichung) materyi.

Bezporównanie ~~[illegible]~~ ~~trudniejsze jest krytyka co do~~ większe trudności nastręcza zrozumienie i dyskusya ~~praw~~ twierdzeń M. i B.

~~Trzymając~~ Zachowując nomenklaturę wprowadzoną przez Bryana w swem cennym British Ass. Rep. 1894 nazywamy ~~[illegible]~~ prawem Maxwella co do rozdziału energii kin. następujące prawo:

Jeżeli w systemie dynamicznym konserwatywnym obierzemy ~~prędkości~~ spółrzędne p_1, p_2, p_3 tak, że energia będzie ~~wyrazić się~~ wyrażona ~~[illegible]~~ jako suma kwadratów ich (momentów) $\dot{q}_1 = \frac{dp_1}{dt}$ - dt prędkości: to w stanie stałym

$$T = \frac{1}{2}\left[\left(\frac{dp_1}{dt}\right)^2 + \left(\frac{dp_2}{dt}\right)^2 + \dots\right]$$

Średnia wartość każdego z tych kwadratów (t. j. części energii przypadającej na ruch p_n) będzie równa: $\overline{\left(\frac{dp_1}{dt}\right)^2} = \overline{\left(\frac{dp_2}{dt}\right)^2} = \dots$

Tych części będzie tyle ile system ma stopni swobody ruchu.

~~Maxwella~~ Maxwell-Boltzmanna twierdzenie zaś opiewa:

Prawdopodobieństwo że w takim systemie mechanicznym spółrzędne ~~[illegible]~~ mają wartości w obrębie $p_1 + dp_1$, $p_2 + dp_2 \ldots$ a prędkości $\dot{q}_1 + d\dot{q}_1 \ldots$ jest proporcyonalne do $e^{-hU} dp_1\, dp_2 \ldots dq_1\, dq_2 \ldots$

gdzie U = energia potencyalna w owym układzie spółrzędnych $p_1 \ldots p_n$

Od czasu co Boltzmann i Maxwell ogłosił ~~swe~~ te twierdzenia (1879), zadziwiające prostotą i ogólnością zastosowania, aż do dziś dnia ciągle jeszcze trwa dyskusya co do ścisłości tych twierdzeń. Oprócz Boltzmanna mianowicie Anglicy ogromne wszystkie czynili aby tę ~~[illegible]~~ kwestyę wyjaśnić ale mimo to jeszcze ~~kwestyę~~ nie można jej uważać za całkiem rozstrzygniętą.

Zarzuty zrobiono niektórym niekiedy rozumowaniom użytym przez Maxwella,* ale te udało się poprawić nie zmieniając wniosków ostatecznych. Słownie jednak przeciwnicy owych twierdzeń opierali się na tem, że można ~~[illegible]~~ wynaleźć systemy mechaniczne, które nie czynią zadość owym twierdzeniom, że zatem ~~[illegible]~~ nie mogą one być ogólnie ważne.

Tak n.p. Lorda Kelvina test case 1).

2).

Burnside Watson i Burbury tymczasem ~~jednak~~ starali się wydoskonalić metodę dowodu użytą przez Boltzmanna; później jednak Burbury przeszedł także na stronę sceptyków ~~[illegible]~~ inne prawo rozdziału prędkości w kilku wypadkach i w tym roku wreszcie ogłosił książkę która otwartą wojnę wypowiada tamtym twierdzeniom.

*) Np. E ... S

Burbury przedewszystkiem zwraca się przeciwko zasadzie użytej przez Boltzm[anna] do udowodnienia trwałości rozkładu prędkości.

~~Stosując~~ Boltzmann przyjmuje że przed spotkaniem dwóch cząsteczek prawdopodobieństwa ich prędkości ~~ich~~ są zupełnie niezależne. Więc wogóle jeśli $f(u,v,w_1)$ jest $du_1 dv_1 dw_1 \, dx_1 dy_1 dz_1$

leżące w obrębie dx dy dz

prawdopodobieństwo że cząsteczka ma prędkości u,v,w, , również $f(u_2 v_2 \dots)$ że jakaś cząsteczka ma prędkości $u_2 v_2 w_2$; to prawdopodobieństwo równoczesnego istnienia tych zjawisk jest oznaczone przez iloczyn $f(u_1 v_1 w_1) f(u_2 v_2 w_2) \, du_1 dv_1 dw_1 \, du_2 dv_2 dw_2$ ale właśnie tylko wtedy jeśli one są od siebie niezależne.

[Dalsze rozumowanie bardzo proste ; ilość spotkań jest proporcjonalna do tego wyrażenia przy spotkaniach zmieni się:

$$\frac{m}{2}(u_1^2+v_1^2+w_1^2)+U_1 + \frac{m}{2}(u_2+\dots)+U_2 = \frac{m}{2}(u_1'^2+v_1'^2+w_1'^2)+U_1'+\dots$$

więc jeśli ~~a ...~~ f ma ową postać eksponencyalną, to $f_1 f_2$ pozostanie $= f_1' f_2'$

z drugiej strony według znanego twierdzenia Liouville'a $\int du_1 dv_1 \dots d\dots = \int \dots$

$f_1' f_2' (du_1' \dots)$ ~~To~~ oznacza jednak równocześnie ilość spotkań przeciwnych t.j. takich które ~~...~~ zamieniają prędkości cząsteczek z $u_1' v_1' \dots$ na $u_1 v_1 \dots$ więc, ponieważ jeden proces będzie równoważony przez drugi to wogóle nie nastąpi zmiana w rozkładzie.]

Burbury jednak powiada że jeśli jakaś cząsteczka ma pewną prędkość, to ~~...~~ ona przez to już wpływa na prawdopodobieństwo prędkości w innych (w mniejszości sąsiednich) cząsteczkach, że tych zjawisk nie można zatem uważać jako od siebie niezależnych.

Zarzut ten zdaje mi się całkiem usprawiedliwionym. Dopóty gaz jest bardzo (nieskończ.) rozrzedzony, więc ~~długa~~ wielkość sfery sił (nieskończ.) mała w porównaniu z odległościami cząsteczek, oczywiście ruchy ich można przyjąć jako niezależne, ale to założenie nie będzie usprawiedliwionem w gęstych gazach (lub cieczach)!

Burbury jednak jeszcze dalej idzie. Usiłuje dowieść, że przyjęcie owego rozkładu B. M. doprowadza do ~~[illegible]~~ ~~owych~~ ~~tu~~ ~~innych~~ sprzeczności, które można tylko usunąć, przyjmując zamiast — nadzwyczajnie skomplikowaną funkcję która zawiera w liczniku sumę kwadratów prędkości takie

Rachunków, bardzo skomplikowanych Burbury'ego, przeciwko którym niejeden można zarzut podnieść, nie mogę tutaj ~~[illegible]~~ szczegółowo roztrząsać; wspomnę tylko że w jednym z ostatnich zeszytów Wiedemanna Annalen Zemplen Győző kwestye podniesione przez Burb. objaśnia w zadawalający sposób, tak że (domniemane sprzeczności znikają) ~~nie~~ ~~potrzebujemy~~ ~~jakiegokolwiek~~ ~~zmieniania~~ ~~do~~ ~~pozmieniania~~ ~~prawa~~ ~~B. M.~~ słuszność zarzutu przeciwko metodzie drugiej użytej przez B.), nie wynika z tego jednak żeby rezultat był fałszywy.

~~Ale~~ – (Możemy ~~także~~ przyznać ~~Zarzut~~ ~~przeciwko~~ metodzie drugiej użytej przez Boltzmanna ~~jest~~ ~~słuszny~~, ~~ale~~ ~~nie~~ ~~najściślej~~ (mamy jeszcze zupełnie odmienny sposób wywodu

Ostrzeżenia nas w tym zapatrywaniu to że)

oparty na pracy Maxwella i wydoskonalony przez ~~Ra~~ Lorda Rayleigha, który zawsze ~~..~~ stawał po stronie obrońców owego prawa.

Wystawmy sobie system mechaniczny oznaczony przez n współrzędnych $p_1 - p_n$. Jeżeli te współrzędne mają wartości dane nazwiemy to „konfiguracyą" systemu; ~~jeśli~~ ~~jeżeli~~ konfiguracya będzie się wciąż ciągle zmieniała, ale może znów stać się tę samą co pierwotnie. W danej konfiguracyi współrzędne ~~mają~~ ~~pewne~~ mogą mieć ~~...~~ prędkości, co oznaczamy przez wyrażeniem ~~..~~ „fazą". Wyobr. sobie jednocześnie wiele wiele systemów wszystkich możl. konf. i faz

Wyobraźmy sobie teraz równocześnie ~~ten sam system~~ niezmiernie wielką ilość takich samych systemów, które wszystkie są w tej samej konfiguracyi [albo właściwie w konfiguracyi wahającej się między granicami $p_1 - p_1+dp_1$ etc.] i ~~..~~ mają tę samą energię całkowitą [albo właściwie z energią ~~..~~ zawartą między E i ~~...~~ $E+dE$]. Wtedy jeszcze sposób w jaki tę energię całkowita składa się z energii kinetycznych pojedynczych ruchów etc. ~~...~~, ~~...~~ jest dowolny więc faza danego stopnia jest dowolna. Ułożymy prędkości w ten sposób, że wszystkie fazy będą zawarte, które ~~zgadzają się~~ odpowiadają dan~~ym~~ granicom energii, ~~i do tego w~~ ~~to w~~ a ilość systemów pewnej fazy oznaczamy przez założenie rozkłada w pewnym sensie równomiernego t.j. ilość systemów fazy zawartej między q_1 i q_1+dq_1 ma być proporcyonalna do $dq_1\, dq_2 \dots dq_n\, dp_1 \dots dp_n$

~~a takich gromad systemów wyobraźmy sobie jeszcze wiele wiele, że wszystkich możliwych~~

~~a ... a te systemy konfigura~~

Wtedy twierdzimy że cały ten układ będzie stały t.j. że ilość systemów pewnej fazy i konfiguracyi ~~pozostaje stała~~ nie zmieni się z czasem.

jeżeli pozostawimy ~~..~~ wszystkie systemy samym sobie, tak że nastąpi w nich ruch ...

Łatwo tego dowieść. Wskutek ruchu nie zmieni się całkowita energia tylko systemy obszaru $p_1 \; p_1+dp_1$ ~~zamienią się~~ przyjmą inną konfigurację i fazę: $p_1' \; p_1'+dp_1'$

$K\,F$ $\quad q_n \; q_n+dq_n$ $\qquad K'F'$ $\quad q_n' \; q_n'+dq_n'$

~~te systemy~~ ~~tło~~ ilość systemów, które dawniej miały ową konf. i fazę $K'F'$ była wedługo zatem $A\,dp_1' \ldots dq_n'$, ~~Wo~~ one wskutek ruchu ~~inne~~ zamieniły się w $K''F''$, ale za to przybyły owe ~~KF~~ systemy KF których ilość jest $A\,dp_1 \ldots dq_n$. Znane twierdzenie Liouvilla jednak opiewa że przy wszystkich ruchach mechanicznych $dp_1 \ldots dq_n = dp_1' \ldots dq_n'$ ~~[illegible]~~ nie zmienią się wskutek ruchu. Zatem następuje tylko cykliczna przemiana konf. i faz, tak że ilość każdego rodzaju pozostaje niezmienna.

Następuje pytanie ~~wiele między temi wszystkiemi systemami jest takich~~ ~~jakie np. q_1 ma pewną wartość $q_1 - q_1+dq_1$ a inne spółrzędne wszystkie mające wartości~~ ~~A~~ ~~jakie~~ jakie będzie prawdopodobieństwo —

Całkowitą ilość systemów znajdziemy: ~~[illegible]~~

$$\frac{\int dq_1 \ldots dq_n}{\ldots}$$

$$\int \ldots dq_n$$

$$T+V=E=V+\tfrac{1}{2}(p_1^2+\ldots p_n^2)$$

$$p_1\,dp_1 = dE \qquad dp_1 = \frac{dE}{p_1} = \frac{dT}{p_1}$$

$$dE\int\frac{dp_2\ldots dp_n}{p_1} = dE\int\frac{dp_2 \ldots dp_n}{\sqrt{E-V-p_2^2 \ldots p_n^2}}$$

przy czem ~~...~~ wiemy należy że $q_1^2 + \dots q_n^2 < T$

Otrzymuje się $N = \frac{[\Gamma(\frac{1}{2})]^n}{\Gamma(\frac{n}{2})} [2E-2V]^{\frac{n}{2}-1}$

a jako ~~stosunek~~ ilości systemów gdzie $p_1 \dots p_1 + dp_1$ do całkowitej liczby:

$$n_{p_1} = \frac{\Gamma(\frac{n}{2})}{\Gamma(\frac{1}{2})\,\Gamma(\frac{n}{2}-\frac{1}{2})} \left[1-\frac{p_1^2}{2T}\right]^{\frac{n}{2}-\frac{3}{2}} \frac{dp_1}{\sqrt{2T}}$$

[Naturalnie $\int_{-\sqrt{2T}}^{+\sqrt{2T}} n_{p_1} = 1$]

jeśli w tym wzorze zrobimy tu $n=\infty$ to

$$n_{p_1} = \frac{e^{-\frac{p^2 n}{4T}}}{\sqrt{2\pi}} \frac{dp}{\sqrt{\frac{2T}{n}}}$$

a jeśli stawiamy $\frac{T}{n}$ = średniej energii kin. jednej współrzędnej =

$$\cancel{\Gamma(\tfrac{n}{2}) = e^{-\frac{n}{2}} \left(\tfrac{n}{2}\right)^{\frac{n}{2}} \sqrt{n\pi}} \qquad \cancel{\Gamma(\tfrac{1}{2}) = \sqrt{\pi}}$$

$$\left(\frac{n}{2e\pi}\right)^{\frac{n}{2}} \sqrt{n\pi}$$

~~[illegible]~~

Trudność przy tym dowodzie polega na tem żeby wykazać ~~pod~~ czy i pod jakimi warunkami ten rozkład jest jedynie możliwym rozkładem.

Warunek konieczny do tego jest w każdym razie założenie Maxwella że system musi być tego rodzaju iż pierwej czy później przejdzie przez każdą konfiguracyę z każdą możliwą fazą. Zdaje się że to jest także warunek wystarczający; ~~możnaby~~ wprawdzie dotąd nie zdołano tego jeszcze dowieść ~~pokazać że~~ taki rozkład $f_n = f(p_1 \dots p_n)\, dp_1 \dots dp_n$ jest stałym, gdzie $f(p_1 \dots p_n) = c$ oznaczałoby jedną z całek ruchu

Nie jest zawsze wcale łatwo orzec czy dany system jest tego rodzaju, ~~tego wcale nie łatwo~~ iż spełnia ów warunek, ~~to wcale nie tak łatwo orzec.~~ Owe ~~dla~~ przedtem wspomniane przykłady które przeciwnicy przytaczali jako contra-argumenty mają to wspólne, że nie czynią zadość temu warunkowi więc nie można je uważać jako trafne. ~~Ostatni przykład to jeszcze~~ Są to ~~jednak~~ w pewnym sensie przypadki osobliwe.

Prosty przykład posłuży do objaśnienia.

ale niemożliwe tylko kąt

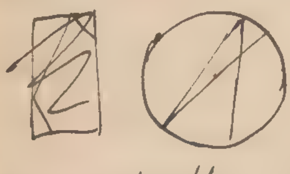
Wyobraźmy sobie ~~wszystkie~~ bilard, idealnie gładki tworzący idealny

stosowanie prawa zderzeń .. ale tylko pod pewnym warunkiem prowadzenia

~~nieskończenie~~. Jeśli rzucimy kulę w kierunku (~~idealnie~~) ~~prostopadłym całej do~~

~~jednej ściany~~, to ruch pozostanie periodyczny, prostodrożny, ona będzie się ~~...~~

ciągle w ten sam sposób odbijała. Jeśli rzucimy pod jakimś kątem, ~~nastąpi~~



odbijanie nastąpi w ten sposób, że kula z czasem po

~~...~~ całem polu koła przejdzie – z wyjątkiem ~~koła~~ obszaru

pewnego w środku — jeśli przypadkiem ten kąt nie był tak obrany

w którym to wypadku punkt

że $\varphi =$

(tak byłoby „zgrabnie" Boltzmann)

powracałby się tylko po przekątnych w bok. Prawdopodobieństwo żebyśmy

właśnie utrafili taki kąt (matematycznie ściśle) jest nieskończenie małe, ponieważ

jest nieskończenie więcej liczb niewymiernych niż wymiernych. Zatem jednak przez

każdy punkt płaszczyzny kula przejdzie tylko w dwóch kierunkach.

brzegiem

Jeśli jednak koło nie jest idealnie regularnym tylko w jakimś miejscu

ma małą nieregularność, ~~to z czasem~~ np. ~~spłaszczenie~~ eliptyczne to ~~z czasem~~

po każdym obiegu kuli kierunek jej trochę się zmieni i z czasem powierzchnia

zostanie pokrytą siecią dróg we wszystkich kierunkach. Wtedy wszędzie

~~rozmieszczenie~~ $\overline{u^2} = \overline{v^2}$ we wszystkich punktach ruchu.

będzie można

~~Ale Hipoteza M. B. co do rozkładu~~ prędkości ~~wszystkie jesteśmy~~ używać ~~nie można~~

~~bo nie ma się ... wszystkie możliwe konf.~~, ale

~~jeśli ... wszystkie kąt zderzające się~~

~~Inny~~ Dajmy ? na to że istnieją teraz wielka ilość takich punktów czyli kul które jednak na siebie nie działają (przechodzą przez siebie jak duchy).

Czy tu można użyć twierdz. O.M.? Oczywiście nie, bo każda kula nie zmieni ~~[illegible]~~ swojej chyżości początkowej więc system nie przyjmie wszystkich faz możliwych.

Dopiero jeśli przyjmujemy że one się zderzają jak kule sprężyste albo w jakiś inny sposób na siebie działają, wtedy następuje wymiana prędkości i wtedy ~~można~~ będzie się stosowało nasze twierdzenie.

Inny przykład: ~~[illegible]~~ Kule w kierunku prostym, odbijające się w A i B prędkości początkowe $c_1 c_2$ ...

A o o o B Jeśli masy równe, to zawsze ~~jedna~~ jakaś kula będzie miała prędkość c_1 jakaś inna c_2 etc. . Dopiero gdy jakaś przyczyna mogąca regularność n.p. mała różnica w masie, albo uderzenie ekscentryczne etc. nastąpi rozkład M.D.

Szczególnie ostrożnym należy być ze zastosowaniem M.D. do systemów drgających.

Można powiedzieć ~~warunek~~ warunkiem jest niestałość (Unstabilität) ruchów składowych, t.j. jeśli warunki początkowe trochę zmienimy to ruch (przy tym [illegible]) nie będzie się wahał koło ~~[illegible]~~ ~~stanu od~~ pierwotnego typu tylko będzie się coraz więcej od niego oddalał.

[Cechą jest [illegible] wartości jakiejś funkcji od [illegible] warunków coraz mniej zależnej]

Zdaje się że w gazach, cieczach etc. w każdym razie te warunki są spełnione z bardzo wielkim przybliżeniem

Że rozkład D.M. jest jedynym możliwym (rozkładem) typu D. dowiódł — ale tylko dla gazów rozrzedzonych zapomocą słynnego swego „Minimum theorem". ~~H funkcyi H.~~ Do gazów zgęszczonych etc. dowodu tego z przyczyn dawniej wspomnianych nie można zastosować, ponieważ używa obliczenia ilości spotkań (itd.) —

Polega on na ~~określeniu stanu~~ (uporządkowaniu) funkcyi $H = \Sigma f \log f$, którą Boltzmann identyfikuje z entropią gazu (z ujemnym znakiem).

Pokazuje on, że $\frac{dH}{dt}$ może mieć tylko znak ujemny, zatem H dąży do Minimum, gdzie $\frac{dH}{dt}=0$, i ~~stąd pokazuje się że z wyglądu~~ a z tego równania otrzymuje się jako ów rozkład stały, do którego gazy dążą : rozkł. D.M.

Przeciwnicy teoryi kinetycznej ~~...~~ zaprzeczali przedewszystkiem ~~...~~ słuszności tej interpretacyi prawa entropii.

Mówiono tak: ~~jeżeli oczywiście dam znak ujemny, to $\frac{dH}{dt}$ stanie się dodatnim, a mogę to uczynić, bo znaczy to tylko że wszystkim przed dążeniom~~ Jeżeli wszystkie dążności ($\frac{dx}{dt}$) obierze się ze znakiem ujemnym (więc tak same wielkie ale w przeciwnym kierunku) to mając (dt) uwzględnić to w owym wzorze dając dt znak ujemny, z czego by wynikało konsekwentnie $\frac{dH}{dt} > 0$.

~~...~~ To jest prawda, gdyby się wszystkim prędkościom dało znak ujemny, świat by się cofał w tył, ~~całe życie~~ wszystkie procesa następowałyby ~~...~~ wstecz

w przeciwnym kierunku i także prawo entropii trzeba by odwrócić.
Maxwella armia demonów mogłaby to urzeczywistnić my jednak nie możemy.
Teraz oni jednak mówią: prędkości dodatnie i ujemne są równie prawdopodobne
więc i równie prawdopodobnym jest żeby się natrafiły wszystkie jak zmniejszenia.

Otąd ten powstaje z tego że nie określamy bliżej co rozumiemy pod prawdopo-
w danym razie — jakie są te możliwe przypadki których ilość jest mianownikiem
we wyrażeniu dla prawdopodobieństwa

Jeżeli my obieramy n.p. cząsteczki mające pewne prędkości i (kładąc je do naczynia same) dajemy im różne kierunki — niezależnie od wielkości prędkości tak tam powiedzieć — to pewnie będzie więcej równie prawdopodobne że $\frac{dH}{dt}$ dodatnie jak ujemne, bo $\frac{dH}{dt}$ będzie właśnie $=0$

Tza najprawdopodobniejszy jest wtedy stan zupełnie „nieporządny", stan stały gdzie panuje rozkład O.M.

Byłoby n.p. rzeczą nadzwyczajnie nieprawdopodobną, żebyśmy przypadkowo do górnej części naczynia włożyli same cząsteczki ~~v. p.~~ obdarzone ruchem w kierunku $+X$ a do dolnej w kierunku $-X$. Jeżeli się to jednak stało, to będzie rzeczą nadzwyczajnie prawdopodobną że $\frac{dH}{dt}$ będzie ujemne

Jeżeli jednak system rzeczywisty fizyczny nam jest faktycznie dany, to ~~...~~
„prawdopodobieństwo" pewnego układu jego
musimy wziąć układ pewnych warunków początkowych jako dany, co do których
nie możemy mówić o żadnym prawdopodobieństwie, bo nie wiemy ~~o skutkach ...~~
jakie czynniki ~~jako~~ rozkład spowodowały, innemi słowami nie wiemy ~~...~~
historyi przeszłej systemu, więc wtedy ruchów może tylko znaczyć
prawdopod. że w pewnym momencie czasu syst. dany przyjmie owe ruchy
a co do tego nie mamy żadnego praw. twierdzić że one będzie równe
dla rozkładu ~~przypadkowi~~ ~~dodatnych i ujemnych~~ i analogicznego przeciwnego.
Teraz wyjaśnia się jeszcze lepiej ~~o skutek dyskusyi~~ przez uwzględnienie
twierdzenia ogólnego mechanicznego Poincaré'go że ~~wszystkie ruchy~~
ruch jakiegoś ~~układu~~ systemu ~~danego~~ (skończonego) konserwatywnego zawsze po upływie jakiegoś czasu
się powtórzy więc że będzie okresowy. Zermelo twierdził że wskutek tego ~~...~~
~~...~~ twierdzenie o entropii nie mogłyby zastosowane do takich systemów więc
że ~~...~~ ono nie da się wytłomaczyć mechanicznie. Boltzmann przeciwnie
~~odpowiedział~~ pokazał jednak że nie zachodzi żadna sprzeczność jeżeli się owe
prawo entropii uważa jako ~~...~~ nie jako twierdzenie ~~nie bezwzględne~~
ścisłe tylko jako twierdzenie o prawdopodobieństwie nadzwyczaj wielkiem tak jak wszystkie
nasze prawa fizyczne empirycznie poznane.

~~Wstały~~ Jeśli 1 cm³ powietrza i 1 cm³ wodoru pozostawimy wzajemnemu
działaniu, to one będą się przenikały wskutek dyfuzji i będą dążyły do
równomiernego rozkładu — ~~[illegible]~~ H będzie się zmniejszało. Prawo rozkładu
Boltz. jest jednak tylko prawem prawdopodobieństwa, więc także ten układ przy
którym połowa naczynia jest H_2, druga połowa powietrze, ma pewne prawdop.
nadzw. mało prawdop., zatem po upływie jakiegoś ogromnie długiego czasu
musi się zdarzyć (raz) znowu całkowite odmieszanie — co również wynika
z owego twierdzenia Liouville'a — ale czas ten jest tak ogromny, prawdop.
owe tak małe, że nie mamy żadnego widoku, ~~stan~~ abyśmy mieli raz sposobność
sprawdzić doświadczalnie takie zjawisko ~~o~~dmieszania choć jednego mm³.
Inny przykład : ~~kula z motocyklu nie [illegible] uderzając o [illegible]~~ ~~tak~~
~~zostaje tam przytrzymana aż [illegible] zamieni się w energię cieplną. Wtedy~~
weźmy na uwagę jakieś ciało stałe zwykłej temperatury. Z czasem, ponieważ
wszystkie kombinacje prędkości (zgadzające się z prawem energii) mają pewne prawdop. — nawet w drodze ~~owego~~
twierdzenia R B ~~[illegible]~~ wyraźnie robimy zastrzeżenie, że system przejdzie przez
wszystkie możliwe konfiguracje, — zdarzy się także że ~~wszystkie~~ prędkości
wszystkich atomów będą miały prędk. równy kierunek, wtedy ciało samo
przez się ruszy się z miejsca, mając temperaturę ~~0~~ bezwzgl. zero — ~~[illegible]~~ prawo entr.
zostanie przełamane, ale to tylko na chwilkę, bo potem znów nastąpi
niezmiernie długi czas ~~[illegible]~~ nieregularnego ruchu cieplnego ogólnie.

Wyż. Stan porządkowy (specjalnie) gazów ~~..~~ przed dyfuzyą, przy nierównomierności temperatury, ciśnień etc ~~itd. procesy~~ (~~przy których doświadczamy powstawanie entropii (zmniejszenie H)~~

~~połączony na~~ jest stosunkowo tak bardzo nieprawdopodobny [je molekular produkt] że w praktyce zawsze będziemy spostrzegali zwiększenie entropii (zmniejszenie H), ale nie wolno ekstrapolować tego prawa na nieskończenie długi przeciąg czasu; ~~i. ta jak to czasem czynią ogólnie~~ takie twierdzenia jak n.p. ~~o stateczności~~ że wszechświat ostatecznie osiągnie stan nieruchomości martwoty wskutek dyssypacyi energii, według Teoryi mechanicznej nie są uzasadnione.

Prawo Maxwella co do rozdziału energii kinetycznej wcześnie zastosowano do interpretacji stosunku ciepła właściwego $\frac{C}{c} = \kappa$ ~~przy gazach~~ gazów.

Jeżeli cząsteczka gazu ma n stopni wolności (n ruchów niezależnych) ~~to znaczy że~~ to przypadnie na nie energia $E = n\alpha$, ~~gdyż~~ ponieważ każdej spółrzędnej odpowiada równa ilość energii kin; z tego 3α na ruch postępowy, co się równa temperaturze θ, zatem $\alpha = \frac{\theta}{3}$, $E = \frac{n}{3}\theta$.

Oprócz tego uwzględnić należy (energię potencyjną) (która się może podwyższyć) przy podwyższeniu temp, n.p. wskutek zmiany odległości atomów składających się, $U = \nu\alpha = \frac{\nu\theta}{3}$ gdzie ν oznacza stałą (pewien stosunek) wielkość, i pracę wykonaną ~~przy~~ przeciwko ciśnieniu zewnętrznemu $\int p\,dv = \frac{m\bar{c}^2}{3} = \frac{2}{3}\theta$

Zatem $\frac{C}{c} = \frac{n+\nu+2}{n+\nu} = 1 + \frac{2}{n+\nu}$

Jak wiadomo Boltzmann stosował za pomocą tego wzoru wartości $k = 1{\cdot}66,\ 1{\cdot}4,\ 1{\cdot}33$ które napotykamy ~~po~~ u gazów jedno, dwu- i złożonych

1. Przyjmujemy jako kształt cząsteczek ~~kuli~~ kule całkiem gładkie ~~Stąd~~ wprawdzie mamy ~~6 stopni swobody ruchu~~ 3 spółrzędne ruchu postępowego i 3 ruchu obrotowego — ale tych ostatnich nie należy uwzględnić ponieważ, ~~jeżeli kule są gładkie~~ ruchy obrotowe wogóle nie będą żadnego miały wpływu, zatem $n = 3$, $\nu = 0$; $k = 1 + \frac{2}{3}$

2). elipsoidy (albo wogóle ciała) obrotowe $n=5, v=0;\ \kappa = 1{\cdot}4$

3). elipsoidy trójosiowe albo wogóle ciało sztywne dowolnego kształtu

$$n=6, v=0;\ \kappa = 1{\cdot}33$$

Mniejsze wartości stosunku κ łatwo wytłumaczyć przyjmując pewną ilość ruchów wewnętrznych, trudności tylko nastręczają te wielkie wartości κ. Trudności polegają na tem:

1). Stosunek osi elipsoidy trójosiowej nie zmienia stosunku κ, ale on ma wpływ na czas relaksacyi t.j. na przeciąg czasu potrzebny do osiągnięcia stanu stałego, czem mniejsza różnica między osiami, tem dłuższego czasu będzie potrzeba aż wszystkie ruchy się wyrównają.

Aby więc n.p. gaz miał $\kappa = 1{\cdot}4$, cząsteczki musiałyby być matematycznie idealnemi ciałami obrotowemi, gładkiemi, najmniejszy brak symetryi zrobi $\kappa = 1{\cdot}33$. Taka symetrya kulista nie jest jednak rzeczą ~~nie~~ ~~bardzo~~ prawdopodobną przynajmniej dla niektórych gazów n.p. N_2, które jest trój- albo pięcio wartościowe.

Więc Boltzmann sądzi, że owe wartości κ mogą być ewentualnie za wielkie, że gazy mają w rzeczywistości mniejsze κ, tylko że to z powodu wielkiego czasu relaksacyi uszło dotychczasowym badaniom.

Trudno sądzić o prawdopodobieństwie tego przypuszczenia dla tamtych gazów, w każdym razie otwiera się tu nowe pole do doświadczeń ~~dla badań doświadczalnych~~: badania zmienności ciepła właściwego wpływu czasu z czasem (Nachwirkung d. g W.), która

z promieniami odgrywa znaczną rolę u gazów wieloatomowych i może przyczynić się do wytłomaczenia licznych sprzeczności dotychczasowych rezultatów różnych badaczy.

2). Gdyby cząsteczki były ciałami sprężystemi, toby ~~jak~~ z czasem cała energia ruchu postępowego musiała się zamienić w drgania sprężyste tych cząsteczek, ponieważ ~~...~~ ~~jak~~ Kelvin powiada ciało sprężyste ma $n=\infty$ (niesk. wiele drgań fundamentalnych). Wprawdzie zdaje mi się iż zachodzi pewna wątpliwość czy założeni Maxwella tutaj jest ściśle spełnione, ale trzeba przyznać że istotnie energia ruchów wewnętrznych musiałaby być wszelkiego bezporównania wyższą aniżeli w rzeczywistości.

Oczywiście jednak owe pojęcie kul materyalnych było tylko wprowadzone do uzmysłowienia sfery działania sił i nie można myśleć o tem aby jemu przypisać objektywną egzystencyę. ~~Zastępując więc one całe sferami~~ Wracając więc do racyonalniejszego pojęcia sfery sił omijamy tę trudność. Ale z drugiej strony musimy przyznać że wśród cząsteczek mogą się odbywać jakieś ruchy drgające innego rodzaju, które powodują promieniowanie, ~~zatem musi posiadać daleko więcej stopni swobody ruchu~~ wiec ilość cząsteczkowych n przelicz ~~istotnie~~ ... u mniejsza.

Tu jednak w grę wchodzi ośrodek, eter, ~~w~~ na który owe ruchy się przenoszą. Jego wpływ nie można sumować w siły konserwatywne supponowane przy wywodzie prawa M., zatem do tych ruchów nie możemy stosować prawa rozdziału równego energii. Wkracza to w pole nieznanego związku promieniowania z teoryą kinetyczną.

A propos dowolnym zrobić uwagę na interesującą

Pod tym względem Stoney podaje myśl że fosforescencya może ~~być~~ polegać na takich ruchach wewnętrznych które mają stosunkowo długi czas relaksacyi. (są tylko w małym stopniu związane z innemi ruchami więc)

3). Jeżeli się sobie wyobrażamy atomy jako punkty materyalne otoczone sferą sił, cząsteczka jednoatomowa będzie miała $n = 3$, tak samo jak kula; cząsteczka dwuatomowa $n = 5$, ale tylko jeżeli odstęp dwuch atomów jest niezmienny; w przeciwnym razie ~~innym przypadku~~ $n = 6$; trzyatomowe w ogólnym razie $n = 9$ etc.

Więc jeżeli połączenie między oba atomami jest idealnie sztywne, matematycznie niezmienne, to będzie $n = 5$, $\kappa = 1{,}4$. Nie mogłoby być n.p. urzeczywistnione przez połączenie praktycznie fizycznie sztywne, zapomocą ciała sprężystego, bo wtedy byłoby $n = 6$ a oprócz tego jeszcze $v = 1$ (zob. później) –
Co to znaczy jednak: poł. matemat. sztywne, w mechanice występujące, gdzie połączenie urzeczywistnione tylko przez siły, a więc nie mamy
Warto ~~jednak~~ dla tego uwagę zwrócić na to, że możemy otrzymać $\kappa = 1{,}4$, (t.j. rezygnując z wprowadzenia połączenia idealnie sztywnego a więc zmienności energii pot. ujemny) także $n = 6$ / $v = -1$. Wielkość v może mieć takie wartości ujemne, bo niekoniecznie musi się powiększać wewnętrzna energia potencyalna przy podwyższeniu temperatury; ~~Ona pójdzie~~ v będzie dodatnie jeżeli siły są tego samego rodzaju jak sprężyste, będzie jednak ujemnem jeżeli n.p. przyciąganie według prawa $\frac{1}{r^2}$ bo wtedy musi się zmniejszyć ~~się~~ odstęp ~~atomy~~ (więc wielkość cząsteczek! zob. porzątek, Sutherland) przy podwyższeniu temperatury. jeżeli układ ~~będzie~~ na prawie stały

Supponując jako przykład ruch po obwodzie koła

$$\alpha = \frac{m}{2} r^2 \omega^2 \quad ; \quad \text{siła odśrodkowa} \quad m r \omega^2 = \frac{A}{r^\mu}$$

$$U = \frac{-A}{(\mu-1) r^{\mu-1}} = -\frac{m r^2 \omega^2}{(\mu-1)} = \frac{-2\alpha}{\mu-1} \qquad \text{więc } v = -\frac{2}{\mu-1}$$

N.p. $v=-1$

dla $\mu=3$

Nie zgadza naturalnie żeby to było wyrazem prawa istniejącego bo przykład ten polega na bardzo uproszczonych podstawieniach. Przy prawie Dulong i Petit'a o tem jeszcze wspomniemy; ale może nie być ścisłym – wspomnieć o [illegible]

Steigmüller ~~[illegible]~~ wprowadza takie potęgowanie absolutnie sztywne (tam gdzie $\kappa = 1\frac{2}{3}$ n.p.), przy innych gazach ~~[illegible]~~ inne znów potęgowanie sprężyste [gdzie $U =$], przez co otrzymuje wielkość $n+v$ zgadzającą się z faktycznem κ. ~~Nie możemy jednak żadnej przyczyny supponować siły wdzięcznej~~ Nie wyjaśnia to jednak wcale kwestji tu poruszonej; do tego znaczenie jego ~~sprężystego~~ niektórych ~~gdzie~~ potęgowania jako ~~się~~ sztywnego ~~[illegible]~~ innych jako sprężystych ~~[illegible]~~ jest bardzo dowolnem.

Powiedzieliśmy że zastosowanie twierdzenia o wiriale i twierdzenia M.B. tworzy jedyną drogę racjonalną do teorji kinetycznej gazów zgęszczonych i cieczy i ewent. ciał stałych.

Jak wiadomo Van d. Waals wywiódł swą znaną formułkę

$$p + \frac{a}{v^2} = \frac{R\theta}{v-b}$$

ze założenia że cząsteczki można ~~[illegible]~~ uważać jako kule sztywne między którymi działają jeszcze siły przyciągające których sfera jest ~~stosunkowo~~ wielka w stosunku do ~~odstępów~~ cząsteczek samych. On posługiwał się przy tem twierdzeniem

Albowiem pierwotny wzór wprawdzie oddaje ~~bardzo~~ w ścisły sposób jakościowo zjawiska zachodzące przy zgęszczaniu i skraplaniu gazów ale ~~dość~~ dalekim jest od ilościowej dokładności.

Pokazuje się to n.p. mianowicie w objętości krytycznej, v odpowiedniego punktu który krytyczny ~~krytyczny~~ ~~który określony~~ ~~jest przez~~ ~~t.j. punktów~~ warunki $\frac{\partial p}{\partial v}=0$, $\frac{\partial^2 p}{\partial v^2}=0$
[t.j. objętości owego punktu]

Z równania otrzymałoby się $v_k =$ t.j. $\frac{1}{2{,}8}$ części objętości, która by odpowiadała zwykłemu prawu Boyle Charles, podczas gdy liczne doświadczenia mian. S. Younga podały jako wartość owego stosunku $\frac{1}{3{,}7}$ – ..

Dieterici pokazuje że także z formy ( ) otrzymałoby się $\frac{1}{3}$, ~~…~~ ~~…~~ a uwzględniając dalsze ~~dodatnie~~ wyrażenia szeregu ( ) jeszcze wyższe wartości. Z tego on wnioskuje że w ogóle równanie formy nie może odpowiadać rzeczywistości.

Ten wniosek zdaje mi się przedwczesnym, bo nie mamy żadnej przyczyny do przypuszczenia że dalsze potęgi muszą być dodatnie; przeciwnie ~~…~~ wyrażenie / o którem dalej mówić przyjdzie p. n. mamy kilka mających wyższe / będzie miało znak – , więc może być że byśmy z ~~zupełnego~~ równania uzupełnionego otrzymali wartość przybliżoną do owego $\frac{1}{3{,}7}$

Dieterici podał dwa inne wzory:

$pv + \frac{a}{v^{k}} = \frac{R\theta}{v-b}$ i $pv =$

z których pierwszy dałby stosunek $\frac{v_k}{v} =$ a drugi

Pierwszy jednak nie jest wcale teoretycznie uzasadniony, a drugi jest oparty na formule z teorii cieczy Dieterici'ego, którą dalej będziemy krytykowali. Ani jeden ani drugi nie czyni zadość warunkowi ( ) $p=\infty$ $v=\frac{b}{3}$

Więc ~~teoretycznie~~ nie można ich uznać ze stanowiska teoretycznego a pod względem doświadczalnym wiemy tylko że zgadzają się w tym jednym punkcie z doświadczeniem, ~~dobr~~ ~~inne~~ ~~koniecznie~~ pod innym względem nie zostały one jeszcze bliżej sprawdzone.

Równanie musi mieć w każdym razie kształt $p + \frac{a}{v^2} = T\varphi(v)$, jeżeli uważamy cząstki za ciała sztywne jakiegokolwiek kształtu; wynika to z tego że można wszystkie prędkości zmniejszyć w pewnym stosunku, przez co nie zmieni się forma ruchu, tylko ciśnienie. (Rayleigh). Z tego atrakcyjnego się ~~pozost~~ pozostaje owe wyrażenie $\frac{a}{v^2}$, niezależnie od prawa działania, jeżeli sfera sił wielka w porównaniu do odległości cząstek. Gdyby była mała, to trzeba by je zastąpić jakąś funkcją v i T.

Jeżeli jednak cząstki nie są idealnie sztywne, ~~tylko~~ ~~...~~ t.j. jeżeli siły odpychające nie stają się ∞ .. to proporcjonalność ciśnienia

całkowitego do temperatury wstępu. Tak n.p. pod założeniem Maxwellowskich sił $\frac{A}{r^5}$ otrzymałoby się równanie (Boltzm.

$$pv + \frac{a}{v} = r\theta\left[1+\frac{b}{v}\right]$$

gdzie jednak b będzie proporcyonalne (do gęstości i, do potęgi $\theta^{-\frac{3}{4}}$.

Tego rodzaju jest także równanie Kastetta .. $\left(p+\frac{a}{v^2}\right)(\cancel{v-b})^4 = \frac{RT\,v^3}{(v-b)^4}$,

~~ponieważ tutaj a i v są zmienne~~ które Reinganum wywiódł jako rezultat empiryczny ze ~~licznych~~ spostrzeżeń*) (Younga co do Izopentanu) gdzie a i b są wielkościami zmiennemi:

$$a = A\,e^{\frac{0{\cdot}0345\,A\left[(v-2\beta)^3 + 12{\cdot}2\,\frac{\beta^4}{v}\right]}{\beta v^3 R\theta}}$$

$$b = \beta\,e^{\frac{0{\cdot}0726\,A\left[(v-2\beta)^3 + 3{\cdot}34\,\frac{\beta^4}{v}\right]}{\beta v^3 R\theta}}$$

Równanie to ~~jest~~ mimo swej skomplikowanej formy zawiera tylko trzy ~~wielkości~~ współczynniki stałe, i czyni zadość ~~spełnia~~ twierdzeniu o stanie corresp. ; ~~jest~~ stosuje się ono z wielką dokładnością do doświadczeń, ale ~~teoretycznie~~ nie ma żadnych teoretycznych podstaw, ~~i~~ więc może być uważane tylko jako empiryczna formułka.

*Rozumowania teoretyczne p. Reinganum są błędne, mianowicie miarą temperatury jest średnia energia kinet. cząsteczek wzdłuż całej drogi, a nie tylko w częściach drogi gdzie siła = 0.

Jeśli nie udało nam się

Z góry można przewidzieć, że zastosowując pojęcie V. d. W. do cieczy nie możemy ~~nam otrzymać~~ się spodziewać pomyślnych rezultatów, dopóki nie znamy ~~[illegible]~~ sposobu ~~obliczenia~~ dokładniejszego wartości ( ) owego szeregu z większą dokładnością, ponieważ tam $\frac{b}{v}$ już znaczną musi mieć wartość (zazwyczaj >1). ~~Wystarcza to do [illegible]~~ Z tej przyczyny też ~~obliczona~~ teorya kinetyczna cieczy w ~~ujęciu Voigta~~, w tej formie jak została rozwinięta na podstawie zasad V.d.W. przez Voigta, Dieterici'ego i Milnera, jest bez praktycznej wartości. Voigta pierwotna teorya była zupełnie teoryą jakichś „idealnych" absolutnie ściśliwych cieczy, ponieważ nie uwzględniała wcale wielkości sfery ich odpychających, tj. uważała cząsteczki jako punkty nie jako kule. W drugiej rozprawie wprowadził Voigt ~~[illegible]~~ zwykłe pojęcie V. d. W. co do mnożnika szeregu $(1+\frac{b}{v}\dots)$ ale przeprowadzenie teoryi — pominąwszy nawet ów ważny zarzut — jest nieścisłe, oparte na przybliżeniach wrodzonych obliczeniu teorii Clausiusa i Jeansa ~~[illegible]~~ których ogólnie trudno ocenić. To samo stosuje się do teoryi Dieterici'ego i Milnera.

Jako słuszną uważamy ~~podstawę~~ zasadę, że także w cieczy średnia energia kinet. jest miarą temperatury, ale Voigt i Diet. nie przyjmują ~~[illegible]~~ jej ~~tego~~

jako rezultat prac Ross.-O. tylko stosuję się jej dowieść — w sposób co prawda stanowczo ~~nieprawidłowy~~ nieścisły.

Celem tych prac jest przedewszystkiem obliczenie ~~jednostek~~ ciepła utajonego parowania. Voigt (i Drude) otrzymuje wzór:

$$L_i = \frac{1}{2} \frac{R\theta}{\mu} \left[\lg \frac{v_f^z - b}{v - b} + \frac{b}{v - b} - \frac{b}{v_f - b} \right]$$

gdzie L_i oznacza ciepło parowania wewnętrzne t. j. po odliczeniu odpowiedniej pracy zewnętrznej, v =

Okazuje się, że w rzeczywistości L_i jest prawie dwa razy tak wielkie z wielkim ciężarem. Milner także rzeczywiście otrzymuje ze swoich obliczeń podwójną wartość owego wyrażenia, ale rezultat ten polega na ~~błędzie~~ omyłce, mianowicie

Można sobie ściśle oszczędzić wszystkie rozumowania kinetyczne co do tej wielkości L_i ~~zważając że~~ robiąc użytek z ~~wzoru~~ warunku termodynamicznego dla ~~stanu~~ równ. parowania ciecz-para: $v_e - v_g = \int_{v_g}^{v_e} v\,dp$ i zważając że

$\frac{a}{v}$ =

co daje nam pod założeniem równania pierwotnego V.d.Waals

Niezgodność z rzeczywistością zaś nie może nas wcale dziwić ponieważ nie można myśleć to [illegible] równanie samo zastosować do stanów gdzie $\frac{b}{v}$ ma wartości 217

Wychodząc z formy która jest już o jeden stopień dokładniejsza,

otrzymuje się wzór Boltzmanna:

ale zapewne i to nie będzie dokładnie spełnione.

~~Jäger zdaje się, jest~~ Ścieśnienie Zgęszczenie nadzwyczaj wielkie cząsteczek w cieczy, w skutek którego $\frac{b}{v}$ staje się tak wielkie zostało przeglądnięte, o ile wiem, jedynie przez Jägera; o równaniu jego wspomnimy przy innej sposobności.

W skutek owych wzorów obliczenie ~ ciepła parowania w ogóle jest rzeczą zadowalającą jednak ~~będzie zależnem od~~ dane jest nam (równanie zasadnicze.

Całkiem nowego obliczenia jednak wymagałyby zjawiska tarcia wewnętrznego przewodnictwa ciepła i dyfuzji dla cieczy i gazów zgęszczonych. Łatwo jednak zrozumiemy że trudności tu powstające nie zostały jeszcze pokonane gdyż nawet dla gazów doskonałych nie zdołano jeszcze tych rachunków wykonać ~~przy~~ ~~przyjmując~~ teorię kul elastycznych (a znów metody Maxwella nie można zastosować jeśli ~~ciśn~~ zgęszczenie jest znaczne). Ale możemy się spodziewać że osiągniemy jakichś rezultatów przynajmniej co do zmienności tych wielkości z temperaturą — którą tam dla gazów dosk. w ~~całkiem~~ prosty sposób można obliczyć.

I tutaj jeśli ciecz zachowuje stałą objętość, wszystkie ruchy pozostaną geometrycznie podobne jeżeli ~~je~~ wydłużeń ich w pewnym stosunku α powiększymy t. zn. ruchy cząsteczkowe i ~~ruchy~~ ruchów ~~widocznych~~ cieczy samej). Wtedy to jednak wszystkie ~~ciśnienia~~ natężenia w elementach cieczy się powiększą w stosunku α^2 a temperatura w stosunku α^2.

Więc również natężenia styczne tarcia wewnętrznego.

N. p. $p_{xy} = \alpha^2 (p_{xy})_0$ $= \cancel{\eta}$ z czego wynika tak samo jak przy gazach idealnych:

$$(p_{xy})_0 = \eta_0\left(\frac{\partial u_0}{\partial y} - \frac{\partial v_0}{\partial x}\right)$$

$$p_{xy} = \eta\left(\frac{\partial u\alpha}{\partial y} - \frac{\partial v\alpha}{\partial x}\right) = \alpha^2 \eta_0\left(\frac{\partial u_0}{\partial y} - \ldots\right)$$

więc $\eta = \alpha\eta_0 = \eta_0\left(\frac{\theta}{\theta_0}\right)^{\frac{1}{2}}$

Tutaj jednak tarcie nie jest niezależne od ~~...~~ gęstości jak przy gazach, więc ~~następuje ...~~ przy stałym ciśnieniu trzeba jeszcze uwzględnić zmienność w skutek powiększenia średnich odległości cząsteczek ~~...~~ z podniesienia temperatury.

$$\frac{d\eta}{d\theta} = \left(\frac{\partial\eta}{\partial\theta}\right)_v + \left(\frac{\partial\eta}{\partial v}\right)_\theta \left(\frac{\partial v}{\partial\theta}\right)_p$$

$\frac{\partial v}{\partial\theta}$ jest współczynnik rozszerzalności termicznej

a $\frac{\partial\eta}{\partial v}$ można obliczyć ~~dla kilku cieczy~~ jeżeli znamy ~~zmienność~~ zależność $\frac{\partial\eta}{\partial p}$ tarcia wewnętrznego od ~~...~~ ciśnienia i ściśliwość cieczy $\frac{\partial v}{\partial p}$: $\frac{\partial\eta}{\partial v} = \frac{\frac{\partial\eta}{\partial p}}{\frac{\partial v}{\partial p}}$

Znamy te wartości ~~...~~ n. p. dla Benzolu i Eteru dla których otrzymujemy n. p. dla $\theta = 20°$: $\frac{d\eta}{d\theta} =$ wz.

podczas gdy w rzeczywistości:

Więc pokazuje się kompletne prawo naszej formułki zatem `zasady V. d. Waals – Clausiusa, t. zn. że nie można cząstek uważać jako kul sprężystych, ~~co się tu [illegible]~~ ścianny, to już z odpowiednich wzorów dla gazów doskonałych, ale tutaj sprzeczność założenia i niezgodności występuje ~~dale~~ jeszcze jaskrawiej.

W dalszym ciągu wrócimy jeszcze do ~~[illegible]~~ omówienia wniosków z tej przyczyny się nasuwających.

Należałoby tu jeszcze obszerniej pomówić o teoryi kinet. Jägera który w licznych ~~rozprawach~~ pracach starał się wybudować teoryę kin. cieczy na podstawie pojęć V. d. W. Jakkolwiek tam w szczegółowych problemach dużo jest myśli zawartych ~~[illegible]~~ ciekawych ~~[illegible]~~ [n. p. ~~[illegible]~~ ciągłe używanie korzystanie ze związku między stałymi [illegible] a wielkością $\frac{a}{v}$ (w którego [illegible] Stefana)] nie myślę jednak wchodzić w ~~[illegible]~~ specyalne jej rozstrzygania bo ~~[illegible]~~ jest w sprzeczności ~~[illegible]~~ z fundamentalną ~~zasadniczą~~ zasadą (B. [illegible]) iż energia kinet. w cieczy i w gazie jest równą. Jäger sądzi że ona w stanie w którym jest znacznie mniejszą & stara się wyprowadzić różnicę na podstawie swej teoryi.

W kinetycznej teorii cieczy więc nie postąpiliśmy jeszcze daleko,
Kinetyczna teorya ciał stałych jednak jeszcze mniej jest wykończona,
~~dodać nam jeszcze o zdaniu~~ nie zgodzono się nawet jeszcze co do
głównych zarysów które tej teorii trzeba nadać.

Pod względem ~~punktem~~ jednak tutaj w korzystniejszej jesteśmy sytuacyi
niż wobec cieczy t.j. że mamy ~~[illegible]~~ prawo tak zadziwiającej
ogólności i prostoty – choć nie dokładnie spełnione – jak prawo Dulong a
i Petita co do ciepła właściwego.
ciepła stał.

A te regularności zdołano wytłumaczyć w sposób zupełnie zadowalający
jedynie z przyjęcia że cząsteczki albo raczej atomy poruszają się koło (średniej pozycji) jakiejś równowagi
wychodząc ~~z ogólnych zasad~~ ~~[illegible]~~
~~zasad teorii kinetycznej~~ –

Można użyć w tym celu albo (Riecke)
~~Riecke w tym celu~~ użyć twierdzenia co do wiriału, z którego wynika, ~~stosując~~:

pomijając pv jako małe:

$$\bar{L} = -\tfrac{1}{2}\Sigma(Xx+Yy+Zz)$$

~~Uwzględniając tarcie cząsteczek~~

$$= +\tfrac{1}{2}\Sigma\left(x\frac{\partial U}{\partial x}+y\frac{\partial U}{\partial y}+z\frac{\partial U}{\partial z}\right)$$

albo też wprost powołać się na prawo Maxwella B., według którego:

$$\bar{U} = \frac{\int U e^{-hU}\,dx\,dy\,dz\ldots}{\int e^{-hU}\,dx\,dy\,dz\ldots}$$

gdzie h = odwrotn. temperatury

Jeżeli cząsteczki poruszają się około pewnej średniej pozycji równowagi to siły w pewnym dostatecznie małym obrębie będzie można przyjąć jako proporc. do wychylenia, więc U jako funkcyę drugiego stopnia współrzędnych, ~~stąd~~ co według jednego i drugiego wzoru daje $\bar{U}=\bar{L}$ (prawo zachodzące przy wszystkich drganiach sprężystych (Huygens)) więc też zmiana energii pot. będzie równa zmianie energii kinet., zatem całkowita zmiana energii będzie dwa razy tak wielka jak gdyby $U=0$; t.j. (dwa razy tak wielkie jak ~~przy~~ w gazach jednoatomowych t.j. ciepło właściwe jednego atomu)

Uzupełniając wzór funkcyami jednorodnemi wyższych stopni otrzymałby się:

$c = 6..$

Czem większe wychylenia cząsteczek poza obręb sfery gdzie siły można uważać za proporcyonalne do odstępów, tem mniej dokładnie będzie się ~~sprawdzać~~ sprawdzało to prawo, więc różnicy największej można oczekiwać u tych pierwiastków

1). które mają najmniejszy ciężar ~~pot.~~ atomowy, bo te przy równej temperaturze muszą mieć odpowiednio większe prędkości, więc i wychylenia ~~mogą~~ większe.

2). które mają małą wartość objętości atomowej, więc małe odstępy atomowe, ponieważ tam siły w małym obrębie mogą być zmienne.

Ta reguła ~~sprawdza~~ zgadza się z ~~rzeczywistością~~ najzupełniej z doświadczeniem jak to wskazuje następująca tablica

Wpływ ciężaru atomowego jeszcze lepiej uwydatnia się w ~~...~~ niższej temperaturze

Dziwne wszakże jest nadzwyczajnie mała wart. dla C, tam trzeba by chyba $\frac{\Delta \Gamma}{d\vartheta}$ przyjąć jako ujemne, co jednak przy ciałach stałych trudniej usprawiedliwić ~~jak~~ niż przy gazach. Boltzmann tłumaczy to zjawisko znów ~~opóźnieniem~~ ~~stałych~~ ~~...~~ zbytnią wielkością czasu relaksacji niektórych ruchów i z tego powstającą „Nachwirkung". Warto by było ~~wyłącznie~~ rozstrzygnąć tę kwestyę doświadczalnie.

że ciepła właściwe się zmniejszają (ze zniżeniem temp. (mimo zmniejszenia prędkości)) wskazuje na przewagę wyrażeń F_3 etc... wyższych potęg więc na bardzo szybkie wzmaganie się sił odpychających przy zbliżeniu się cząsteczek.

Staigmüller ~~podaje~~ wywodzi także prawo Koppa-Neumanna co do ciepła właściwego związków chemicznych w ten sam sposób jak prawo Dul. Pet.

nie tłumaczy jednak wcale czemu tutaj trzeba obierać dla niektórych pierwiastków zupełnie inne wartości ciepła właściwego np.

aniżeli wtedy gdy one są ~~nieoznaczone~~ niepotęgowe.

Według mojego zdania ~~wskazuje~~ zmusza nas to ~~bezpośrednio znowu do na potrzebę~~ do przyjęcia asymetryi sił działających, wskutek której siła (i potencyał) zależy nie tylko od odległości cząsteczek ale też od orientacyi ich względem pewnych kierunków. O tem jeszcze więcej będziemy mówili.

~~Opiszę tego pr.~~ ~~Zastosowania~~ Wyszczególniwszy Pominąwszy te zjawiska dyfuz. [illegible] resultaty dotychczasowe ~~na~~ ~~nasze~~ teoryi kinet. na innych polach fizyki ciał stałych są prawie równe zeru.

~~Sutherland próbował wyjaśnić równanie zasadnicze ciał stałych~~

Zdaje się że jedyna próba zastosowania ~~teoryi~~ pojęć kinet. do ciał stałych została podjęta właśnie przez Sutherlanda, ~~kt.~~ oczywiście mówi pod zetknięciem cząsteczek à la Van der Waals.

Poprawiając ~~[illegible]~~ kilka błędów w jego rozumowaniu możemy tak wnioskować:

Z twierdzenia o virialu wynika:

$$p + \frac{a}{v^2} = R\theta + \overline{\Sigma\Sigma\, r f(r)}$$

przyczem $\overline{\Sigma\Sigma\, r f_r} = \frac{1}{t}\int_0^t \Sigma\Sigma\, r f_r\, dt = \cancel{\sigma \Sigma \rho}\ \sigma\, \Sigma m\rho$

$\sigma =$ ~~[illegible]~~ średnica

gdzie $\Sigma m\rho$ oznacza sumę momentów (ilości ruchu) wymienionych (prędkości ~~[illegible]~~ względnych składowych w kierunku osi zderzenia

między cząsteczkami ~~podczas~~ przez zderzenie podczas jedności czasu.

~~Σy możemy zastąpić przez $N\bar{y}$ gdzie N = ilość zderzeń~~

Wprowadzając średnią wartość $\bar{y}$, ilość zderzeń N, i drogę swobodną λ będziemy jednak mieli:

$$\Sigma y = N\bar{y} = \frac{\bar{c}}{\lambda}\bar{y} \quad \text{a wstawiając} \quad \bar{y} = \frac{\sigma}{\sigma_3} = \frac{\sigma\sqrt{2}}{3} \quad \text{(Boltzm..}$$

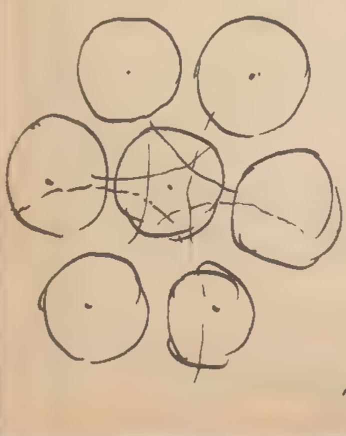
otrzymujemy:

$$pv + \frac{a}{v\,p} = R\vartheta\left[1 + \frac{\sqrt{2}}{3}\,\frac{\sigma}{\lambda}\right]$$

chodzi teraz o wyrażenie ~~...~~ wartości średniej drogi λ;

co do tego Sutherland to samo założenie robi jak Jäger w odpowiednim rachunku dla cieczy t.j. że $\lambda = d - \sigma$.

Jest to urzeczywistnione, jeżeli wystawiamy sobie wszystkie cząsteczki w porządku normalnym z wyjątkiem jednej ruchomej, i udzielając im podwójne promienie a uważając ruchomą cząsteczkę jako punkt otrzymujemy rysunek



[2] którego pokazuje się że ~~miejsce wolne dla ruchu~~ miejsce wolne dla ruchu ruchomej cząsteczki przybliżenie będzie kulą o owym promieniu. (Jäger Winkelm

W rzeczywistości trzeba sobie jednak cząsteczki sąsiednie wyobrazić we wszystkich możliwych, nie tylko tych normalnych porządkach; λ w skutek tego ~~...~~

otrzyma inną wartość, ale zawsze jeszcze przybliżenie podobnej wielkości.

Wstawiając $\frac{d}{\sigma} = \sqrt[3]{\frac{v}{v_0}} = 1 + \frac{1}{3}\frac{v-v_0}{v_0}$ otrzymujemy wreszcie uważając $v-v_0$ jako małe:

$$p + \frac{a}{v^2} = \frac{r\theta\sqrt{2}}{v-v_0}$$

(p. 544)

Sutherland ~~sprawdza się przytacza explicite swoje równanie~~ równanie jest, ale symbole ~~jego~~ równoważne ~~ze~~ do

$$p + \frac{a}{v^2} = \frac{3r\theta}{v-v_0}$$

, co polega na przeoczeniu czynnika $\frac{\sqrt{2}}{3}$.

Z tego równania otrzymuje się związek między współczynnikiem rozszerzalności α i ściśliwością $\frac{1}{\kappa}$:

$$\kappa = \frac{\varrho}{\sqrt{2}} \frac{\theta}{R\rho} \alpha^2$$

który jednak zupełnie nie sprawdza się w rzeczywistości, obliczone wartości ~~ze~~ ściśliwości są kilkadziesiąt ~100 razy za małe!

Sutherland myśli wtedy poprawić zgodzenie zakładając promień cząsteczek zależny od ~~zmienny z~~ temperatury, ale obliczenia i hipoteza jego są już bardzo problematycznej wartości.

Uderzającym może się wydać fakt, że jego ~~[illegible]~~ równanie dla cieczy [illegible] ~~[illegible]~~ z [illegible] otrzymuje wychodząc z [illegible] jego ~~[illegible]~~ powinien był otrzymać zupełnie te same równanie i [illegible] tego równania (a naszego) polega tylko na innym, zdaje mi się mniej ścisłym, sposobie ~~[illegible]~~ pominięcia wielkości [illegible] ~~[illegible]~~ tego pracy. Jeśli jednak to równanie jest ważne dla cieczy, jak wtedy Sutherland wytłumaczy sprężystość postaciową?

Istotnie [illegible] dotyczy [illegible]

[illegible] ktore. Jest to [illegible] już ~~[illegible]~~ [illegible]

~~[illegible]~~ [illegible] stosunku [illegible]

[illegible] w [illegible] wewnętrzne [illegible]

[illegible] jako jeden [illegible] jednak [illegible]

które [illegible] to [illegible]

stały [illegible]. [illegible] ciągle [illegible]

ułożenie [illegible] — [illegible]

[illegible] raz.

Możnaby sobie ze ~~[illegible]~~ [illegible] do rozmaitych

[illegible] ale [illegible] ~~[illegible] od~~

[illegible] ~~[illegible]~~ [illegible]

[illegible]. Tak samo także [illegible] rezultaty

[illegible]

[illegible]

~~[illegible]~~ Potem punkt ten sam i [illegible] ~~[illegible]~~ ułożenie [illegible] do

~~[illegible]~~ [illegible].

[illegible] ~~[illegible]~~

[illegible] dopiero tylko dwie stałe [illegible] doświadczalnie [illegible] ~~[illegible]~~ [illegible] które by [illegible]

[illegible] pozwolił [illegible] że [illegible] tylko [illegible]

[illegible] ~~[illegible]~~

[illegible]

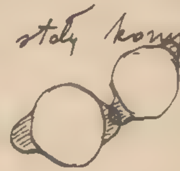
Pasmują się jednak pewne wątpliwości, czy jesteśmy do tego także uprawnieni. [illegible] mamy [illegible] ~~[illegible]~~ [illegible] [illegible] które są [illegible] statycznemi; byłoby rzeczą nadzwyczaj ważną rozstrzygnąć czy ~~[illegible]~~ ([illegible]) forma ~~którą formę~~ umieściłby także zachodzić w [illegible] teorji cząsteczkowej.

Należy bowiem podkreślić że sam akt krystalizacji nie [illegible] jeszcze do wyjścia [illegible] ~~[illegible]~~ [illegible] na siebie. ~~[illegible]~~ Średnia przyczyna cząsteczek stanowi ważną [illegible] [illegible] silnego [illegible] i sił [illegible] [illegible] bardzo silnie ~~[illegible]~~ (przy zbliżeniu) ~~[illegible]~~ (z danej przestrzeni) ~~[illegible]~~ będą usuwane [illegible] Stąd [illegible] (jak jest) [illegible] ~~[illegible]~~ [illegible] [illegible] w ~~[illegible]~~ nadzwyczajnym [illegible] [illegible] rozmaitych [illegible]

zajmujemy się takiemi układami. Jeśli cząsteczki są kulami, to powstają t.zw. agregaty [illegible], składające się z czworościanów i ośmiościanów  w których każda cząsteczka [illegible] z 12oma sąsiedniemi i ten system ~~[illegible]~~ posiada [illegible] [illegible] osi równo-prawnione przedstawia nam trzeci przykład systemu regularnego, bez przyjęcia asymetrji w cząsteczkach składających samych.

Z drugiej strony przyznać należy że ~~tak~~ pomijając krystalizację dużo innych zjawisk nas i tak już skłania do przyjęcia sił kierunkowych, przedewszystkiem zjawisko chemiczne. Chemicy, mówiąc o różnej wartościowości pierwiastków którą to czasem widać ~~schem~~ schematycznie-naiwnie reprezentowaną przez haczyki , supponują w rzeczywistości takie atrakcyjne siły kierunkowe.

Boltzmann te pojęcia ujął we formę matematyczną w swej teorji dysocyacji gazów; wystawia on sobie wykreślone na powierzchni kuli, która ma przedstawiać cząsteczkę, pewne obręby „empfindliche Bezirke", które na siebie wywierają siły zwykle nieznaczne ale przy zupełnym zbliżeniu nadzwyczaj wielkie, tak że dwa atomy ~~ulegną~~ połączą się i utworzą stały kompleks, jeżeli ich „empfindliche Bezirke" się przecinają :



Boltzmann pokazuje też na przykładach, że teorya jego zgadza się z doświadczeniem a z drugiej strony stwierdza że nie możnaby otrzymać teoryi dysoc. na podstawie sił które by były tylko f.(r), bo ~~wtedy~~ w takim razie jeżeli n.p. ilość cząsteczek dwuatomowych ~~[illegible]~~ przeważa nad ilością jednoatomowych ~~to~~ również musiałaby być także wielka ilość cząsteczek trzy, cztery etc. atomowych, — ~~[illegible] doświadczenie~~ co się sprzeciwia z doświadczeniem.

Jeśli się nie przyjmuje sił kierunkowych w cząstce, trudno także sobie wytłumaczyć, co właściwie spowodowuje nagłe przejście ze stanu ciekłego do stanu krystalicznego ~~przy~~ w punkcie ~~topliwości~~ topn. krzepnięcia.

Przyjmując asymetrię sił łatwo zaś pojąć że przy ~~tt~~ w tej samej temperaturze, więc przy tej samej energii kinetycznej, dwa mogą istnieć układy ~~stałe~~ o różnej wartości funkcji U, jeden gdzie osi są nieregularnie rozdzielone (ciecz) a drugi gdzie mają pewne kierunki (kryształ) i gdzie ta nieregularność zostaje utrzymaną przez ruchy nieregularne cząsteczek) przez co ~~siły~~ powstają siły orientacyjne które ograniczają swobodę ruchu cząsteczek.

~~Ciepło~~ Ciepło utajone krzepnięcia odpowiada stąd zmianie U wskutek orientacji osi – również jak ciepło utajone skroplenia odpowiada zmianie U wskutek sił włoskowatych ($\frac{a}{v}$).

Za tą hipotezą przemawia też fakt że istnieją t. zw. kryształy ciekłe

n. p.

t. j. ~~substancje~~ kryształy które ~~przy pewnej~~ wysokiej ~~temperaturze mają~~ z podniesieniem temperatury tak się zmieniają że przedstawiają ~~nieraz~~ nieraz nawet tak potężne jak lepkie ciecze) tarcie wewnętrzne ~~tak małe jak lepkie ciecze~~ ale zachowują jeszcze wszelkie (które) optyczne właściwości struktury krystalicznej. Dopiero przy pewnej oznaczonej temperaturze naraz (podobnie zatem) ustaje, ~~[illegible]~~ z czem jest połączone pochłonięcie pewnej ilości ciepła utajonego

Także Tammann, któremu zawdzięczamy wiele nadzwyczaj cennego materyału co do ~~stanu~~ zjawisk przejścia ze stanu ciekłego do stałego, ~~[illegible]~~ podtrzymuje hipotezę kierunkowości itd. i wnioskuje z niej, że nie istnieje żaden punkt krytyczny dla tej przemiany, ponieważ w takim punkcie ~~nie tylko~~ równocześnie ~~ciepło~~ ciepło utajone i zmiana objętości ~~ale~~ musiałoby być $=0$, co ~~[illegible]~~ są zjawisko do pewnego stopnia nieciągłe, jeżeli się przyjmuje że to ostatnie zależy także od orientacyi cząsteczek. Rzeczywiście doszedł on przy jej bardzo blisko do punktu gdzie ~~[illegible]~~ $v_1 - v_2 = 0$

a r - r wcale nie dążyło do wartości 0.

Trzeba się tu starać jak najwięcej o dalszy materyał doświadczalny.

Nie mówiliśmy wcale jeszcze o ciałach stałych t.zw. ~~[illegible]~~ bezpostaciowych. To co zwykle tą nazwą określamy są tylko agregaty krystaliczne, ciała mikrokrystaliczne jak n.p. metale w zwykłej formie. Dużo innych substancyj jak n.p. wosk, szkło, żywice itd. ~~[illegible]~~ są mieszaninami różnych ciał. Ale istnieją także kilko ciał ~~elementarnych~~ czystych, jednolitych, bezpostaciowych które można uważać jako ciecze n.p.

Według ~~X~~ trafnego wyrażenia Tammana należy je sobie wystawić jako ciecze przechłodzone (unterkühlte Flüssigkeiten). Nie różnią się zasadniczo od cieczy, nie mają żadnej struktury ani kierunkowości, ale tarcie wewnętrzne stało się tak wielkie że (swoboda ruchu jest tak mała) nie mogą ~~więcej nabrać~~ (cząsteczki) już nabrać orientacji regularnej. Takie ciała istotnie nie mają oznaczonego punktu topliwości. One mięknę stopniowo przy ogrzaniu i nie ma żadnej temperatury gdzie by nastąpiło nagłe pochłonięcie ciepła utajonego.

Vogt - Jamuszkiewicz

Mówię o teorji kinet. ciał stałych nie dlatego ponieważ tej sposobności aby wskazać na doświadczenia nadzwyczajnie ważne i interesujące

Sir R. Austena i Springa co do dyfuzji metalów stałych.

Nie ma zjawiska któreby wyraźniej przemawiało za kinetyczną naturą ~~wszystkich~~ ciał stałych jak fakt że one dyfundują w siebie ze szybkością którą można nawet dokładnie mierzyć.

Tak n. p. Roberts Austen ~~położył~~ przycisnął słup ołowiu do płyty ~~złotej~~ jako podstawy i ~~trzymając~~ zachowując cały system w pewnej temperaturze aż do 30 dni, mógł się przekonać potem za pomocą analizy chemicznej że złoto przeniknęło, czasami w znacznych ilościach aż do danego końca (

Przytaczam kilka wartości dla współczynnika dyf.

Jest to zupełnie naturalne według teorji kinet. ponieważ cząsteczki mają wszystkie możliwe drgania się czasem także tak wielkie, że opuszczają swe pozycje średniej równowagi i wędrują dalej jak cząsteczki cieczy.

Spring pokazał znów że proces ten zwykle bardzo leniwy przyspieszony przez wielkie ciśnienie n. p. Pb — Sn przyłożone do siebie i poddane ciśnieniu 10000 atm. przy. utworzyły warstwę stopu grubą 6 mm w ciągu kilku godzin.

Z proszków takich metalów które (się wogóle ściskać dadzą) tworzą ~~wogóle~~ ciało można otrzymać drut przez samą kompresyę bez wpływu ogrzania.

Można więc powtórzyć stare filozofa w dosłownym sensie:

Παντα ῥεῖ.

Jeżeli ~~tu~~ na koniec rzucimy raz jeszcze okiem na rezultaty pozytywne tu-tej omawiane nowej teoryi kinetycznej materyi — pomimo że plan właściwie jest dokonały — to będzie się nam wydawało może iż plon nie był zbyt obfitym. Ale myślę, że i negatywne rezultaty mają swoją wartość i również też liczne znaki zapytania, które ~~sam~~ przy omówieniu różnych ~~kwestyi~~ teoryi musieliśmy umieszczać, i które nam wskazują, jakie kwestye należy uważać za najważniejsze ~~najpilniejsze~~ i w którym kierunku należy głównie pracować.

Pod tym względem chciałem ~~nieco~~ wskazać na problematy (teoryi kinetycznej ciał

stałych, ponieważ sądzę że racyonalne teorye kin. materyi od tego muszą rozpocząć, i ~~[illegible]~~ myślę że tu (pierwej) dojdziemy do ważniejszych rezultatów niżeli w teoryi cieczy. ~~Ale~~ ~~Ważnie~~ Nadzwyczaj ważnem byłoby, co do tego przedewszystkiem nagromadzenie dalszego materyału doświadczalnego mianowicie co do ~~stałych~~ sprężystości kryształów i ciał właściwych bezpostaciowych (nie krypto-krystalicznych, te są bez wartości) przy bardzo niskich (wysokich) temperaturach, dalej ~~badanie~~. [illegible]; badania [illegible] bo ~~[illegible]~~ teraz najmniej nam jest znaną: fizyka ciał stałych.

$-\frac{1}{v}\frac{\partial v}{\partial p} = 0.000170$ 0.000650

ith. Om.

424
315
110 : 37 = 3
Om. 15 % 0.015

19.3 14.5
18.9 11.7
0.4 2.8
Ath.

28 : 20
14 : ~~14~~ 13.1 - .1

$\alpha = 0.00118$ 0.00148

0.00073

$\frac{\partial \eta}{\partial T} = 0.00093$

$-\frac{\cancel{0.00093}}{\cancel{0.00065}} \cdot \cancel{0.00118}$ 0.00009
5

$-\frac{\cancel{0.00073}}{17} \cdot 0.00148$

837 : 5

~~0.00118~~
- 0.00167
+ 0.00182
+ 0.00015

148.73
1036
444
1080 : 17 = 63
60

- 0.0063
+ 0.0018
- 0.0045

Przewodnictwo cieplne proszków musi zależeć od ~~stałe~~, jakości gazu

→  otaczającego i od temp. spręż. [illegible] zatem od ciśnienia.

Można sobie to wystawić w przybliżeniu pod obrazem rzędu blaszek o grubości podobnej jak ziarnka proszku:

→  

k przewodn. ciała stałego

K " gazu

χ " proszku

$$k \frac{d\theta_s}{dx} = K \frac{d\theta_g}{dx} = \chi \frac{d\theta_p}{dx}$$

$$\delta \frac{d\theta_s}{dx} + \varepsilon \frac{d\theta_g}{dx} = \frac{d\theta_p}{dx}(\delta+\varepsilon)$$

$$\delta + \varepsilon \frac{k}{K} = \frac{k}{\chi}(\delta+\varepsilon) \qquad \frac{\delta}{k} + \frac{\varepsilon}{K} = \frac{\delta+\varepsilon}{\chi}$$

albo nazywając μ procentową zawartość (względem objętości) ciała stałego, zatem $\frac{\varepsilon}{\delta+\varepsilon} = 1-\mu$:

$$\frac{\mu}{k} + \frac{1-\mu}{K} = \frac{1}{\chi}$$

Wskutek skoku temp. zmieni się to na:

$$\delta \frac{d\theta_s}{dx} + (\varepsilon + 2\gamma)\frac{d\theta_g}{dx} = (\delta+\varepsilon)\frac{d\theta_p}{dx}$$

$$\frac{\delta}{k} + \frac{\varepsilon+2\gamma}{K} = \frac{\delta+\varepsilon}{\chi} \qquad \frac{\mu}{k} + \frac{1-\mu}{K} + \frac{\frac{2\gamma}{\delta+\varepsilon}}{K} = \frac{1}{\chi}$$

N. p. dla powietrza przy 760 mm: $\lambda = 10^{-5}$ cm

$\gamma = 1{\cdot}7 . 10^{-5}$ cm

Jeżeli kule to przybliżenie: $\mu = \frac{3}{4}$

$$\frac{1}{\lambda} = \frac{1}{K}\left[\frac{1}{4} - \frac{3{\cdot}4 .\ 10^{-5}\,\mathrm{cm}}{\delta+\varepsilon}\right]$$

N. p. $\delta+\varepsilon = 0{\cdot}1$ mm

$$\frac{1}{\lambda} = \frac{1}{K}\left[\frac{1}{4} - 3{\cdot}4 .\ 10^{-3}\right] = \frac{1}{4K}\left[1 - 1{\cdot}3 .\ 10^{-2}\right] \qquad 1{\cdot}3\,\%$$

$\delta+\varepsilon = 0{\cdot}01$ mm $\qquad \frac{1}{\lambda} = \frac{1}{4K}\left[1 - 1{\cdot}3 . 10^{-1}\right] \qquad 13\,\%$

~~$\delta+\varepsilon = 0{\cdot}001$ mm~~

$\delta+\varepsilon = 0{\cdot}001$ mm tutaj λ byłoby 10 razy większe zatem już nie można zastosować takich obliczeń, ale ~~odtąd zaczyna się~~ to jest granica ~~gdzie~~ poniżej której przewodnictwo będzie się zmniejszało proporcyonalnie do ciśnienia gazu, a nie będzie zależnem od grubości ziarn (jeśli są $< 0{\cdot}001$) i ciśnienia proszku

Jaka jest średnia prędkość ~~[illegible]~~ środka ciężkości dwuch cząstek?

$$f(u_1 v_1 w_1) = A\, e^{-h(u_1^2+v_1^2+w_1^2)}\, du_1\, dv_1\, dw_1$$

$$\iiint\!\!\iiint A^2 e^{-h\left[(u_1^2+v_1^2+w_1^2)+(u_2^2+v_2^2+w_2^2)\right]}\, du_1\, dv_1\, dw_1\, du_2\, dv_2\, dw_2\, \frac{1}{2}\sqrt{(u_1+u_2)^2+(v_1+v_2)^2+(w_1+w_2)^2}$$

$$e^{-h(V_1^2+V_2^2)}\quad V_1^2\, V_2^2\, \sin\theta_1 \sin\theta_2\, d\theta_1\, d\theta_2\, dV_1\, dV_2\, d\varphi_1\, d\varphi_2 \sqrt{V_1^2+V_2^2-2V_1V_2 \ldots}$$

Jeśli przybliżenie: średnia prędkość c

$$\iiiint \sin\theta_1 \sin\theta_2\, d\theta_1\, d\theta_2\, d\varphi_1\, d\varphi_2\, \frac{1}{\sqrt{2}}\sqrt{1-\cos\begin{smallmatrix}\theta_1\varphi_1\\ \theta_2\varphi_2\end{smallmatrix}}$$

$$\iiint$$

Emulsye można uważać jako roztwory, kropelki drobne w nich zachowują się jako molekuły. Dyfuzya.

A zatem kropelki same jako gas.

$\frac{1}{\rho} dp = -g\,dz \qquad p = \frac{p_0}{\rho_0} \rho$

$\rho = \rho_0 e^{-\frac{g \rho_0}{p_0} z}$

$\frac{980 \cdot 0{\cdot}0012}{980 \cdot 76 \cdot 13{\cdot}6} = 10^{-6} z_{cm} = 10^{-4} z_m$

$\frac{p}{\rho} = R\theta \qquad \theta$ to samo

R w stosunku ciężarów molek.

jeżeli n. p. $\delta = 0{\cdot}1\mu$ (kropelki) to będzie w nich cca 10^9 molekułów

zatem stała gazowa

$\rho = \rho_0 e^{-10^3 z_{cm}}$

więc już w 1 mm wysokości ρ prawie zero

jeżeli jednak $\delta = 0{\cdot}01\mu$ to będzie $\rho = \rho_0 e^{-z_{cm}}$ zatem cząsteczki tej wielkości wogóle nie ~~…~~ osadzą się, jeszcze w odległości kilku cm $\rho > 0$

„Fällen" przez dodawanie sól etc. polega na tem że się zmienia elektrolityczne, że one się łączą w większe kropelki.

Rudzki w ostatnich latach ogłosił szereg prac ~~w których rozwija matematyczną teorię ciężkości ziemi z uwzględnieniem~~

~~Praca poniższa~~ Jest to czwarta z szeregu prac, w których Rudzki rozwija matematyczną teorię ciężkości ziemi, tworząca dalszy ciąg wyprawy

~~[illegible]~~ O kształcie fali sprężystej w pokładach ziemskich. Rozp. XXXIII t.

Rudzki ~~rozważa uważa~~ zauważył, że to pokłady że ziemi nie można uważać ~~jako ciała~~ za ośrodki izotropowe. ~~[illegible]~~ ziarnistości ~~[illegible]~~ ze względu na ich strukturę ~~[illegible]~~ które skąd, przy Nie polega to na tego rodzaju zjawiskach, gdzie chodzi o stosunkowo długie fale, nie wchodzi w rachunek, ale na uwarstwowaniu ich i na wpływie ciśnienia górnych warstw na dolne, wskutek którego sprężystość w kierunku pionowym będzie różna od sprężystości w kierunku poziomym, (a zatem równanie ~~[illegible]~~ ~~[illegible]~~ sprężystości, ~~[illegible]~~ 5 spółczynników musiały będą zawierały ~~5 spółczynników spręż.~~ (zamiast 2 jak w ośrodkach izotrop.)

~~wynika że fale powstające wskutek drgania w jednym punkcie nie będą miały kształtu kuli jak w ośrodkach izotr. Lecz będą one składały z~~

Ruch drgający który powstanie wskutek wstrząśnienia w jednym punkcie nie będzie się rozchodził równomiernie we wszystkich ~~[illegible]~~ kierunkach, ~~jako~~ tworząc fale kuliste jak w ośrodkach izotropowych, tylko z prędkością różną w różnych kierunkach.

Obliczenie ~~kształtu~~ wykazuje, że powierzchnia fali ~~będzie się składała z dwóch~~ rozpada się na dwie części

D. elipsoidę obrotową i ~~III~~ II inną powierzchnię obrotową, której kształt

określony jest przez równanie ~~bardzo skomplikowane, to~~ 6go stopnia.

~~Nie znając jeszcze z doświadczenia~~

Nie rozporządzając jeszcze doświadczalnemi danemi co do wielkości

owych 5 spółczynników, ~~Rudzki~~ nie można ~~ścisłego~~ badania dalej

~~Rudzki oblicza jednak kształt owej~~

przeprowadzić w ścisły sposób; na podstawie bardzo prawdopodobnych

założeń co do owych wielkości ~~jako przykład~~ (wykonywuje ~~jednak~~ Rudzki) jednak ~~jednak Rudzki~~

dyskusyę owego równania ~~ten rachunek~~ (na podstawie

wynika z niego, że owa druga powierzchnia znów rozpada się na dwie

części, wewnętrzną która się rozchodzi ~~z prędkością podług~~ mniej więcej tak samo prędko jak [illegible]

fala i zewnętrzną rozchodzącą się z prędkością ~~prawie podwójną~~ tegoż

zgoła wyższą (mniej więcej podwójną!) Jednoczesne wstrząśnienie środka sprowadzi zatem we wszystkich (...) punktach

~~skutkiem wstrząśnienia jednoczesnego punkt~~ ziemi trzy (...) wstrząśnienia

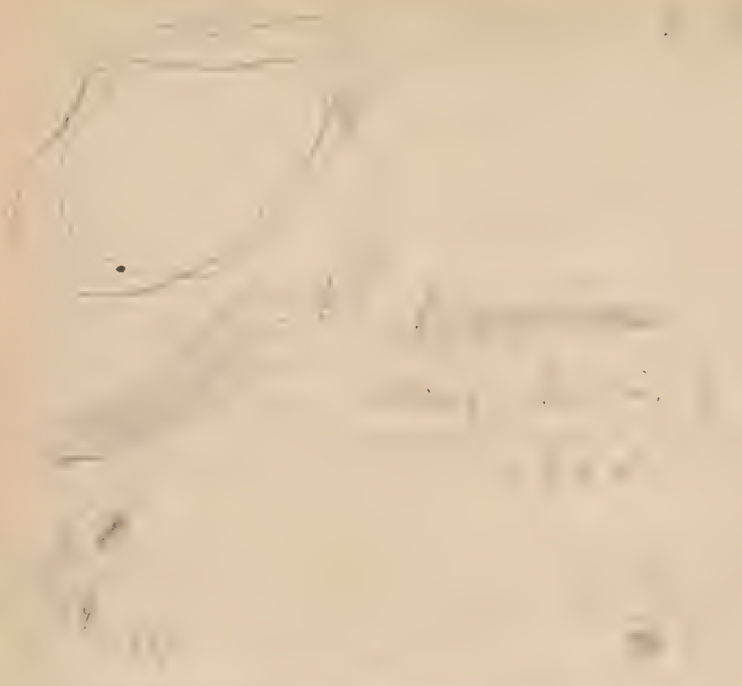
~~Każdy punkt ziemi (z wyjątkiem punktów osobliwych) dozna więc trzech punktów~~

~~jednoczesnego wstrząśnienia środka,~~

(~~uwzględniając odbicie~~) wstrząśnień ~~skutek~~

~~z których każde się różni~~ które nie będą zresztą (ani wyłącznie dylatacyjne

(jak fale głosu) ani torsyonalne (jak ~~fala~~ fale światła) tylko charakteru mieszanego.



Le mouvement brownien est donc un phénomène général d'autant plus sensible que la viscosité du liquide est plus petite. Le point le plus important est la régularité du phénomène ; des milliers de part. ont été examinées et dans aucun cas on n'a vu une part. en suspension qui n'offrît pas le mouvement habituel, avec son intensité ordinaire, eu égard à la grosseur de la particule.

Lumière n'a pas d'influence

Des particules de même grosseur mais de nature diverse solides, liquides, gazeuses, sont animées de mouvements peu différents.

Très rapide à la limite de visibilité, beaucoup plus que pour $\delta = 1\mu$

vitesses env. quelques μ/sec, ce qui répond peut être à l'objection qu'on pourrait tirer de la loi des grands nombres, en considérant l'extrême petitesse des molécules.

p_{H_2O} =	0.39	−30°
	0.91	−20°
	2.08	−10°
	4.60	+0°
	9.16	10°

$\log p = 0.6556 + 0.6938$

$= 0.9618 - 10$

$= 0.916$

$$p = 4.525 \cdot 10^{\frac{7.4475\,t}{234.69+t}} = a \cdot 10^{\frac{bt}{c+t}}$$

n.p. $t = -20$

$\frac{148.95}{214.69} =$

$$\begin{array}{r} 1731 \\ -\ 3318 \\ \hline 0.8413 - 1 \end{array}$$

$10^{0.6938} = 4.941$

$$\frac{dp}{dt} = a \cdot 10^{\frac{bt}{c+t}} \cdot {}^{e}\log 10 \cdot \frac{b}{(c+t)^2}[(c+t)-t]$$

$$= p \cdot {}^{e}\log 10 \cdot \frac{bc}{(c+t)^2}$$

$$s = \frac{1}{p} \qquad \log s = -\log p$$

$$\frac{d\log s}{dt} = -\frac{d\log p}{dt} = -\frac{1}{p}\frac{dp}{dt} = -{}^{e}\log 10 \cdot \frac{bc}{(c+t)^2}$$

$$J\lambda = T\frac{dp}{dt} \qquad V = \frac{v_0}{273} \cdot T$$

$$\frac{dv}{dt} = -\frac{JC - \lambda J V \frac{d\log s}{dt}}{p + \lambda J} = -\frac{JC + T\left(\frac{dp}{dt}\right)^2 \frac{V}{p}}{p + T\frac{dp}{dt}}$$

$$\frac{dx}{dt} = p\frac{dv}{dt} - \frac{pv}{T}$$

$$= -P\left\{\frac{JC + T\left(\frac{dp}{dt}\right)^2\frac{V}{p}}{p + T\frac{dp}{dt}} + \frac{V}{T}\right\}$$

$$= -P\left\{\frac{JC + \left(T\frac{dp}{dt}\right)^2 \frac{1}{p}\frac{V_0}{273}}{p + T\frac{dp}{dt}} + \frac{V_0}{273}\right\}$$

70716
−97676
73040

53752
2824
56.576 . 1034
1697
226
$\boxed{58.500} = \frac{585^m}{8100}$
88705
43616
45089

$\frac{771}{273}\Big\} \log =$

$JC = 9.897$

$C = 0.2377$

$J = \frac{42200 \cdot 10^3}{980 \cdot 76 \cdot 13.6} = 4164$

$\left(\frac{dp}{dt}\right)_0 = p_0 \, {}^{e}\log 10 \cdot \frac{b}{c} = 4.525 \cdot 2.3026 \cdot \frac{7.4475}{234.69}$

88081	62531	65562	51936		91104
99149	00584	87201	43616		45089
13354	61947	~~27046~~	95552		36193
00584	37603	36222			−65562
	99550	~~24034~~	$T\frac{dp}{dt} =$	90.265	70631
		−88985		4.525	50853
		51936		94790	9.9
					5095.2

$=2M$ $\quad\quad\quad$ $\cdot\, I$

$$X_x = a\frac{\partial\xi}{\partial x} + b\,\theta \qquad X_y = Y_x = \frac{a}{2}\left(\frac{\partial\xi}{\partial y} + \frac{\partial\eta}{\partial x}\right)$$

$$Y_y = a\frac{\partial\eta}{\partial y} + b\theta \qquad Y_z = Z_y = \frac{a}{2}\left(\frac{\partial\eta}{\partial z} + \frac{\partial\zeta}{\partial y}\right)$$

$$Z_z = a\frac{\partial\zeta}{\partial z} + b\theta \qquad Z_x = X_z = \frac{a}{2}\left(\frac{\partial\zeta}{\partial x} + \frac{\partial\xi}{\partial z}\right)$$

$$\rho\frac{\partial^2\xi}{\partial t^2} = \frac{\partial X_x}{\partial x} + \frac{\partial X_y}{\partial y} + \frac{\partial X_z}{\partial z} = a\frac{\partial^2\xi}{\partial x^2} + \frac{a}{2}\left(\frac{\partial^2\xi}{\partial y^2} + \frac{\partial^2\eta}{\partial x\partial y} + \frac{\partial^2\zeta}{\partial x\partial z} + \frac{\partial^2\xi}{\partial z^2}\right) + b\frac{\partial\theta}{\partial x}$$

$$= \frac{a}{2}\nabla^2\xi + \frac{a}{2}\frac{\partial\theta}{\partial x} + b\frac{\partial\theta}{\partial x} = \frac{a}{2}\nabla^2\xi + \left(b+\frac{a}{2}\right)\frac{\partial\theta}{\partial x}$$

$$\rho\frac{\partial^2\xi}{\partial t^2} = \frac{a}{2}\nabla^2\xi + \left(\frac{a}{2}+b\right)\frac{\partial\theta}{\partial x} \qquad \Big|\ \frac{\partial}{\partial x}$$

$$\rho\frac{\partial^2\eta}{\partial t^2} = \frac{a}{2}\nabla^2\eta + \left(\frac{a}{2}+b\right)\frac{\partial\theta}{\partial y} \qquad \Big|\ \frac{\partial}{\partial y}$$

$$\rho\frac{\partial^2\zeta}{\partial t^2} = \frac{a}{2}\nabla^2\zeta + \left(\frac{a}{2}+b\right)\frac{\partial\theta}{\partial z} \qquad \Big|\ \frac{\partial}{\partial z}$$

$$\Big\}\ \rho\frac{\partial^2\theta}{\partial t^2} = (a+b)\nabla^2\theta$$

~~Podłużne~~ Dylatacyjne prędkość $\sqrt{\dfrac{a+b}{\rho}}$

$$\rho\frac{\partial^2}{\partial t^2}\left(\frac{\partial\xi}{\partial y} - \frac{\partial\eta}{\partial x}\right) = \frac{a}{2}\nabla^2\left(\frac{\partial\xi}{\partial y} - \frac{\partial\eta}{\partial x}\right)$$

$\Big\}$ Torsyjonalne prędkość $\sqrt{\dfrac{a}{2\rho}} = \sqrt{\dfrac{T}{\rho}}$

$$\frac{a}{2} = T$$

$$x_{n-m}\frac{d^2x_{n-m}}{dt^2} + x_{n-m+1}\frac{d^2x_{n-m+1}}{dt^2} + \dots + x_{n+m}\frac{d^2x_{n+m}}{dt^2} =$$

$$= E\Big[x_{n-m}(x_{n-m-1} - 2x_{n-m} + x_{n-m+1}) + x_{n-m+1}(x_{n-m} - 2x_{n-m+1} + x_{n-m+2}) + \Big]$$

$$= -E\Big[(x_{n-m} - x_{n-m+1})^2 + (x_{n-m+2} - x_{n-m+1})^2 + \Big] +$$

$$+ E\Big[x_{n-m}x_{n-m-1} - x_{n-m}^2 + x_{n+m}x_{n+m+1} - x_{n+m}^2\Big]$$

$$= \frac{d}{dt}\Big[x_{n-m}\frac{dx_{n-m}}{dt} + \dots\Big] - \Big[\left(\frac{dx_{n-m}}{dt}\right)^2 + \left(\frac{dx_{n+m}}{dt}\right)^2\Big]$$

$$\frac{d^2 x_n}{dt^2} = E\left[(x_{n+1} - x_n - l) + (l - (x_n - x_{n-1}))\right]$$

$$= E\left[x_{n+1} - 2x_n + x_{n-1}\right]$$

$$\frac{d^2 x_1}{dt^2} = E(x_2 - x_1 - l)$$

$$\frac{d^2 x_2}{dt^2} = E(x_3 - 2x_2 + x_1) \qquad \frac{d^2(x_2 - x_1)}{dt^2} = E(x_3 - x_2 - l)$$

$$\frac{d^2 x_3}{dt^2} = E(x_4 - 2x_3 + x_2) \qquad \frac{d^2(x_3 + x_2)}{dt^2} = E(x_4 - x_3 - x_2 + x_1)$$

$$\frac{d^2 x_4}{dt^2} = E(x_5 - 2x_4 + x_3)$$

$$\dots$$

$$\frac{d^2 x_n}{dt^2} = E(x_{n+1} - 2x_n + x_{n-1})$$

$$\frac{d^2 x_{n+1}}{dt^2} = E(x_{n+2} - 2x_{n+1} + x_n)$$

$$\frac{d^2 x_n}{dt^2}\frac{dx_n}{dt} = E\Big(\underbrace{x_n \frac{dx_{n+1}}{dt} + x_{n+1}\frac{dx_n}{dt}}_{\frac{d}{dt}(x_n x_{n+1})} - x_{n-1}\frac{dx_n}{dt} - x_n \frac{dx_{n-1}}{dt} + x_{n-2}\frac{dx_{n-1}}{dt}$$

$$- 2x_n \frac{dx_n}{dt} - \underbrace{x_{n+1}\frac{dx_1}{dt} - x_1 \frac{dx_{n+1}}{dt}}_{} - 2x_{n+1}\frac{dx_{n+1}}{dt} - x_{n+2}\frac{dx_{n+1}}{dt}$$

$$+ \frac{d^2 x_{n+1}}{dt^2}\frac{dx_{n+1}}{dt}$$

$$\frac{1}{2}\frac{d}{dt}\left[\left(\frac{dx_{n-m}}{dt}\right)^2 + \left(\frac{dx_{n-m+1}}{dt}\right)^2 + \dots \left(\frac{dx_{n+m}}{dt}\right)^2\right] = E\Big[\frac{d}{dt}\left(x_{n-m}x_{n-m+1} + x_{n-m+1}x_{n-m+2} + \dots\right)$$

$$- \frac{d}{dt}\left[x_{n-m}^2 + x_{n-m+1}^2 + \dots\right] + x_{n-m-1}\frac{dx_{n-m}}{dt} + x_{n+m+1}\frac{dx_{n+m}}{dt}\Big]$$

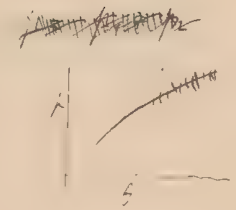
$$= -E\Big[\frac{d}{dt}\frac{1}{2}\left[(x_{n-m+1} - x_{n-m})^2 + (x_{n-m+2} - x_{n-m+1})^2 + \dots\right] + x_{n-m-1}\frac{dx_{n-m}}{dt} + x_{n+m+1}\frac{dx_{n+m}}{dt}\Big]$$

$dQ = A\,p\,dv + dU$ pro 1 gr.

$0 = A\,p\,dv + c\,d\theta + r\,dx$ $\qquad dx = \frac{\partial x}{\partial p}\,dp + \frac{\partial x}{\partial \theta}\,d\theta$

$\frac{1}{\varrho}\frac{dp}{dz} = g$

$\frac{p}{\varrho} = R\theta$



$x = ?$

wenn 1 kg. x kg ... temp. θ p, wie viel wird sich ausscheiden wenn $\begin{matrix} p + dp \\ \theta + d\theta \end{matrix} = dx$?

r ... θ ... P

... cond. ... $\qquad x = \rho d.$

... $\frac{1}{\rho}$ ... P ... $\qquad x = \frac{1}{s\,\rho}$

$s = \frac{RT}{P} \qquad s = s_0 \frac{T}{T_0}\cdot\frac{P_0}{P}$

~~750~~ $\frac{18.5}{750}$ $\frac{99.67}{2405}$. ~~55746~~ $\qquad \frac{p}{\rho} = R\theta$

$R = \frac{760.}{0.001293\ .\ 273}$

2553

$s = \frac{1}{0.627}\,\frac{R_{Lft}\,T}{P}$

$x = \frac{P}{\rho\,R_w\,\theta}$ $\qquad r = T\,\frac{dP}{dT}\,s = \frac{1}{A}\,\frac{\theta^2 R_w}{p}\,\frac{dP}{d\theta}$

$=$

$$v\,dp = g\,dz$$
$$pv = R\theta$$
$$p\,dv = R\,d\theta - g\,dz$$

$$x = \frac{P}{R v \theta} v$$

$$dx =$$

$$p = \frac{R\theta}{v}$$

$$\cancel{v\,dp = \frac{R\theta}{v} dv}$$

$$p\,dv + c\,d\theta = +A\, d\left(\frac{Pv}{R\theta}\right) \cdot \frac{\theta^2 R}{P}\frac{dP}{d\theta}$$

$$pv = R\theta$$
$$\frac{v}{R\theta} = \frac{1}{p}$$

$$v\,dp = g\,dz$$
$$pv = R\theta$$

$$\frac{AR\theta}{v} dv + c\,d\theta = A\frac{\theta^2 R}{P}\frac{dP}{d\theta} \quad d\left(\frac{Pv}{R\theta}\right) = -A\frac{\theta^2 R}{P}\frac{dP}{d\theta}\left[P\,d\left(\frac{v}{R\theta}\right) + \frac{v}{R\theta}dP\right]$$

$$v\,d\left(\frac{R\theta}{v}\right) = -g\,dz$$

$$AR\,d\theta + c\,d\theta + Ag\,dz = -A\frac{\theta^2 R}{P}\frac{dP}{d\theta}\left[d\left(\frac{1}{p}\right) + \frac{1}{p}\frac{dP}{P\,d\theta}d\theta\right]$$

$$\frac{R\theta}{p}dp = -g\,dz$$

$$d\theta\left[AR + c + A\frac{\theta^2 R}{pP}\left(\frac{dP}{d\theta}\right)^2\right] = -Ag\,dz - A\,\theta\frac{dP}{d\theta}\frac{1}{p}g\,dz$$

$$A\frac{k}{k-1}$$

$$= -\left(A + \frac{\theta}{p}\frac{dP}{d\theta}\right)g\,dz$$

$$\frac{d\theta}{dz} = \frac{g}{R}\frac{1 + \frac{\theta}{p}\frac{dP}{d\theta}}{\frac{k}{k-1} + \frac{\theta^2}{pP}\left(\frac{dP}{d\theta}\right)^2} = -\underbrace{\frac{k-1}{k}\frac{g}{R}}_{=1}\frac{1 + \frac{\theta}{p}\frac{dP}{d\theta}}{1 + \frac{k-1}{k}\frac{\theta^2}{pP}\left(\frac{dP}{d\theta}\right)^2}$$

$$= \frac{1 + \left(\frac{1}{A}\frac{\theta}{p}\frac{dP}{d\theta}\right)}{1 + \left(\frac{1}{A}\frac{\theta}{p}\frac{dP}{d\theta}\right)\left(\frac{k-1}{k}\frac{\theta}{P}\frac{dP}{d\theta}\right)}$$

−25° −15° −5° 0° +5° +10° +15°

~~0·309~~ 0·340 0·451 0·609

$\frac{dP}{d\theta}$

	$\frac{dP}{d\theta}$	θ	P	p
−30°				
−25°				
−20°	0·071	.253	0·91	519
−15°	0·1185	.258	1·44	554
−10	0·1095	.263	2·15	591
−5	0·238	.268	3·16	629
0	0·340	.273	4·52	671
5	0·451	.278	6·51	715
10	0·609	.283	9·14	760

35

9·767
8·548
1·219 : 2

6·971
069
0·902 : 2

7·909
569
340

4·569 ·340
3·9 ·2930
0·619 0·471

1·562
325
237

2·327
1·988
0·339

$\theta = 273 \quad 1 + 273 \frac{0·34}{760}$

$+ \frac{273 \cdot 0·34}{760} \cdot \frac{14}{1·4} = \frac{\cancel{223}\ 0·34}{\cancel{4·52}}\ 2·26$

$\frac{p}{p_0} = \cancel{\left[1 - \frac{k-1}{k}\frac{gx}{R\theta_0}\right]^{\frac{k}{k}}} \left(\frac{\theta}{\theta_0}\right)^{\frac{k}{k-1}}$

$p = 760 \cdot \frac{283}{273} \left(\frac{\theta_0 - \Delta}{\theta_0}\right)^{\frac{k}{k-1}}$

$= 760 \left[1 - \frac{k}{k-1}\frac{\Delta}{\theta_0} + \frac{\alpha(\alpha-1)}{1.2}\left(\frac{\Delta}{\theta_0}\right)^2 - \frac{\alpha(\alpha-1)(\alpha-2)}{1.2.3}\left(\frac{\Delta}{\theta_0}\right)^3 \cdots\right]$

$\frac{7}{2} \cdot \frac{5}{283} = 3500 : 566 = 0·0618$
104
47

$\frac{7}{2} \cdot \frac{30}{283} = \cancel{210}\ 105 : 283 =$

$\frac{2}{7}$

30103
84510
45593

$\frac{7}{2} \cdot \frac{5}{2} \cdot \frac{1}{2} = \frac{35}{8}$

47712
45179
0.02533
0.05066
0.07599

1.01354
− 0.18551
0.82803 . 760
579621
49682

0.106005
0.742035
0.371017
1855085
309181

519

029

760

110

131

0.01124
0.00281
843
2405

0.05417
− 0.37102
0.08315 . 760
478205
40989
519194

0.001191

0.05417
135425
33856

1.13762
− 0.309181
0.72844 . 760
509908
737064
553614

$$\frac{k-1}{k}\,\frac{J}{u_0 g} = \frac{2}{7}\cdot\frac{0.0033 \cdot 0.05 \cdot 425 \cdot 980 \cdot 10^2}{0.02 \cdot 10 \cdot 980}$$

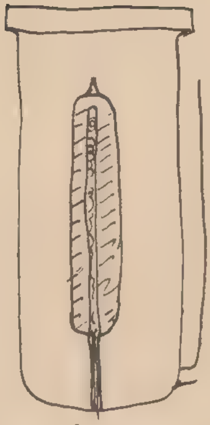
$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^k \qquad \frac{\frac{p}{\rho}}{\frac{p_0}{\rho_0}} = \left(\frac{\rho}{\rho_0}\right)^{k-1} = \frac{\theta}{\theta_0}$$

$$= \left(\frac{\theta}{\theta_0}\right)^{\frac{k}{k-1}} \qquad \left(\frac{1}{760}\right)^{\frac{k-1}{k}}\theta_0 = \theta \qquad = \left(\frac{1}{760}\right)^{\frac{2}{7}} \cdot 283$$

2.4362
− 0.6511
1.7851

2.2788 . 2
4.5576 . 7
− 0.6511
+ 2.4518
1.8607

273
63.2
210



$100\,cm^2 \cdot \frac{50°}{1\,cm}\ 0{\cdot}0001 =$

$= 0{\cdot}5\ \frac{cal.}{sec.} = 2$ Volt Am

50 Ohm 10 Volt

$\theta = 3$
$\omega = 10$

$\frac{7}{3}\ (1{\cdot}4)^2\ \cancel{980}^2\ \frac{\cancel{0.00}1293}{(76.\ 13{\cdot}6)^2\ \cancel{980}^2}\ \left(\frac{\cancel{273}}{3}\right)^{\cancel{91}\ \cancel{4}6}\ .\ 1{\cdot}07\ .\ \cancel{10}\ .\ \cancel{10}$

$(9{\cdot}1)^6$

$\underbrace{V\rho c}_{10}\ \frac{\partial\theta}{\partial t} = -0{\cdot}01\ \theta.$

$\theta = \theta_0 e^{-\alpha t}$

$\left(\frac{1{\cdot}4}{76.\ 13{\cdot}0}\right)^2 \cdot \frac{4.\ 1293.\ 107}{3} \cdot (9{\cdot}1)^6$

$\alpha = 0{\cdot}001$

$\frac{1}{74}^2$

$\left(\frac{9{\cdot}1^3}{740}\right)^2 \cdot\ 4.\ 1{\cdot}07.\ 0{\cdot}431$

$e^{-\alpha t} = 1\%$

$\cancel{1}$

$100 = e^{\alpha t}$

$\cancel{91}$
819
$\underline{\quad 91}$
828_1
$\underline{\quad}$
74529
$\underline{\ \ 8281}$
75357

$\left(\frac{754}{740}\right)^2 = 1{\cdot}04\ .\ \cancel{1{\cdot}07.\ 1{\cdot}72}$
$(1{\cdot}7)^2$

$\lg 100 = \alpha\ t$

$t = \frac{\lg 100}{\alpha}$

$= \frac{4{\cdot}6}{10^{-3}}$

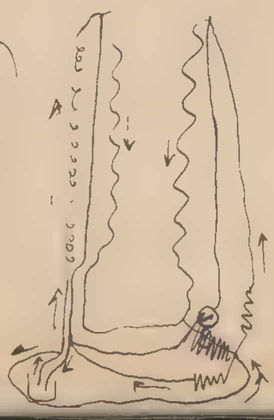
$= 4{\cdot}6 . 10^3$

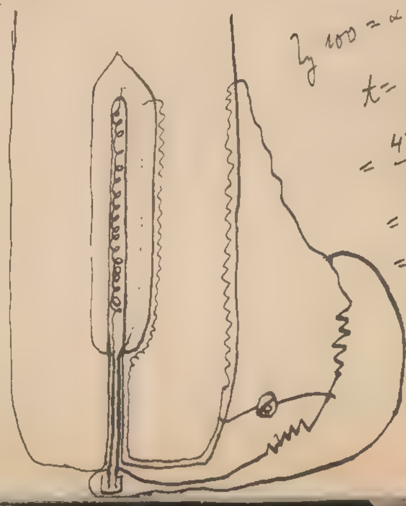
$= 0{\cdot}7 . 10^2$ min

$= 70$ min

$= 1$ [illegible]

$0.034.\ 13{\cdot}6$
$\quad 408$
$\quad\ 544$
$\overline{0{\cdot}46}$





$$\theta = a\int_0^{\frac{x}{\sqrt{t}}} e^{-\frac{c\rho}{4k}y^2}\,dy + b \qquad \Bigg| \quad \begin{array}{l} x=0 \ : \ \theta = \theta_2 \\ t=0 \ : \ \theta = \theta_1 \end{array}$$

$$\theta_1 = a\underbrace{\int_0^{\infty} e^{-\frac{c\rho}{4k}y^2}\,dy}_{} + b$$

$$\left.\begin{array}{l} \theta_1 = a\,\cancel{2}\sqrt{\frac{k}{c\rho}}\,\frac{\sqrt{\pi}}{\cancel{2}} + b \\ \theta_2 = \qquad\qquad\quad b \end{array}\right\} \quad a = (\theta_1-\theta_2)\sqrt{\frac{c\rho}{k\pi}}$$

$$\theta = (\theta_1-\theta_2)\sqrt{\frac{c\rho}{k\pi}}\int_0^{\frac{x}{\sqrt{t}}} e^{-\frac{c\rho}{4k}y^2}\,dy + \theta_2$$

$$= (\theta_1-\theta_2)\,\frac{2}{\sqrt{\pi}}\int_0^{\frac{x}{\sqrt{t}}\sqrt{\frac{4k}{c\rho}}} e^{-y^2}\,dy + \theta_2 = \frac{2}{\sqrt{\pi}}(\theta_1-\theta_2)\int_0^{\frac{2x}{\sqrt{t}}\sqrt{\frac{k}{c\rho}}} e^{-y^2}\,dy + \theta_2$$

więc zależnem będzie od

$$\left.\frac{\partial\theta}{\partial x}\right|_{x=0} = \frac{2}{\sqrt{\pi}}(\theta_1-\theta_2)\left.e^{-\frac{4x^2}{t}\frac{k}{c\rho}}\cdot\frac{2}{\sqrt{t}}\sqrt{\frac{k}{c\rho}}\right|_{x=0} = \frac{4(\theta_1-\theta_2)}{\sqrt{t}}\sqrt{\frac{k}{\pi c\rho}}$$

$$k\frac{\partial\theta}{\partial x} = \frac{4}{\sqrt{t}}(\theta_1-\theta_2)\frac{k}{\sqrt{\pi}}\sqrt{\frac{k}{c\rho}} = \text{na powierzchni absorbowane ciepło}$$

Ściana zimna w kontakcie z parą wodną, jeśli np. parę wpuszczamy do cylindra.

$$\frac{\partial \theta}{\partial t} = \frac{k}{c\rho}\frac{\partial^2 \theta}{\partial x^2}$$ Warunek początkowy $t=0$ $\theta = 0 \begin{cases} x=0 \\ x=l \end{cases}$

$t \dots$ $\theta = \theta_1 \mid x=0$

$\sqrt{t} = x z$

$$\theta = f.\left(\underbrace{\frac{\sqrt{t}}{x}}_{z}\right) \qquad \frac{\partial\theta}{\partial t} = f'(z)\frac{1}{2\sqrt{t}\cdot x} = f'(z)\cdot\frac{z}{2t} = f'(z)\frac{1}{2z x^2}$$

$$\cancel{\frac{\partial\theta}{\partial t} = f'(z)\frac{\partial\theta}{x^2 \, \partial x}} \cdot \quad \frac{\partial\theta}{\partial x} = -f'(z)\frac{\sqrt{t}}{x^2}$$

$$\theta = \qquad \frac{\partial^2\theta}{\partial x^2} = +f''(z)\frac{t}{x^4} + 2f'(z)\frac{\sqrt{t}}{x^3} = \frac{f''(z)\, z^2 + 2f'(z)\, z}{x^2}$$

$$\frac{f'(z)}{2z\cancel{x^2}} = \frac{k}{c\rho}\, \frac{f''(z)\, z^2 + 2f'(z)\, z}{\cancel{x^2}} \qquad f'(z) = e^{\varphi(z)}$$

$$f''(z) = \varphi' e^{\varphi(z)}$$

$$\varphi' \cdot z^2 + 2z - \frac{c\rho}{2kz} = 0$$

$$\varphi' + \frac{2}{z} - \frac{c\rho}{2kz^3} = 0$$

$$\varphi = \frac{c\rho}{4kz^2} - 2\log z \quad \Big\| \quad f'(z) = \frac{e^{\frac{c\rho}{4kz^2}}}{z^2}$$

$$\theta = A\int \frac{e^{-\frac{c\rho}{4kz^2}}}{z^2}\,dz \qquad \frac{1}{z} = y \qquad = -A\int e^{-\frac{c\rho}{4k}y^2}\,dy = A\int_{\frac{x}{\sqrt{t}}} e^{-\frac{c\rho}{4k}y^2}\,dy + \text{const}^{\theta_1}$$

$$\frac{\partial\theta}{\partial t} = -e^{-\frac{c\rho}{4k}\frac{x^2}{t}}\frac{x}{2\sqrt{t^3}} \qquad \frac{\partial\theta}{\partial x} = e^{-\frac{c\rho}{4k}\frac{x^2}{t}}\frac{2}{\sqrt{t}} \qquad \cancel{\frac{\partial^2\theta}{\partial x^2} = e^{\dots}\left[\frac{2}{t} + \frac{4x^2}{t^2}\right]}$$

$$\frac{\partial^2\theta}{\partial x^2} = \frac{c\rho}{4k}\frac{2x}{\sqrt{t^3}}\, e^{\frac{c\rho}{-}}$$

Ilość ciepła przewodzonego

przez gaz

$$0{\cdot}000066 \frac{.100}{0{\cdot}8} \cdot 35 = \frac{0{\cdot}231}{0{\cdot}8} = 0{\cdot}3 \frac{g\,cal}{sec} = 18 \frac{g\,cal}{min}$$

przez szkło:

$$\frac{0{\cdot}001 \cdot 100}{0{\cdot}8} \cdot 2 = 0{\cdot}25 \frac{g\,cal}{sec}$$

przez ustoję żelazną $q = 10\,mm^2$

$$\frac{0{\cdot}15 \cdot 100}{0{\cdot}8} \cdot 0{\cdot}1 = 2 \frac{g\,cal}{sec}$$

$$\gamma_0 (1+\varepsilon\theta)\frac{d\theta}{dz} = \text{const} = a$$

$$\gamma_0\theta + \gamma_0\varepsilon\frac{\theta^2}{2} = az + \text{const}$$

$$\cancel{\gamma_0\theta_0 + \gamma_0\varepsilon}\,\theta \qquad z=0:\ \theta=0 \qquad \text{const}=0$$

$$\cancel{\gamma_0\theta+\gamma}\quad \gamma_0\theta\left[1+\varepsilon\frac{\theta}{2}\right] = az$$

$$\gamma_0\theta_1\left[1+\varepsilon\frac{\theta_1}{2}\right] = ah$$

$$\theta\left(1+\varepsilon\frac{\theta}{2}\right) = \frac{z}{h}\,\theta_1\left(1+\varepsilon\frac{\theta_1}{2}\right)$$

N.p. $z=\frac{h}{2}$ $\qquad \theta\left(1+\varepsilon\frac{\theta}{2}\right) = \frac{1}{2}\theta_1\left(1+\varepsilon\frac{\theta_1}{2}\right)$ $\qquad \theta$.

$$\cancel{\left(\frac{\theta}{2} - \theta_1\right) + \frac{\varepsilon}{2}\left(\frac{\theta^2}{2} - \theta_1^2\right) = 0}$$

$$\underbrace{\theta-\frac{\theta_1}{2}}_{\delta} + \frac{\varepsilon}{2}\left(\theta^2 - \frac{\theta_1^2}{2}\right) = 0$$

$$\int\theta\,dz = \int_0^{\theta_1}\theta\,\frac{\gamma_0(1+\varepsilon\theta)\,d\theta}{a} = \frac{\gamma_0}{a}\left(\frac{\theta_1^2}{2}+\varepsilon\frac{\theta_1^3}{3}\right) = \frac{\gamma_0}{a}\,\frac{\theta_1^2}{2}\left[1+\frac{2\varepsilon}{3}\theta_1\right]$$

$$= h\,\frac{\frac{\theta_1^2}{2}\left[1+\frac{2\varepsilon\theta_1}{3}\right]}{\theta_1\left[1+\frac{\varepsilon\theta_1}{2}\right]} = \frac{h\theta_1}{2}\left[1+\frac{1}{6}\varepsilon\theta_1\right]$$

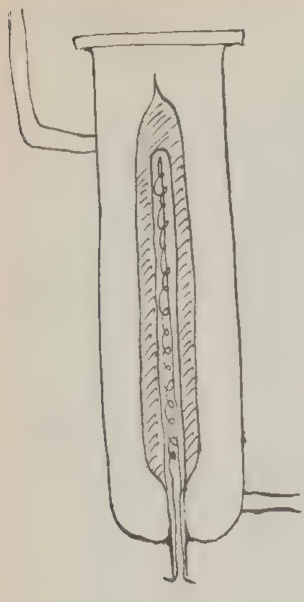
więc różnica przeciętnej $\frac{1}{6}\varepsilon\theta_1$ od średniej temp.

N.p. $\varepsilon = \frac{1}{273} = 0{\cdot}0036$

$\theta_1 = 100$ $\qquad \frac{\varepsilon\theta_1}{6} = 0{\cdot}06 = 6\%$ więc przeciętna temperatura o 3°C ($=53°C$) wyższa od średniej 50°

Temu odpowiada różnica w ciśnieniu lub objętości $\frac{3}{273} = \frac{1}{90}$

t.j. 8 mm Hg = ~~80~~ 87 mm glyceryny



$$K \cdot \frac{30\ cm^2}{1\ cm}\ \theta^{=50}$$

smewen

k H_2O = 0·0015 | 0·045 . 50 = 2·5

Glas

k Luft = 0·00006 | 0·0018 . 50 = 0·09

1 Volt Ampere = 0·24 $\frac{g\,cal}{sec}$ 10 - - - · 0·3 Voltage

$i^2 w = \frac{e^2}{w}$ $w = 2$ Ohm

$i^2 =$ 5 . . . 0·15

$i =$ 2 - - - · 0·4 Amp

i = Cl pro cm²

$$\varphi = i^2 w = \frac{i^2}{\lambda_\theta} = -\frac{\partial}{\partial x}\left(\kappa \frac{\partial \vartheta}{\partial x}\right)$$

$$= -\frac{d\kappa}{dx}\frac{d\theta}{dx} - \kappa \frac{d^2\vartheta}{dx^2}$$

$$= -\frac{d\kappa}{d\theta}\left(\frac{d\theta}{dx}\right)^2 - \kappa \frac{d^2\vartheta}{dx^2}$$

$\kappa = \kappa_0(1+\alpha\theta)$

$\lambda = \lambda_0(1+\beta\theta)$

Jeśli I przybliżenie $\kappa = \kappa_0$

$$i^2 = -\lambda_0(1+\beta\theta)\kappa_0 \frac{d^2\vartheta}{dx^2}$$

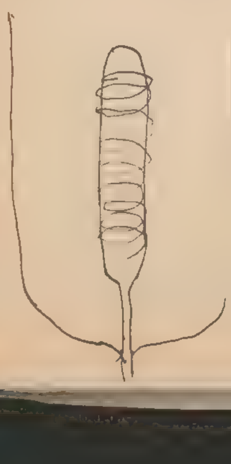
W pierwszym przybliżeniu: $i^2 = -\lambda_0\kappa_0 \frac{d^2\vartheta}{dx^2}$

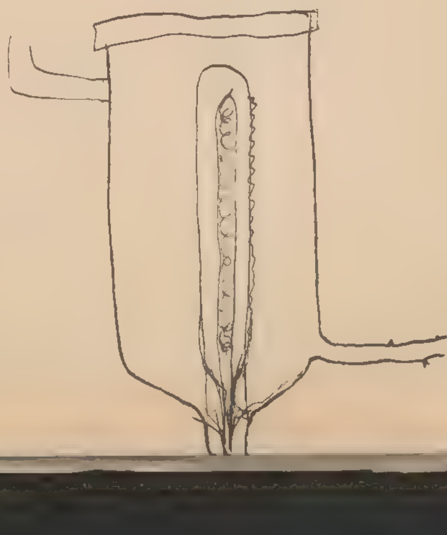
$$\vartheta = \theta_0 - \frac{i^2}{\lambda_0\kappa_0}x^2 + \frac{i^2}{\lambda_0\kappa_0}x l$$

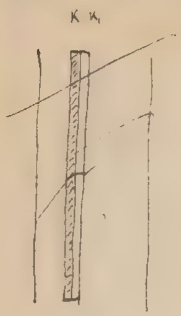
kiedy $\theta_0 = 0$

$$V = \int \frac{dx}{\lambda_\theta} = \int \frac{dx}{\lambda_0\left(1+\beta\theta_0 + \frac{\beta i^2}{\lambda_0\kappa_0} l x - \frac{\beta i^2}{\lambda_0\kappa_0}x^2\right)} = \frac{1}{\lambda_0}\int \frac{dx}{1 + \frac{\beta i^2 l}{\lambda_0\kappa_0}x - \frac{\beta i^2}{\lambda_0\kappa_0}x^2}$$

$$= \frac{1}{\lambda_0}\,\frac{}{1 - \sqrt{\frac{\beta}{\lambda\kappa}}\, i x}$$







$$\frac{c}{A} f(r) \frac{\partial\theta}{\partial x} = R f(r) \frac{\partial\theta}{\partial x} - w \frac{\partial p}{\partial x} + \mu\left(\frac{\partial u}{\partial r}\right)^2 + K \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)$$

$$\left(\frac{c}{A} - R\right) f(r)\frac{\partial\theta}{\partial x} = -\frac{w}{\rho}\frac{2\mu}{r^2-R^2} f(r) + \mu\left(\frac{\partial u}{\partial r}\right)^2 + K\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)$$

$$\cancel{-2(x+y+z)+6x}$$

$$\cancel{4x-2y-2z}$$

$$-(x+y+z)^2+3\left[x^2+y^2+z^2\right]$$

$$\cancel{2x^2 - 2xy + 2y^2 + 2z^2 - 2xz - 2xy}$$

$$-(1+\alpha+\beta)^2+3\left[1+\alpha^2+\beta^2\right]$$

$$= 2\left[1+\alpha^2+\beta^2-\alpha-\beta-\alpha\beta\right]$$

$$= \left[(\alpha-\beta)^2+(1-\alpha)^2+(1-\beta)^2\right]$$

$$\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)^2 + \dots \qquad = \left(\frac{\partial y}{\partial x}\right)^2 +$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial z} = \frac{\eta}{\rho}\left[\frac{4}{3}\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right] - \frac{1}{\rho}\frac{\partial p}{\partial z}$$

$$\frac{c}{A}\rho w \frac{\partial \theta}{\partial z} = -p\frac{\partial w}{\partial z} + \mu\left[-\frac{2}{3}\left(\frac{\partial w}{\partial z}\right)^2 + 2\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial r}\right)^2\right] + K\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right)$$

$\theta, \rho, w = f_n(r, z)$

$$\frac{p}{\rho} = R\theta$$

$\rho w = f_n(r)$

$$\frac{\partial p}{\partial x} = \mu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right)$$

$\rho w = f_n(r)$ $\qquad \frac{\partial}{\partial x}(\rho w) = 0$ $\qquad \frac{p}{\rho} = Rt$

$$\frac{c}{A}\rho w \frac{\partial \theta}{\partial x} = -p\frac{\partial w}{\partial x} + \mu\left(\frac{\partial w}{\partial r}\right)^2 + K\underbrace{\left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r}\right)}_{\frac{1}{r}\frac{\partial \theta}{\partial r}}$$

$p = f_n(x)$

$$R\frac{\partial \theta}{\partial x} = \frac{w}{f(r)}\frac{\partial p}{\partial x} - p\frac{\partial}{\partial x}\left(\frac{w}{f(r)}\right)$$

$$= \frac{w}{f(r)}\frac{\partial p}{\partial x} - \frac{p}{f(r)}\frac{\partial w}{\partial x}$$

$$R f(r)\frac{\partial \theta}{\partial x} = w\frac{\partial p}{\partial x} - p\frac{\partial w}{\partial x}$$

$$\frac{\partial}{\partial r}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) = 0$$

$$\underbrace{\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}}_{\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}r\right)} = \psi(x)$$

$$= r\,\psi(x)$$

$$r\frac{\partial u}{\partial r} = \frac{r^2}{2}\psi(x) + \psi(x)$$

$$u = \frac{r^2}{2}\psi(x) + \cancel{\psi(x)\log(r)} + \lambda(x)$$

$$u = \frac{r^2 - R^2}{2}\psi(x) = \frac{r^2 - R^2}{2}\frac{1}{\mu}\frac{\partial p}{\partial x}$$

$$\rho\frac{r^2 - R^2}{2\mu}\frac{\partial p}{\partial x} = f(r) \qquad \rho\frac{\partial p}{\partial x} = \frac{2\mu}{r^2 - R^2}f(r)$$

C, c ciał stałych, Dulong Petit, związki, ciecze! temper.!

Przy przekroczeniu wytrzymałości ciało staje się płynnem:

magnetyz.

elektroliza, opór zmniejszony?

podchłodzenie przy ~~[illegible]~~ mieszaniu cieczy!

Unterkühlung durch Bewegung: Despretz CR II Pogg Ann. XLI

Leitungsvermögen von Salzen: Grätz Wied. Ann. 40 p. 18 (1890)

KCl St Andrews Edinb. Proc. 1884/5 13 p. 275

([illegible]) NaCl bei 850°

Salzkrystalle in verschied. Richtung [illegible]!, dann bei Druck bis zur Zerdrückung

---

Kann die Capillaritätsconstante β Eis-Wasser theoretisch berechnet werden?

Capillaritätsconstante α von Eis?

Emulsionen: Quincke Pflügers Archiv 1879 Bd. 19

Ctbl. 1880 p. 110

Brücke " " p. 109

[illegible]

[illegible]

Wied. Ann. 67 p. 702 (1899) [illegible]

1). [illegible] = 20 [illegible]

[illegible] = [illegible]

2) [illegible] :

$c_p - c_v = \frac{0.0691}{d}$ $c_{\text{[illegible]}} = \frac{3 \cdot 0.0691}{2d}$ $d = \frac{A}{2 \cdot 14.5}$

[illegible]

$A \cdot c_v = 3 \cdot 0.0691 \cdot 14.5 = 3.006$

C_v [illegible] $= 2c_v = 6.012$

$A \cdot C_p = 6.072 - 6.252$

[illegible]

p. 670 of "theorie de la cond. [illegible] Dulong Petit"

$C_v = \frac{6 v c_v}{\vartheta}$ w v = [illegible] = [illegible] Atom

$c_v = 0.98835 \frac{\vartheta}{\mu}$] $= \frac{5.93}{\frac{\mu}{N} = \alpha = \text{At.g}}$

p. [illegible]: C a+b+c Atom $\mu = \text{Mol.g} = a\alpha + b\beta + c\gamma$

$\mu c_v = (a+b+c)\, 5.93$

$\frac{a \mu c_v}{a+b+c} = a\alpha A$

$\frac{b \mu c_v}{a+b+c} = b\beta B$

$\frac{c \mu c_v}{} = c\gamma C$

$\mu c_v = a\alpha A + b\beta B + c\gamma C$

[illegible]

Therm [illegible] 67 p. 695 [illegible]

0°	4.55
−200°	0.284
500°	0.029
600°	0.008
700°	0.002
800°	0.0004

266

1.293

4249

1335

1584

1116

4468

0156

4624

$\frac{283}{273} =$ 4518 − 4362 = 0156

Wied. Ann. 65 p. 055 (1898)

Staigmüller [illegible]

[illegible] Verhältnis [illegible] Atome:

$x\,y\,z$ [illegible] Atoms [illegible] Mol. [illegible] $(9,4)$ [illegible]

[illegible] $k = 6$ äussere Coord.

[illegible] $(3n-6)$ [illegible] Coord.

[illegible]

$$\left.\begin{array}{ll} 3\varepsilon & = [illegible] \\ 3\varepsilon & \text{rot.} \\ 3n-6 & [illegible] \\ 3n-6 & \text{pot.} \; [illegible] \end{array}\right\} = \underbrace{E = 6(n-1)\varepsilon}_{\vartheta = \text{Wärmedimension}}$$

[illegible] Coord. [illegible] $\vartheta = 0$ [illegible] Relat. [illegible] Var. [illegible]

[illegible] Coord. [illegible] $\vartheta = a + 2i$

$$k = \frac{\vartheta + 2}{\vartheta}$$

$\vartheta = 5$: H_2, O_2, N_2, CO, NO, HCl, HBr, HJ $\quad (a = 5,\ i = 0)$

$= 7$: Cl_2, Br_2, J_2, ClJ, $\underbrace{H_2O,\ N_2O,\ CO_2,\ H_2S,\ SO_2}_{[illegible]}$

$a = 5 \quad i = 1$

NH_3, CH_4

$\underbrace{\qquad}_{6}$ $a = 0$, $i = 0$ [illegible]

$$\frac{1}{\eta}\frac{d\eta}{d\theta} = \frac{1}{\eta}\frac{\partial\eta}{\partial\theta} + \left(\frac{v}{\eta}\frac{\partial\eta}{\partial v}\cdot\left(\frac{\partial v}{\partial\theta}\frac{1}{v}\right)\right)$$

$$= \frac{1}{\eta}\frac{\partial\eta}{\partial p} : \frac{1}{v}\frac{\partial v}{\partial p}$$

$$= \frac{1}{\eta}\frac{\partial\eta}{\partial p}\cdot\frac{\alpha}{\beta}$$

Äther $\theta = 20°$: $\frac{1}{\eta}\frac{\partial\eta}{\partial p} = 7{\cdot}30 . 10^{-4}$ | Benzol $9{\cdot}3 . 10^{-4}$

f. Schwingung

1·1139
2·4116
$-$ 3·5255
$+$ 1·5185
0·9930 -3

2·8096
1·2553
$-$ 4·0649
$+$ 2·2504
0·1855 -2

$\frac{1}{\eta}\frac{d\eta}{d\theta} =$ 0·00127 8
245
33 : 13 . 258 0·0098

für 185°

0·00739
561
178 : 18 . 645 0·0153

für 21°

$\beta =$ 254 13·5 190 168 $. 10^{-6}$
Amagat
$1{\cdot}78 . 10^{-6}$ (Atm)

0 : 83·0
15·4 : 87·1
17·9 : 91·7
50 : 111
94
90 , $92 . 10^{-6}$ (Atm)

0·00116
0·00008
0·00124

$\alpha =$ 0·00148
13
0·00161

350 . 34 . 45
105
140
119 .

35 . 37
105
245
1295

0·0018
$+$ 0·0066
$-$ 0·0048
zu. 0·0098

$$\frac{7{\cdot}3 . 10^{-4} . 0{\cdot}0016}{1{\cdot}78 . 10^{-6}}$$

73
738
1168 : 1·78 656
100
11

$$\frac{9{\cdot}3 . 10^{-4} . 0{\cdot}00124}{0{\cdot}92 . 10^{-6}}$$

$-$ 0·0125
$+$ 0·0018
$-$ 0·0107
$-$ 0·0153 zu.

Benzol $\dfrac{286 \cdot (0{\cdot}00116)^2 \cdot 39}{R \quad 0{\cdot}892}$

$0{\cdot}645 \cdot 2$
$0{\cdot}1290 - 6$
$2{\cdot}4564$
$1{\cdot}5911$
$0{\cdot}1765 - 2$
$- 6{\cdot}5443$
$0{\cdot}6322 - 9$

$0{\cdot}9504 - 1$
$5{\cdot}5939$
$6{\cdot}5443$

ob. $\cancel{4}\,4{\cdot}288 \cdot 10^{-9}$

zn. $8{\cdot}3 \cdot 10^{-11}$

Alk. $\dfrac{293 \cdot (0{\cdot}00111)^2 \cdot 23}{R \quad \cdot 0{\cdot}794}$

$0{\cdot}0453 - 3 \quad 2$
$0{\cdot}0906 - 6$
$2{\cdot}4669$
$1{\cdot}3617$
$0{\cdot}4061 - 7$
$0{\cdot}3253 - 9$
$- 0{\cdot}8998 \quad - 1$
$0{\cdot}4255 - 9$

$- 6{\cdot}5939$
$= 0{\cdot}4061 - 7$

$2{\cdot}66 \cdot 10^{-9}$

$1 \cdot 10^{-10}$

Hg: $\dfrac{273 \cdot \left(\frac{1}{550}\right)^2 \cdot \cancel{100}}{R \cdot 13{\cdot}6}$

$2{\cdot}7404$
$5{\cdot}4808$
$1{\cdot}1335$
$6{\cdot}4434$
$13{\cdot}0577$

$2{\cdot}4362$
$- 13{\cdot}0577$
$0{\cdot}3785 - 11$

$2{\cdot}39 \cdot 10^{-11}$

zn.: $3{\cdot}9 \cdot 10^{-12}$

Pentan: $\dfrac{288 \cdot (0{\cdot}00159)^2 \cdot 36}{R \cdot 0{\cdot}626}$

$0{\cdot}2014 - 3 \quad 2$
$0{\cdot}4028 - 6$
$1{\cdot}5563$
$2{\cdot}4594$
$0{\cdot}4185 - 2$
$- 6{\cdot}2400$
$0{\cdot}1785 - 8$

$0{\cdot}7966 - 1$
$6{\cdot}4434$
$6{\cdot}2400$

$1{\cdot}51 \cdot 10^{-8}$

zn $2{\cdot}9 \cdot 10^{-10}$

$\beta = -\frac{\alpha^2 \vartheta}{\sqrt{2}\, R\rho} = -\frac{\alpha^2 \vartheta}{\sqrt{2}.\, \kappa}\left(\frac{\mu}{\rho}\right)$

$\alpha = 0{\cdot}00116$

Benzol ca 13°

$\mu = \underline{78}, \ \rho = 0{\cdot}892$

$\beta = 0{\cdot}000085$

0·00170

Äther 0–50°

$\mu = 74$ $\rho = 0{\cdot}736$,

0·000189

~~0·00127~~ 0·00111

Alk 0–40° $\mu = \frac{46}{2}$ $\rho = 0{\cdot}794$

0·000100

Petroleum 0·00099

0·000069

$\rho = 0{\cdot}76$

Pentan 0–30° 0·00159

0·000292

$\mu = \frac{72}{2}$ $\rho = 0{\cdot}6263$

98% H_2SO_4 : 18° 0·00055 $= \alpha$

$\beta = 0{\cdot}0000462 \cdot 0{\cdot}730$

3234
1386

$\rho = 1{\cdot}841$ $3{\cdot}3726.10^{-11}$

$\mu = \begin{matrix}2\\32\\64\end{matrix} = \frac{98}{2}$

Hg : $\frac{1}{5500}$

(Amagat) 0·0000039

$\rho = 13{\cdot}6$

$\mu = \frac{200}{2}$

$R = \frac{\kappa}{\mu}$

$R = \frac{1000}{0{\cdot}001293 \cdot 273}$ ~~76. 13·6.~~ 980

9912
5478
4434

1116
4362
5478

$= 2{\cdot}776 \cdot 10^6$

$\lg R = 6{\cdot}4434$

$+\lg\sqrt{2} = 0{\cdot}1505$

6·5939

$\frac{0{\cdot}00055^2 \cdot 291 \cdot 49}{2{\cdot}78 \cdot 10^6 \cdot 1{\cdot}84 \cdot 1{\cdot}42}$

121.25

3025 .29
6050
27225
87725

87725.49
350900
789525
42985

2·78. 184
2224
111
5115.142
20460
102
7263

$42985 : 7263 = 5{\cdot}918 \cdot 10^{-11}$
6070
133
60

CO_2: 0.000196
188
8

$\frac{40.3}{32.6}$ $\frac{7.7}{2.305.6}$ =

$8 : 188 =$

$2 : 47 = 0.042$

$3 : 94 = 0.033$

$7.7 : 611 = 0.0126$

5

$\rho = \frac{1}{v} \frac{\partial v}{\partial p}$

$v\beta = -\frac{\delta^2}{\sqrt{2}R\theta}$

$-\frac{2a}{v^3}\frac{\partial v}{\partial \theta} = \frac{R\sqrt{2}}{v-v_0} - \frac{R\theta\sqrt{2}}{(v-v_0)^2}\frac{\partial v}{\partial \theta}$

$v.\beta = -\frac{\delta^2}{\sqrt{2}R\theta}$ cub.

$v.\alpha = \frac{\delta}{\theta}$

$\left[-\frac{2a}{v^3} + \frac{R\theta\sqrt{2}}{(v-v_0)^2}\right]\frac{\partial v}{\partial \theta} = \frac{R\sqrt{2}}{v-v_0}$

$\beta = -\frac{\theta\alpha^2 v}{\sqrt{2}R}$

$\left[-\frac{2R\theta\sqrt{2}}{(v-v_0)v} + \frac{R\theta\sqrt{2}}{(v-v_0)^2}\right]\frac{\partial v}{\partial \theta} =$

$\boxed{\beta = -\frac{\alpha^2\theta}{\sqrt{2}R_p}}$

$\therefore R\theta\sqrt{2}\left[-2(v-v_0)+v\right]\frac{\partial v}{\partial \theta} = R\sqrt{2}.\ v(v-v_0)$

$2v_0 - v$

$\frac{\partial v}{\partial \theta} = \frac{\delta}{\theta} = \frac{v-v_0}{\theta}$

$$\int_{-\infty}^{+\infty} (a x^2 + b y^2 + \cdot) e^{-h(x^2+y^2)} \, dx \, dy \ldots$$

$$\int_{-\infty}^{+\infty} e^{-hx^2} dx = \sqrt{\frac{\pi}{ah}} \qquad a\int x^2 e^{-ahx^2} dx = \cancel{ax \frac{e^{-hx^2}}{-2h}} + \int \frac{e^{-ahx^2}}{2h} dx = \frac{1}{2h}\sqrt{\frac{\pi}{ah}}$$

$$\frac{d}{dh}\left(\sqrt{\frac{\pi}{h}}\right)^n \qquad \int_{-\infty}^{+\infty} e^{-h(x^2+y^2\ldots)} dx \ldots dy = \left(\sqrt{\frac{\pi}{h}}\right)^n$$

$$\frac{c}{A}\frac{d\theta}{dx} + \frac{p}{b}\frac{du}{dx} = \frac{\sigma\rho}{b}$$

$$\frac{1}{\rho}\frac{dp}{dx} = -g$$

$$\rho u = b$$

$$\frac{p}{\rho} = R\theta$$

$$\frac{p}{\rho u}\frac{du}{dx} = R\theta \cdot \frac{1}{u}\frac{du}{dx}$$

$$= -R\theta \frac{1}{\rho}\frac{d\rho}{dx}$$

$$\boxed{\rho = \frac{p}{R\theta}}$$

$$= -\frac{R\theta}{\rho}\left(\frac{1}{\theta}\frac{dp}{dx} - \frac{p}{\theta^2}\frac{d\theta}{dx}\right)$$

$$= -\frac{1}{\rho}\frac{dp}{dx} + \frac{p}{\rho\theta}\frac{d\theta}{dx}$$

$$= g + R\frac{d\theta}{dx} \qquad \frac{k-1}{k} = \frac{2}{7}$$

$$d\rho\sigma = \rho g u_0 = \rho g \frac{dx}{dt} = dp$$

$$\sigma\rho = R\rho \frac{d\theta}{dx}\frac{dx}{dt}$$

$$\delta\varphi = \alpha . \phi . \delta t$$

$$= c\phi . \delta x . \frac{g}{R}\ \frac{k-1}{k}$$

$$\frac{\delta x}{\delta t} = \frac{\alpha}{c}\frac{R}{g}$$

$$\cancel{\beta = \frac{\alpha^2 \cdot \ldots \cdot \theta}{2 . \rho . R}}$$

$$\frac{42 . 10 . 0{\cdot}00033}{4 . 10^7 \quad 20} \cdot \frac{10^6}{10^3} \quad \frac{}{b}$$

$$= \frac{0{\cdot}6 . 10^3}{b} \text{ cm} =$$

$$= \frac{0{\cdot}6 . 10^3}{0{\cdot}001 . u} \text{ cm} = \frac{6 . 10^5}{u} \text{ cm}$$

$$= \frac{6 . 10^3}{u_0} \text{ m} = \frac{6}{u_0} \text{ km}$$

Müller p. 298 [illegible]

Met. Ztschr. 10 (1893)

[illegible]

[illegible]

[illegible]

[illegible] ~~[illegible]~~

Durchl.:	Σ – Δt	[illegible]	Nach den [illegible]
		0.61	49
0–1000 m	0.58	0.54	50
200	0.56	0.54	54
3	0.54	0.54	53
4	0.53	0.64	64
5	0.64	0.69	64
6	0.69	0.68	68
7	0.68	0.72	72
8	0.~~72~~ 67	0.90	80
[illegible]00–9000	0.90		80
			63[illegible]

B. N. :.................. Sygnatura: ....

Dzieło: (Autor) ....

(Tytuł) ....

....

(Stan dzieła) ....

tom wol.

pożyczyłem z c. k. biblioteki uniwersyteckiej i zwrócę je w ciągu tygodni .... przyjmując odpowiedzialność za wszelkie uszkodzenia

We Lwowie dnia

Mieszkanie: Podpis:

G. Cantoni: Su alcune condizioni fisiche dell' affinità e sul moto browniano. Rendic. Lomb. (2) I, p. 56, Cimento XXVII. 156.

[illegible] Affinität [illegible]

[illegible]

P. Mol. [illegible]

[illegible]

[illegible]

[illegible] H_2 reducirte [illegible] Ag Cu Fe | S | C

[illegible]

[illegible] $\frac{1}{1600}$ mm [illegible]

[illegible] Alkohol [illegible] Äther [illegible]

[illegible] H_2O

[illegible] Atom [illegible]

B. N. .... Sygnatura: ....

Dzieło: (Autor) ....

(Tytuł) ....

....

(Stan dzieła) ....

tom wol.

pożyczyłem z c. k. biblioteki uniwersyteckiej i zwrócę je w ciągu tygodni .... przyjmując odpowiedzialność za wszelkie uszkodzenia.

We Lwowie dnia

Mieszkanie: Podpis:

Tytuł dzieła:

Sygnatura:

Data:	Nazwisko pożyczającego:

[illegible]

$v = \int p^2 \cos$ [illegible]

[illegible]

$0 = e^{-x}$ [illegible]

[illegible]

5:

[illegible]

Próba czy $\Delta^2 \ldots = \mathrm{div}'$

$$\Delta^2 \left[\frac{27}{32}\frac{a}{r^2} + \frac{3}{4}\left(\frac{x^2}{r^3} - \frac{15}{8}\frac{ax^2}{r^4} - \frac{1}{4}\frac{a^3x^2}{r^6}\right)\right] =$$

$$-6\frac{x^2}{r^5}$$

$$\frac{27}{32}\frac{2a}{r^4} + \frac{3}{4}\left[2\frac{1}{r^3} - \frac{15a}{8}\left(\frac{2}{r^4} - \frac{4x^2}{r^6}\right) - \frac{1}{4}a^3\left(\frac{2}{r^6} + 6\frac{x^2}{r^8}\right)\right]$$

$$= \frac{27}{16}\frac{a}{r^4} + \frac{3}{2}\frac{1}{r^3} - \frac{45a}{16r^4} + \frac{45}{8}\frac{ax^2}{r^6} - \frac{3a^3}{8r^6} - \frac{9}{8}\frac{a^3x^2}{r^8} - \frac{9}{2}\frac{x^2}{r^5}$$

$$- \cancel{\frac{18}{16}}\frac{9}{8}\frac{a}{r^4}$$

$$= \frac{3}{2}\frac{1}{r^3}\left[1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right] - \frac{9}{2}\frac{x^2}{r^5}\left[1 - \frac{5}{4}\frac{a}{r} + \frac{1}{4}\frac{a^3}{r^3}\right]$$

zatem $\quad k\,\mathrm{div}' = \Delta^2\left\{+\mu c^2 a\left[\frac{27}{32}\frac{a}{r^2} + \frac{3}{4}\left(\ldots \right)\right]\right\}$

$$\mathrm{div}' = \Delta^2 \psi$$

Rozwiązanie systemu równań

$$\begin{cases} \dfrac{\partial p'}{\partial x} = \dfrac{\mu}{3}\dfrac{\partial\,\mathrm{div}'}{\partial x} + \mu\,\Delta^2 u' \\ \dfrac{\partial p'}{\partial y} = \\ \dfrac{\partial p'}{\partial z} = \end{cases}$$

$$\Delta^2 p' = \frac{4\mu}{3}\Delta^2\,\mathrm{div}'$$

$$\underline{\underline{p' = \frac{4\mu}{3}\,\mathrm{div}' + \varphi}} \qquad \Delta^2\varphi = 0$$

$$\mu\frac{\partial\,\mathrm{div}'}{\partial x} + \frac{\partial\varphi}{\partial x} = \mu\,\Delta^2 u' \qquad u' = \frac{\partial\psi}{\partial x} + U$$

$$\begin{cases} \dfrac{\partial\varphi}{\partial x} = \mu\,\Delta^2 U \\ \dfrac{\partial\varphi}{\partial y} = \mu\,\Delta^2 V \\ \dfrac{\partial\varphi}{\partial z} = \mu\,\Delta^2 W \end{cases} \qquad \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0$$

D.U. Z wzorów poprzednich

$$U = -\frac{27\mu c^2 a}{32\,k}\left[-\frac{x}{r^3} + \frac{1}{3}\frac{x^3}{r^5} - \frac{a^2 x}{r^5}\left(3 - 5\frac{x^2}{r^2}\right)\right]$$

$$V = -\frac{27\mu c^2 a}{32\,k}\left[-\frac{y}{r^3} + \frac{1}{3}\frac{x^2 y}{r^5} - \frac{a^2 y}{r^5}\left(1 - 5\frac{x^2}{r^2}\right)\right]$$

$$W =$$

$$\mu\Delta^2 U = -\frac{27\mu^2 c^2 a}{32\,k}\left(\frac{2x}{r^5} - \frac{10}{3}\frac{x^3}{r^7}\right) = \frac{\partial}{\partial x}\left[-\frac{27\mu^2 c^2 a}{32k}\,\frac{2}{3}\left(-\frac{1}{3r^3} + \frac{x^2}{r^5}\right) = \frac{9}{16}\frac{\mu^2 c^2 a}{k}\left[\frac{1}{3r^3} - \frac{x^2}{r^5}\right]\right.$$

$$\mu\Delta^2 V = -\frac{27\mu^2 c^2 a}{32\,k}\left(\frac{2}{3}\frac{y}{r^5} - \frac{10}{3}\frac{x^2 y}{r^7}\right) = \frac{\partial}{\partial y}$$

$$\left.\begin{aligned}\frac{2}{3}\frac{\partial}{\partial x}\left(\frac{x^2}{r^5}\right) &= \frac{4x}{3r^5} - \frac{10x^3}{3r^7}\\ -\frac{2}{9}\frac{\partial}{\partial x}\left(\frac{1}{r^3}\right) &= -\frac{3x}{r^5}\cdot\frac{2}{9}\end{aligned}\right\}\quad \begin{aligned}&\frac{\partial}{\partial x}\left|\frac{2}{3}\left(-\frac{1}{3r^3} + \frac{x^2}{r^5}\right)\right.\\ &\frac{\partial}{\partial y} = \frac{2}{3}\left(+\frac{y}{r^5} - \frac{5x^2 y}{r^7}\right)\end{aligned}$$

$$\varphi = \frac{9}{16}\,\frac{\mu^2 c^2 a}{k}\left[\frac{1}{3r^3} - \frac{x^2}{r^5}\right]$$

$$p = P + (p_0) - \frac{3}{2}\mu c a\frac{x}{r^3} + \frac{1}{P}\left\{\frac{4\mu}{3k}\left[\frac{3}{2}\mu a c^2\frac{1}{r^3}\left(1 - \frac{3}{4}\frac{a}{2} - \frac{1}{4}\frac{a^3}{r^3}\right) - \right.\right.$$

$$\left.\left. -\frac{9}{2}\mu c^2 a\,\frac{x^2}{r^5}\left(1 - \frac{5}{4}\frac{a}{2} + \frac{1}{4}\frac{a^3}{r^3}\right)\right] + \frac{9}{16}\frac{\mu^2 c^2 a}{k}\left[\frac{1}{3r^3} - \frac{x^2}{r^5}\right]\right\}$$

$$\{\,\} = \frac{\mu^2 c^2 a}{k}\left|\begin{aligned}&\frac{2}{r^3}\left(1 - \frac{3}{4}\frac{a}{2} - \frac{1}{4}\frac{a^3}{r^3}\right) + \frac{9}{16}\frac{1}{3r^3}\\ &-\frac{6x^2}{r^5}\left(1 - \frac{5}{4}\frac{a}{2} + \frac{1}{4}\frac{a^3}{r^3}\right) - \frac{9}{16}\frac{x^2}{r^5}\end{aligned}\right|$$

$\frac{\partial}{\partial x}$

$+\frac{\partial}{\partial y}$

$+\frac{\partial}{\partial z}$

$$u_0\frac{\partial u_0}{\partial x}+v_0\frac{\partial u_0}{\partial y}+w_0\frac{\partial u_0}{\partial z}=\frac{9}{16}c^2a^2\frac{x}{r^3}\left(1-\frac{a^2}{r^2}\right)\left[x^2\left(M+\frac{2}{r^3}-\frac{2a^2}{r^5}\right)+y^2M+z^2M\right]$$

$$-\frac{3}{4}c^2a\frac{x}{r^3}\left(1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right)\left(1-\frac{3x^2}{r^2}-\frac{3a^2}{r^2}+\frac{5a^2x^2}{r^4}\right)$$

$$=\frac{9}{16}c^2a^2\frac{x}{r^3}\left(1-\frac{a^2}{r^2}\right)\Big[\underbrace{-\frac{1}{r}-\frac{3x^2}{r^3}-\frac{a^2}{r^3}+\frac{5a^2x^2}{r^5}+\frac{2x^2}{r^3}-\frac{2a^2x^2}{r^5}}_{-\frac{1}{r}-\frac{x^2}{r^3}-\frac{a^2}{r^3}+\frac{3a^2x^2}{r^5}}\Big]-\uparrow$$

$$=\cancel{\frac{3}{4}c^2\left\{\frac{3}{4}\frac{a^2x}{r^2}\right.}$$

$$=\frac{3}{4}\frac{c^2ax}{r^3}\left\{\frac{3}{4}\left[-\frac{\cancel{a}}{\cancel{r}}-\frac{ax^2}{r^3}-\frac{a^3}{r^3}+\frac{3a^3x^2}{r^5}+\frac{a^3}{r^3}+\cancel{\frac{a^3x^2}{r^5}}+\frac{\cancel{a^5}}{\cancel{r^5}}-\frac{3a^5x^2}{r^7}\right]\right.$$

$$-1+\frac{3x^2}{r^2}+\frac{3a^2}{r^2}-\frac{5a^2x^2}{r^4}+\frac{3}{4}\frac{\cancel{a}}{\cancel{r}}-\frac{9}{4}\frac{x^2a}{r^3}-\frac{9a^3}{4r^3}+\frac{15a^3x^2}{4r^5}+\frac{a^3}{4r^3}-$$

$$\left.-\cancel{\frac{3a^3x^2}{4r^5}}-\frac{3\cancel{a^5}}{4\cancel{r^5}}+\frac{5a^5x^2}{4r^7}\right\}$$

$$=\frac{3}{4}\frac{c^2ax}{r^3}\left\{-1+\frac{3x^2}{r^2}+\frac{3a^2}{r^2}-\frac{5a^2x^2}{r^4}-3\frac{ax^2}{r^3}-2\frac{a^3}{r^3}+6\frac{a^3x^2}{r^5}-\frac{a^5x^2}{r^7}\right\}$$

$$\left.\begin{aligned}&\frac{\partial}{\partial x}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)\\&+\frac{\partial}{\partial y}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)\\&+\frac{\partial}{\partial z}\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)\end{aligned}\right\}=\begin{aligned}&\left(\frac{\partial u}{\partial x}\right)^2+\frac{\partial v}{\partial x}\frac{\partial u}{\partial y}+\frac{\partial w}{\partial x}\frac{\partial u}{\partial z}\quad\underbrace{+u\frac{\partial}{\partial x}\operatorname{div}+v\frac{\partial}{\partial y}\operatorname{div}+w\frac{\partial}{\partial z}\operatorname{div}}_{=0}\\&+\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}+\left(\frac{\partial v}{\partial y}\right)^2+\frac{\partial w}{\partial y}\frac{\partial v}{\partial z}\\&+\ \cdot\end{aligned}$$

$$=\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2+2\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}+2\frac{\partial v}{\partial z}\frac{\partial w}{\partial y}+2\frac{\partial w}{\partial x}\frac{\partial u}{\partial z}$$

$$-\frac{1}{2}\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)^2+\cdot$$

$$=\frac{1}{2}\left[\Phi-\xi^2-\eta^2-\zeta^2\right]$$

Czy $f_1 = \frac{\partial U}{\partial x}$ $\quad f_2 = \frac{\partial U}{\partial y}$?

W takim razie mamy $\quad \frac{\partial f_1}{\partial y} = \frac{\partial f_2}{\partial x}$

$$\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) - \frac{\partial}{\partial x}\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) =$$

$$= u\left(\frac{\partial^2 u}{\partial x \partial y} - \frac{\partial^2 v}{\partial x^2}\right) + v\left(\frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 v}{\partial x \partial y}\right) + w\left(\frac{\partial^2 u}{\partial x \partial y} - \frac{\partial^2 v}{\partial x \partial z}\right)$$

$$+ \frac{\partial u}{\partial x}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + \frac{\partial v}{\partial y}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + \frac{\partial w}{\partial y}\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\frac{\partial v}{\partial z}$$

$$= -\frac{\partial w}{\partial z}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)$$

t.zn. czy $\operatorname{curl}\underbrace{(\sigma\nabla)\sigma}_{\frac{1}{2}\nabla(\sigma^2) + \nabla\sigma \operatorname{curl}\sigma} = 0 \quad \Big\} = \operatorname{curl} \nabla\sigma \operatorname{curl}\sigma = (\sigma\nabla)\operatorname{curl}\sigma - (\operatorname{curl}\sigma\,\nabla)\sigma$

To byłoby oczywiście w każdym razie tam spełnione gdzie ruch taki sam, jaki by miał ośrodek bez tarcia ~~t.zn. to byłoby razem w tych warunkach~~

$$u\frac{\partial \xi_1}{\partial x} + v\frac{\partial \xi_1}{\partial y} + w\frac{\partial \xi_1}{\partial z} - \left(\xi\frac{\partial u}{\partial x} + \eta\frac{\partial u}{\partial y} + \zeta\frac{\partial u}{\partial z}\right)$$

$$\eta\frac{\partial u}{\partial y} + \zeta\frac{\partial u}{\partial z} = 0$$

$$u\frac{\partial \eta}{\partial x} + v\frac{\partial \eta}{\partial y} + w\frac{\partial \eta}{\partial z} - \left(\cancel{\xi\frac{\partial v}{\partial x}} + \eta\frac{\partial v}{\partial y} + \zeta\frac{\partial v}{\partial z}\right) =$$

$$\frac{\partial \eta}{\partial x} = +\frac{3}{2}\frac{ca\,xz}{r^5} \qquad \frac{\partial \eta}{\partial y} = +\frac{3}{2}\frac{ca\,yz}{r^5} \qquad \frac{\partial \zeta}{\partial z} = -\frac{1}{2}\frac{ca}{r^3} + \frac{3}{2}\frac{ca\,z^2}{r^5}$$

$$= \frac{\mu^2 c^2 a}{k}\left[\frac{1}{r^3}\left(\frac{35}{16} - \frac{3}{2}\frac{a}{r} - \frac{1}{2}\frac{a^3}{r^3}\right) - 3\frac{x^2}{r^5}\left(\frac{35}{16} - \frac{5}{2}\frac{a}{r} + \frac{1}{2}\frac{a^3}{r^3}\right)\right]$$

$$p = P - \frac{3}{2}\mu a c \frac{x}{r^3} + \frac{\mu^2 c^2 a}{kP}\frac{1}{2r^3}\left[\left(\frac{35}{8} - \frac{3a}{r} - \frac{a^3}{r^3}\right) - \frac{3x^2}{r^2}\left(\frac{35}{8} - \frac{5a}{r} + \frac{a^3}{r^3}\right)\right]$$

Rozmiary $P \ldots \frac{\mu c}{a}$ $\mu c \propto aP$

Tożsamo: $\frac{\partial p}{\partial x}$ $\mu \frac{\partial^2 u}{\partial x^2}$ zgadza się

Równanie termiczne [illegible]:

$$k \operatorname{div}' = -\left[u_0 \frac{\partial p_0}{\partial x} + v_0 \frac{\partial p_0}{\partial y} + w_0 \frac{\partial p_0}{\partial z}\right] + (k-1)\mu\left[\underbrace{\left(\frac{\partial u_0}{\partial x}\right)^2 + \cdots}_{\Phi} + \kappa \Delta^2 \theta\right]$$

Jeżeli się nie robi $\kappa = \infty$ izotermicznie to Φ tegosamego rzędu wielkości co .. zatem adiabatyczne rozwiązanie nieracjonalne!

$$\frac{\partial u_0}{\partial x} = -\frac{3}{4} ca\left[\frac{2x}{r^3} - \frac{3x^3}{r^3} - \frac{2a^2 x}{r^5} + \frac{5a^2 x^3}{r^7}\right] + c\left[\frac{3}{4}\frac{ax}{r^3} + \frac{3}{4}\frac{a^3 x}{r^5}\right]$$

$$\frac{\partial u_0}{\partial x} = -\frac{3}{4} ca\left[\frac{x}{r^3} - \frac{3x^3}{r^5} - \frac{3a^2 x}{r^5} + \frac{5a^2 x^3}{r^7}\right]$$

$$\frac{\partial v_0}{\partial y} = -\frac{3}{4} ca\left[\frac{x}{r^3} - \frac{3xy^2}{r^5} - \frac{a^2 x}{r^5} + \frac{5a^2 x y^2}{r^7}\right]$$

$$\frac{\partial w_0}{\partial z} = -\frac{3}{4} ca\left[\frac{x}{r^3} - \frac{3xz^2}{r^5} - \frac{a^2 x}{r^5} + \frac{5a^2 x z^2}{r^7}\right]$$

$$\frac{\partial u_0}{\partial y} = -\frac{3}{4} ca\left[-\frac{3x^2 y}{r^5} + \frac{5a^2 x^2 y}{r^7}\right] + c\left[\frac{3ay}{4r^3} + \frac{3a^3 y}{4r^5}\right]$$

$$\frac{\partial u_0}{\partial y} = -\frac{3}{4} ca\left[-\frac{3x^2 y}{r^5} + \frac{5a^2 x^2 y}{r^7} - \frac{y}{r^3} - \frac{a^2 y}{r^5}\right]$$

$$\frac{\partial u_0}{\partial z} = -\frac{3}{4} ca\left[-\frac{3x^2 z}{r^5} + \frac{5a^2 x^2 z}{r^7} - \frac{z}{r^3} - \frac{a^2 z}{r^5}\right].$$

$$\frac{\partial v_0}{\partial x} = -\frac{3}{4} ca \left[\left(1-\frac{a^2}{r^2}\right)\frac{y}{r^3} - \frac{3x^2y}{r^5}\left(1-\frac{a^2}{r^2}\right) + \frac{2a^2x^2y}{r^7}\right]$$

$$\frac{\partial v_0}{\partial x} = -\frac{3}{4} ca \left[\left(1-\frac{a^2}{r^2}\right)\frac{y}{r^3} - \frac{3x^2y}{r^5} + \frac{5a^2x^2y}{r^7}\right]$$

$$\frac{\partial v_0}{\partial z} = -\frac{3}{4} ca \left[\frac{2a^2zxy}{r^7} - \frac{3xyz}{r^5}\left(1-\frac{a^2}{r^2}\right)\right]$$

$$\left.\begin{array}{l}\dfrac{\partial v_0}{\partial z} = \\[2ex] \dfrac{\partial w_0}{\partial y} = \end{array}\right\} = -\frac{3}{4} ca \left[-\frac{3xyz}{r^5} + \frac{5a^2xyz}{r^7}\right] =$$

$$\frac{\partial w_0}{\partial x} = -\frac{3}{4} ca \left[\left(1-\frac{a^2}{r^2}\right)\frac{z}{r^3} - \frac{3x^2z}{r^5} + \frac{5a^2x^2z}{r^7}\right]$$

$$\frac{\partial u_0}{\partial x} = -\frac{3}{4} ca \; x\left[\frac{1}{r^3} - \frac{3x^2}{r^5} - \frac{3a^2}{r^5} + \frac{5a^2x^2}{r^7}\right]$$

$$\frac{\partial u_0}{\partial y} = -\frac{3}{4} ca \; y\left[-\frac{1}{r^3} - \frac{3x^2}{r^5} - \frac{a^2}{r^5} + \frac{5a^2x^2}{r^7}\right]$$

$$\frac{\partial u_0}{\partial z} = -\frac{3}{4} ca \; z\left[\qquad\qquad\right]$$

$$\frac{\partial v_0}{\partial x} = -\frac{3}{4} ca \; y\left[\frac{1}{r^3} - \frac{3x^2}{r^5} - \frac{a^2}{r^5} + \frac{5a^2x^2}{r^7}\right]$$

$$\frac{\partial v_0}{\partial y} = -\frac{3}{4} ca \; x\left[\frac{1}{r^3} - \frac{3y^2}{r^5} - \frac{a^2}{r^5} + \frac{5a^2y^2}{r^7}\right]$$

$$\frac{\partial v_0}{\partial z} = -\frac{3}{4} ca \cdot \frac{xyz}{r^5}\left[-3 + \frac{5a^2}{r^2}\right]$$

$$\frac{\partial w_0}{\partial y} =$$

$$\frac{\partial w_0}{\partial x} = -\frac{3}{4} ca \; z\left[\frac{1}{r^3} - \frac{3x^2}{r^5} - \frac{a^2}{r^5} + \frac{5a^2x^2}{r^7}\right]$$

$$\frac{\partial w_0}{\partial z} = -\frac{3}{4} ca \; \cancel{x}\left[\frac{1}{r^3} - \frac{3z^2}{r^5} - \frac{a^2}{r^5} + \frac{5a^2z^2}{r^7}\right]$$

Przybliżone uwzględnienie inercyi:

$\frac{p}{\rho} = R\theta$ czem większe R, tem mniejsze ρ zatem tem mniejszy wpływ inercyi. ~~Jeżeli R~~ Zatem rozwinięcie według

$\frac{p}{\rho} = \frac{p_0}{\rho_0}$ potęg $n \sim \frac{1}{R} \sim \rho_0$

$$\begin{matrix} u \\ v \\ w \end{matrix} = u_0 + \rho_0 u' + \frac{\rho_0^2}{2} u'' +$$

$$\rho_0 \frac{p}{p_0}\left\{u_0 \frac{\partial u_0}{\partial x} + \cdots + \rho_0\left[u_0 \frac{\partial u'}{\partial x} + u' \frac{\partial u_0}{\partial x} + \cdots\right]\right\} + \frac{\partial p_0}{\partial x} + \rho_0 \frac{\partial p'}{\partial x} =$$
$$= \mu\left[\Delta^2 u_0 + \rho_0 \Delta^2 u' + \cdots\right]$$

$$p_0 \operatorname{div}_0 + \rho_0 (p' \operatorname{div}_0 + p_0 \operatorname{div}') + k\left(u_0 \frac{\partial p_0}{\partial x} + \cdots\right) + k\rho_0\left[u' \frac{\partial p_0}{\partial x} + u_0 \frac{\partial p'}{\partial x} + \cdots\right] =$$
$$= (k-1)\Phi_0 + \rho_0 \Psi$$

$$\begin{cases} \frac{\partial p_0}{\partial x} = \frac{\mu}{3} \frac{\partial}{\partial x} \operatorname{div}_0 + \mu \Delta^2 u_0 \\ p_0 \operatorname{div}_0 + k\left(u_0 \frac{\partial p_0}{\partial x} + \right) = (k-1)\Phi_0 \end{cases}$$

Poprawka:

$$\left.\begin{aligned} \left(u_0 \frac{\partial u_0}{\partial x} + \cdots\right)_{f_1} + \frac{\partial p'}{\partial x} &= \frac{\mu}{3} \frac{\partial}{\partial x} \operatorname{div}' + \mu \Delta^2 u' \\ f_2 + \frac{\partial p'}{\partial y} &= \frac{\mu}{3} \frac{\partial}{\partial y} \operatorname{div}' + \mu \Delta^2 v' \\ \text{Jednocześnie: } f_3 + \frac{\partial p'}{\partial z} &= \frac{\mu}{3} \frac{\partial}{\partial z} \operatorname{div}' + \mu \Delta^2 w' \end{aligned}\right\}$$

$$p' \operatorname{div}_0 + p_0 \operatorname{div}' + \left(u' \frac{\partial p_0}{\partial x} + \cdots\right) + \left(u_0 \frac{\partial p'}{\partial x} + \cdots\right) = 0$$

$$\Delta^2 p' + \underbrace{\left(\frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}\right)} = \frac{4\mu}{3} \Delta^2 \operatorname{div}'$$

$$\underbrace{\operatorname{div}\left[(\sigma\nabla)\sigma\right]}_{= F} = \operatorname{div}\left[\nabla \frac{\sigma^2}{2} + \overline{V} \sigma \operatorname{curl} \sigma\right] = \nabla^2 \frac{\sigma^2}{2} + \sigma \operatorname{curl}^2 \sigma - (\operatorname{curl} \sigma)^2$$

$$p' = \frac{4\mu}{3} \operatorname{div}' - \int \frac{F \, dv}{4\pi r} + \varphi$$

Dla cieczy, albo wejdzie jeśli p $\overset{\text{lndro}}{\text{otrze}}$ w porównaniu do $\frac{\partial p}{\partial x}$ etc.

$$f_1 + \frac{\partial p'}{\partial x} = \mu \Delta^2 u'$$
$$f_2 + \frac{\partial p'}{\partial y} = \mu \Delta^2 v'$$
$$f_3 + \frac{\partial p'}{\partial z} = \mu \Delta^2 w'$$

$$\underbrace{\frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}}_{F} + \Delta^2 p' = \mu \Delta^2 \cancel{\operatorname{div}'} = 0$$

$$p' = \underbrace{\int \frac{F\,dv}{4\pi r}}_{\Omega} + \varphi$$

$$u' = u + U$$

$$f_1 = \mu \Delta^2 u$$

$$u = \frac{1}{\mu}\int \frac{f_1\,dv}{4\pi r}$$

etc.

$$\frac{\partial p'}{\partial x} = \mu \Delta^2 U$$
$$\frac{\partial p'}{\partial y} = \mu \Delta^2 V$$
$$\frac{\partial p'}{\partial z} = \mu \Delta^2 W$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = -\left(\frac{\partial u}{\partial x} + \dots\right)$$

$$\mu \Delta^2 \operatorname{div} \mathfrak{U} = \Delta^2 p' = -\mu \Delta^2 \operatorname{div} u$$

$$p' = -\mu \operatorname{div} u + \varphi \quad \| \quad \Delta^2 \varphi = 0$$

$$\cancel{u' = \int \frac{\operatorname{div} u}{r}\,\omega - \operatorname{curl} \int \frac{\operatorname{curl} u'}{r}}$$

$$\cancel{u' = \frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y} + \frac{\partial \chi}{\partial z}}$$

$$-\mu \frac{\partial}{\partial x} \operatorname{div} u + \frac{\partial \varphi}{\partial x} = \mu \Delta^2 U$$
$$-\mu \frac{\partial}{\partial y} \operatorname{div} u + \frac{\partial \varphi}{\partial y} = \mu \Delta^2 V$$
$$-\mu \frac{\partial}{\partial z} \operatorname{div} u + \frac{\partial \varphi}{\partial z} = \mu \Delta^2 W$$

$$\Delta^2 \varphi = 0$$

$$-\mu \nabla \operatorname{div} u + \nabla \varphi = \mu \nabla^2 \mathfrak{U}$$

$$f_1 + \frac{\partial \Omega}{\partial x} + \frac{\partial \varphi}{\partial x} = \mu \Delta^2 u'$$
$$f_2 + \frac{\partial \Omega}{\partial y} + \frac{\partial \varphi}{\partial y} = \mu \Delta^2 v'$$
$$f_3 + \frac{\partial \Omega}{\partial z} + \frac{\partial \varphi}{\partial z} = \dots$$

$$F_1 + p' = \Delta^2 \varphi_1$$
$$F_2 + p' = \Delta^2 \varphi_1$$

$$\frac{\partial f_1}{\partial y} - \frac{\partial f_2}{\partial x} = \mu \Delta^2 \zeta$$

$$\zeta = \int \frac{\frac{\partial f_1}{\partial y} - \frac{\partial f_2}{\partial x}}{\quad} +$$

$$\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = \cdot\ 2\frac{yz}{r^5}x\left[-3+\frac{5a^2}{r^2}\right]$$

$$\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 2z\left[-\frac{3x^2}{r^5} - \frac{a^2}{r^5} + \frac{5a^2x^2}{r^7}\right] = 2\frac{z}{r^3}\left[-\frac{a^2}{r^2} + \frac{x^2}{r^2}\overbrace{\left[-3+\frac{5a^2}{r^2}\right]}^{N}\right]$$

$$\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = 2y\left[\underbrace{\qquad\qquad}_{M}\right]$$

$$\cancel{\frac{\partial u}{\partial x} = x\left(M + \frac{1}{r^3} - \frac{2a^2}{r^5}\right)}$$

$$\cancel{\frac{\partial v}{\partial y} = x\left(M + \frac{1}{r^3}\right)}$$

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2 =$$

$$\frac{x^2}{r^6}\left\{\left[\left(1-\frac{3a^2}{r^2}\right) - x^2\left(\frac{3}{r^2}-\frac{5a^2}{r^4}\right)\right]^2 + \left[\left(1-\frac{a^2}{r^2}\right) - y^2\left[\frac{3}{r^2}-\frac{5a^2}{r^4}\right]\right]^2 + \left[\left(1-\frac{a^2}{r^2}\right) - z^2\left(\quad\right)\right]^2\right\}$$

$$= \frac{x^2}{r^6}\left\{\begin{aligned} &1 - 6\frac{a^2}{r^2} + 9\frac{a^4}{r^4} \qquad -2r^2\left(\frac{3}{r^2}-\frac{5a^2}{r^4}\right) \\ &+1 - \frac{2a^2}{r^2} + \frac{a^4}{r^4} \qquad +2\left(\frac{3a^2}{r^2}x^2 + \frac{a^2y^2}{r^2} + \frac{a^2z^2}{r^2}\right)\left(\frac{3}{r^2}-\frac{5a^2}{r^4}\right) \\ &+1 - \frac{2a^2}{r^2} + \frac{a^4}{r^4} \qquad +(x^4+y^4+z^4)\left(\frac{3}{r^2}-\frac{5a^2}{r^4}\right)^2 \end{aligned}\right.$$

$$= \frac{x^2}{r^6}\left\{3 - \frac{10a^2}{r^2} + \frac{11a^4}{r^4} - 6 + 10\frac{a^2}{r^2} + \frac{4a^2x^2}{r^2}\left(\frac{3}{r^2}-\frac{5a^2}{r^4}\right) + 6\frac{a^2}{r^2} - \frac{10a^4}{r^4}\right.$$
$$+(x^4+y^4+z^4)\left(\frac{3}{r^2}-\frac{5a^2}{r^4}\right)^2$$

$$= \frac{x^2}{r^{10}}\left\{-3r^4 + 6a^2r^2 + a^4 + 4a^2x^2\left(3 - \frac{5a^2}{r^2}\right) + (x^4+y^4+z^4)\left(3 - \frac{5a^2}{r^2}\right)^2\right\}$$

$$\left(\frac{\partial v}{\partial y} + \right)^2 + (\ )+(\ )^2 = \frac{4}{r^6}\left\{\frac{N^2x^2y^2z^2}{r^4} + (z^2+y^2)\left[-\frac{a^2}{r^2} + \frac{x^2N}{r^2}\right]^2\right\}$$

$$= \frac{4}{r^{10}}\left\{N^2x^2y^2z^2 + a^4(z^2+y^2) - 2a^2x^2(z^2+y^2)N + x^4(z^2+y^2)N^2\right\}$$

$$\overline{\Phi} = \frac{2}{r^{10}} \Big\{ \overset{-3x^2r^4}{4a^4x^2} + 6a^2r^2x^2 - 4a^2x^4N + x^2(x^4+y^4+z^4)N^2 +$$

$$2a^4(z^2+y^2) \underbrace{-4a^2x^2(z^2+y^2)N + 2x^4(z^2+y^2)N^2}\ \underbrace{+2N^2x^2y^2z^2} \Big\}$$

$$-4a^2Nx^2\Big[\underbrace{x^2+z^2+y^2}_{\cancel{(\ )}\,r^2}\Big] + N^2x^2\Big\{\underbrace{x^4+y^4+z^4+2x^2z^2+2x^2y^2+2y^2z^2}_{=N^2x^2r^4}\Big\}$$

$$= \frac{2}{r^{10}}\Big\{ \overset{-3x^2r^4}{a^4(x^2+2r^2-2x^2)} + 6a^2r^2x^2 - 4a^2Nx^2r^2 \cancel{\ } + N^2x^2r^4 =$$

$$\overset{-3x^2r^4}{=} 2r^2a^4 - a^4x^2 + 6a^2r^2x^2 - 4a^2r^2x^2N \cancel{\ } + x^2r^4N^2$$

$$N = -3 + 5\frac{a^2}{r^2}$$

$$\overset{-3x^2r^4}{=} +2a^4r^2 - a^4x^2 + 6a^2r^2x^2 + 12a^2r^2x^2 - 20a^4x^2 - \cancel{\ } +$$

$$+9x^2r^4 - 30a^2x^2r^2 + 25a^4x^2 =$$

$$= +2a^4r^2 + 4a^4x^2 - 12a^2x^2r^2 - \cancel{\ } + 6x^2r^4$$

$$\Phi = 2\cdot\frac{9}{16}\frac{c^2a^2}{r^{10}}\Big\{\qquad\Big\}$$

$$= \frac{9}{8}\frac{c^2a^2}{r^4}\Big\{\Big(\overset{6\frac{x^2}{r^2}+}{2\frac{a^4}{r^4}} - 12\frac{a^2x^2}{r^4} + 4\frac{a^4x^2}{r^6}\Big) \cancel{\ } \Big\}$$

$$k\,\mathrm{div}' + \left(u_0\frac{\partial\mu_0}{\partial x} + \dots\right) = (k-1)\,\Phi\,\mu$$

$$\mathrm{div}' = -\frac{1}{k}\left(u_0\frac{\partial\mu_0}{\partial x} \dots\right) + \frac{k-1}{k}\,\mu\,\Phi \quad + u\ (\ )$$

zatem chodzi o wyrażenie Φ jako $\Delta^2\chi$

Do dyspozycji:

$$\alpha \quad \Delta^2\left(\frac{x^4}{r^{10}}\right) = 12\,\frac{x^2}{r^{10}} + 10\,\frac{x^4}{r^{12}}$$

$$\beta \quad \Delta^2\left(\frac{x^4}{r^8}\right) = 12\,\frac{x^2}{r^8} - 8\,\frac{x^4}{r^{10}}$$

$$\gamma \quad \Delta^2\left(\frac{x^2}{r^8}\right) = \frac{2}{r^8} + 24\,\frac{x^2}{r^{10}}$$

$$\delta \quad \Delta^2\left(\frac{x^2}{r^6}\right) = \frac{2}{r^6} + 6\,\frac{x^2}{r^8}$$

$$\varepsilon \quad \Delta^2\left(\frac{x^2}{r^4}\right) = \frac{2}{r^4} - 4\,\frac{x^2}{r^6}$$

$$\zeta \quad \Delta^2\left(\frac{1}{r^4}\right) = \frac{12}{r^6}$$

$$\eta \quad \Delta^2\left(\frac{1}{r^2}\right) = \frac{2}{r^4}$$

$$\vartheta \quad \Delta^2\left(\frac{1}{r^6}\right) = \frac{30}{r^8}$$

~~10a~~

$$10\alpha = 10a^6$$
$$+8\beta = 6a^4$$
$$12\alpha + 24\gamma = 9a^6$$
$$12\beta + 6\delta = -20a^4$$
$$2\gamma + 30\vartheta = 2a^6$$
$$-4\varepsilon = 9a^2$$
$$2\delta + 12\zeta = 0$$
$$2\varepsilon + 2\eta = 0$$

$$4\alpha + 8\gamma = 3a^6$$
$$\gamma = \frac{3a^6 - 4a^6}{8}$$
$$3\delta = -10a^4 - 6\beta = -10a^4 - \frac{9}{2}a^4 = -\frac{29}{2}a^4$$
$$30\vartheta = 2a^6 + \frac{1}{4}a^6 = \frac{9}{4}a^6$$

$$\begin{cases} \alpha = a^6 \\ \beta = \frac{3}{4}a^4 \\ \gamma = -\frac{1}{8}a^6 \\ \delta = -\frac{29}{6}a^4 \\ \vartheta = \frac{3}{40}a^6 \\ \varepsilon = -\frac{9}{4}a^2 \\ \eta = +\frac{9}{4}a^2 \\ \zeta = \cancel{-\frac{1}{6}} = \frac{29}{36}a^4 \end{cases}$$

$$\Phi = \Delta^2\chi$$

$$\chi = \frac{9c^2}{8}\left[\frac{a^6x^4}{r^{10}} + \frac{3}{4}\frac{a^4x^4}{r^8} - \frac{1}{8}\frac{a^6x^2}{r^8} - \frac{29}{6}\frac{a^4x^2}{r^6} - \frac{9}{4}\frac{a^2x^2}{r^4} + \frac{29}{36}\frac{a^4}{r^4} + \frac{9}{4}\frac{a^2}{r^2} + \frac{3}{40}\frac{a^6}{r^6}\right]$$

$$\frac{k-1}{k}\Gamma\chi = \frac{k-1}{k}\mu\,\frac{qc^2}{8}\Big[\frac{a^6x^4}{r^{10}} + \frac{3}{4}\frac{a^4x^4}{r^8} - \frac{1}{8}\frac{a^6x^2}{r^8} - \frac{29}{6}\frac{a^4x^2}{r^6} - \frac{9}{4}\frac{a^2x^2}{r^4}$$

$$+\frac{3}{40}\frac{a^6}{r^6} + \frac{29}{36}\frac{a^4}{r^4} + \frac{9}{4}\frac{a^2}{r^2}\Big]$$

$$\frac{\partial}{\partial x} = \frac{k-1}{k}\mu\,\frac{qc^2}{8}\Big[\frac{4a^6x^3}{r^{10}} + \frac{3a^4x^3}{r^8} - \frac{1}{4}\frac{a^6x}{r^8} - \frac{29}{3}\frac{a^4x}{r^6} - \frac{9}{2}\frac{a^2x}{r^4} +$$

$$-10\frac{a^6x^5}{r^{12}} - 6\frac{a^4x^5}{r^{10}} + \frac{a^6x^3}{r^{10}} + 29\frac{a^4x^3}{r^8} + 9\frac{a^2x^3}{r^6}$$

$$-\frac{9}{20}\frac{a^6x}{r^8} - \frac{29}{9}\frac{a^4x}{r^6} - \frac{9}{2}\frac{a^2x}{r^4}\Big]$$

$r=a$: $\big[\ \big] = -16\frac{x^5}{a^6} + 46\frac{x^3}{a^4} - \frac{2033}{90}\frac{x}{a}$

$$\begin{array}{r} 3 \\ +54 \\ 116 \\ \hline 173 \end{array}$$

$\frac{173}{12} + \frac{99}{20} + \frac{29}{9} =$

$$\begin{array}{r} 173 \\ 865 \\ \hline 2595 \end{array}$$

$= \frac{2595 +}{180}$

$$\begin{array}{r} 2595 \\ 891 \\ 580 \\ \hline 4066 \end{array}$$

$\frac{4066}{180} = \frac{2033}{90}$

$$(\Delta D)_y =$$

$$- \frac{9}{8} \frac{c^2 a^2 x^3 z}{r^8} \left(1 - \frac{a^2}{r^2}\right) + \frac{3}{2} \frac{c^2 a x z}{r^5} \left(1 - \frac{3}{4} \frac{a}{r} - \frac{1}{4} \frac{a^3}{r^3}\right) +$$

$$- \frac{9}{8} \frac{c^2 a^2 x y^2 z}{r^8} \left(\quad \right) - \frac{9}{8} \frac{c^2 a^2 x z^3}{r^8} \left(\quad \right) + \frac{3}{8} \frac{c^2 a^2 x z}{r^6} \left(1 - \frac{a^2}{r^2}\right)$$

$$= - \frac{9}{8} \frac{c^2 a^2 x z}{r^6} \left(\quad \right) + \frac{3}{8} \cdots$$

$$= - \frac{3}{4} \frac{c^2 a^2 x z}{r^6} \left(1 - \frac{a^2}{r^2}\right) + \frac{3}{2} \frac{c^2 a x z}{r^5} \left(\qquad \right)$$

$$- \left\{ \frac{3}{8} \frac{c^2 a^2 x z}{r^3} \left[\frac{1}{r^3} - \frac{3 y^2}{r^5} - \frac{a^2}{r^5} + \frac{5 a^2 y^2}{r^7} \right] - \frac{3}{8} \cancel{\frac{c^2 a^2 x y^2 z}{r^8} \left[-3 + \frac{5 a^2}{r^2} \right]} \right\}$$

$$= \frac{3}{4} \frac{c^2 a^2 x z}{r^6} \left\{ -1 + \frac{a^2}{r^2} + \frac{2 r}{a} - \frac{3}{2} - \frac{1}{2} \frac{a^2}{r^2} - \frac{1}{2} + \frac{a^2}{2 r^2} \right.$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\frac{9}{16}c^2a^2\left(1-\frac{a^2}{r^2}\right)\frac{x}{r^3}\left\{xy\left[\frac{1}{r^3}-\frac{3x^2}{r^5}-\frac{a^2}{r^5}+\frac{5a^2x^2}{r^7}\right]+\right.$$

$$\left.xy\left[\frac{1}{r^3}-\frac{3y^2}{r^5}-\frac{a^2}{r^5}+\frac{5a^2y^2}{r^7}\right]+\frac{xyz^2}{r^5}\left[-3+\frac{5a^2}{r^2}\right]\right\}-\frac{3c^2}{4}a\,y\uparrow\left(1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right)$$

$$=\frac{9}{16}c^2a^2\left(1-\frac{a^2}{r^2}\right)\frac{x}{r^3}\left\{\underbrace{2xy\left(\frac{1}{r^3}-\frac{a^2}{r^5}\right)-3\frac{xy}{r^3}+\frac{5a^2xy}{r^5}}_{-\frac{xy}{r^3}+\frac{3a^2xy}{r^5}}\right\}-\nearrow$$

$$=\frac{9}{16}c^2a^2\left(1-\frac{a^2}{r^2}\right)\frac{x^2y}{r^6}\left[-1+\frac{3a^2}{r^2}\right]-\frac{3}{4}\frac{c^2ay}{r^3}\left[1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right]\left[1-\frac{3x^2}{r^2}-\frac{a^2}{r^2}+\frac{5a^2x^2}{r^4}\right]$$

$$=-\frac{3}{4}\frac{c^2ay}{r^3}\left[1-\frac{3x^2}{r^2}-\frac{a^2}{r^2}+\frac{5a^2x^2}{r^4}\right]+\frac{3}{4}\frac{c^2ay}{r^6}\left\{\frac{3}{4}ax^2\left(-1+\frac{2a^2}{r^2}-\frac{6a^4}{r^4}\right)+\right.$$

$$\left.+\frac{3}{4}a\left(r^2-\cancel{3x^2-a^2}+\frac{5a^2x^2}{r^2}\right)+\frac{1}{4}a^3\left(1-\frac{3x^2}{r^2}-\frac{a^2}{r^2}+\frac{5a^2x^2}{r^4}\right)\right\}$$

$$\{\,\}=+\frac{3}{4}ar^2-3ax^2-\frac{1}{2}a^3+\frac{a^3x^2}{r^2}-\frac{1}{4}\frac{a^5}{r^2}-\frac{13}{4}\frac{a^5x^2}{r^4}$$

$$=-\frac{3}{4}\frac{c^2ay}{r^3}\left[1-\frac{3x^2}{r^2}-\frac{a^2}{r^2}+\frac{5a^2x^2}{r^4}\right]+\frac{3}{4}\frac{c^2ay}{r^6}\{\ \}$$

$$u\frac{\partial w}{\partial x}+\dots=-\frac{3}{4}\frac{c^2az}{r^3}\left[1-\frac{3x^2}{r^2}-\frac{a^2}{r^2}+\frac{5a^2x^2}{r^4}\right]+\frac{3}{4}\frac{c^2az}{r^6}\{\ \}$$

$$\Phi=c^2a^2\,\frac{1}{4}\left(\frac{z^2}{r^6}+\frac{y^2}{r^6}\right)+\frac{3}{2}c^2a\left[-\frac{1}{r^3}+\frac{3x^2}{r^5}+\cancel{\frac{a^2}{r^5}}-\cancel{\frac{5a^2x^2}{r^7}}-\cancel{\frac{3ax^2}{r^6}}-\cancel{\frac{a^3}{r^6}}+\cancel{\frac{6a^3x^2}{r^8}}-\right.$$

$$-\cancel{\frac{a^5}{r^{10}}}-\frac{3x^2}{r^5}\left[-\cancel{1}+\frac{3x^2}{r^2}+\cancel{\frac{a^2}{r^2}}-\frac{5a^2x^2}{r^4}-\frac{3a^2}{r^2}-\cancel{\frac{2a^3}{r^3}}+\cancel{\frac{6a^3x^2}{r^5}}-\cancel{\frac{a^5x^2}{r^7}}\right]+\frac{x}{r^3}\left[\frac{6x}{r^2}-\right.$$

$$\left.-\frac{6x^3}{r^4}-\cancel{\frac{6x}{r^4}}-\cancel{\frac{10a^2x}{r^4}}+\frac{20a^2x^3}{r^6}-\cancel{\frac{6ax}{r^3}}+\frac{9ax^3}{r^5}+\cancel{\frac{6a^3x}{r^5}}+\cancel{\frac{12a^3x}{r^5}}-\cancel{\frac{30a^3x^3}{r^7}}-\cancel{\frac{2a^5x}{r^7}}+\frac{7a^5x^3}{r^9}\right]$$

$$-\frac{2}{r^3}\left[\cancel{1}-\frac{3x^2}{r^2}-\frac{a^2}{r^2}+\cancel{\frac{5a^2x^2}{r^4}}\right]+\frac{2}{r^6}\left[\frac{3}{4}ar^2-\cancel{3ax^2}-\cancel{\frac{1}{2}a^3}+\cancel{\frac{a^3x^2}{r^2}}-\frac{1}{4}\frac{a^5}{r^2}-\frac{13}{4}\cancel{\frac{a^5x^2}{r^4}}\right]$$

$$-\underset{=(x^2-r^2)}{(y^2+z^2)}\left[\frac{15x^2}{r^7}+\frac{5a^2}{r^7}-\frac{35a^2x^2}{r^9}+3\frac{a}{r^6}-18\frac{ax^2}{r^8}+3\frac{a^3}{r^8}-8\frac{a^3x^2}{r^{10}}+2\frac{a^5}{r^{10}}+\frac{65}{2}\frac{a^5x^2}{r^{12}}\right]$$

$$\Phi = \frac{c^2a^2}{4}\left(\frac{r^2-x^2}{r^6}\right) + \frac{3}{2}c^2 a$$

$$\frac{85}{2}\frac{a^5x^4}{r^{12}} - 40\frac{a^5x^2}{r^{10}} - 56\frac{a^3x^4}{r^{10}} + 43\frac{a^3x^2}{r^8} - \cancel{\ldots} - \frac{5}{2}\frac{a^5}{r^8} - 30\frac{a x^2}{r^6}$$

$$-6\frac{a^3}{r^6} - \frac{3}{2}\frac{a}{r^4} + \cancel{\frac{a^2}{r^9}}\ \cancel{\frac{a^2x^2}{r^7}} + \frac{3x^2}{r^5} - \frac{3}{r^3}$$

Przy powierzchni $u\,v\,w = 0$ więc warunek:

$$\rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + \cancel{u\frac{\partial \rho}{\partial x}} + \cancel{v\frac{\partial \rho}{\partial y}} + \cancel{w\frac{\partial \rho}{\partial z}} = 0 \qquad \boxed{\mathrm{div} = 0}$$

istotnie $\mathrm{div} = 0$ dla $r = a$

to znaczy; wybierając x, y jako styczne do powierzchni w kierunek prądu i normalnej

$$\frac{\partial w}{\partial z} = 0 \quad | \text{ ponieważ } \quad v = 0, \quad \frac{\partial v}{\partial x} = 0, \quad \frac{\partial v}{\partial z} = 0, \quad \frac{\partial v}{\partial y} = 0$$

$$u = 0 \quad \frac{\partial u}{\partial x} = 0 \quad \frac{\partial u}{\partial y} = 0 \quad \frac{\partial u}{\partial z} \gtrless 0$$

$$w = 0 \quad \frac{\partial w}{\partial x} = 0 \quad \frac{\partial w}{\partial y} = 0 \quad \frac{\partial w}{\partial z} = 0$$

Jeśli jednak zastosujemy $\overline{III}$:

$$\cancel{k\rho\,\mathrm{div} + \left(u\frac{\partial u}{\partial x} \ldots\right)} = (k-1)\left[\underset{\wedge}{\Phi} + \kappa\,\Delta^2\theta\right]$$

$$0 = \cancel{\ldots}\left(\frac{\partial u}{\partial z}\right)^2 + \kappa\,\Delta^2\theta$$

Tzn. że przy powierzchni będzie $\kappa\frac{\partial^2\theta}{\partial n^2} = \cancel{\mu}\left(\frac{\partial u}{\partial z}\right)^2$

więc gaz ciepłym się stanie? A jeśli κ się może $= 0$ wtedy musi być $\frac{\partial u}{\partial n} = 0$

κ w mierze mechanicznej : $\kappa = 0{\cdot}000058 \cdot 42.10^6 = \underline{42.58} = 243$

Nie zaniedbując κ otrzyma się jako równanie III: Φ

$$\theta = \theta_0 + \frac{1}{p}\theta'$$

$$k\,\mathrm{div}' = -\left(u_0 \frac{\partial p_0}{\partial x} + v_0 \frac{\partial p_0}{\partial y} + w_0 \frac{\partial p_0}{\partial z}\right) + (k-1)\left[\Phi_0 + \kappa \Delta^2 \theta_0\right]$$

$$p = P + p_0 + \frac{1}{p} p' + \frac{1}{2}\frac{1}{p^2} p'' + \dots$$

$$\begin{cases} \dfrac{\partial p_0}{\partial x} = \cancel{\dfrac{\mu}{3}\dfrac{\partial}{\partial x}\mathrm{div}_0} + \mu \Delta^2 u_0 \\ - - - - \end{cases} \qquad \frac{\partial p'}{\partial x} = \frac{\mu}{3}\frac{\partial}{\partial x}\mathrm{div}' + \mu \Delta^2 u'$$

$$\mathrm{div}_0 = 0$$

wystarczy do oznaczenia p, u, v, w,

$$\frac{\partial}{\partial x}(\rho_0 u_0) + \frac{\partial}{\partial y}(\rho_0 v_0) + \frac{\partial}{\partial z}(\rho_0 w_0) = 0$$

$$\cancel{\rho_0\,\mathrm{div}_0} + u_0 \frac{\partial \rho_0}{\partial x} + v_0 \frac{\partial \rho_0}{\partial y} + w_0 \frac{\partial \rho_0}{\partial z} = 0$$

będzie spełnione przez $\rho_0 = \mathrm{const}$

(ale czy koniecznie?)

Rozwiązaniami:

$$\frac{dx}{u_0} = \frac{dy}{v_0} = \frac{dz}{w_0}$$

określa położenie linij prądu

zatem owo równanie wypowiada, że wzdłuż linij prądu ρ niezmienne, ale od jednej do drugiej mogłoby być zmienne, tutaj jednak nie, bo w nieskończoności ρ będzie jednakowe wszędzie

zatem wszędzie $\rho_0 = \mathrm{const}$

$$\left(P + p_0 + \frac{1}{p}p' \dots\right) = R\left(\theta_0 + \frac{1}{p}\theta' \dots\right)\left(\mathrm{const} + \frac{1}{p}\rho' + \dots\right)$$

zatem $\theta_0 = \underline{\mathrm{const}}$

$$\frac{dp}{p} = \frac{d\theta}{\theta} + \frac{d\rho}{\rho}$$

$$\text{IV).}\quad p = R\,\theta\,\rho$$

$$\text{I).}\quad \frac{\partial p}{\partial x} = \frac{\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \right) + \mu \nabla^2 u$$

$$\text{II).}\quad \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

$$\text{III).}\quad k\,p\,\mathrm{div} + \left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right) = (k-1)\left[\mu\,\Phi + \kappa\,\Delta^2\theta\right]$$

$$\text{II).}\quad \rho\,\mathrm{div} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} = 0$$

$$\text{II).}\quad \mathrm{div} + \frac{u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}}{p} - \frac{u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z}}{\theta} = 0$$

$$\text{III).}\quad (1-k)\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right) + k\frac{p}{\theta}\left(u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z}\right) = (k-1)\left[\mu\,\Phi + \kappa\,\Delta^2\theta\right]$$

$$\frac{kR}{k-1}\rho\left(u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z}\right) = \mu\,\Phi + \kappa\,\Delta^2\theta + \left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right)$$

jeżeli Δp mało w porównaniu do p to także div bardzo mało $\qquad \rho = \rho_0 + \rho'$ (const)

$u\ v\ w\ p$ .. itd. $= u_0 + \frac{1}{\rho_0} u' + \dots$

$$\text{II}).\quad (\rho_0 + \rho')(\operatorname{div}_0 + \frac{1}{\rho_0}\operatorname{div}' + \dots) + (u_0 + \frac{1}{\rho_0}u' + \dots)\frac{\partial \rho'}{\partial x} + (v_0 + \frac{1}{\rho_0}v' \dots)\frac{\partial \rho'}{\partial y} + \dots = 0$$

1). $\operatorname{div}_0 = 0$

2). $\cancel{\rho' \operatorname{div}_0} + \operatorname{div}' + u_0 \frac{\partial \rho'}{\partial x} + v_0 \frac{\partial \rho'}{\partial y} + \dots = 0$

$$\operatorname{div}' = -\left(u_0 \frac{\partial \rho'}{\partial x} + v_0 \frac{\partial \rho'}{\partial y} + w_0 \frac{\partial \rho'}{\partial z}\right)$$

$$\text{IV}).\quad p_0 + \frac{1}{\rho_0} p' + \dots = R\,(\theta_0 + \frac{1}{\rho_0}\theta' + \dots)(\rho_0 + \rho')$$

Jeżeli zakładamy że $\frac{p_0}{R\rho_0}$ tego samego rzędu co p_0 itd.

1). $p_0 = R\theta_0\rho_0$

2). $p' = R\theta' + R\theta_0\rho'$

$$\text{I}).\quad 1).\ \frac{\partial p_0}{\partial x} = \cancel{\frac{\mu}{3}\frac{\partial}{\partial x}\operatorname{div}_0} + \mu \nabla^2 u_0$$

$$2).\ \frac{\partial p'}{\partial x} = \frac{\mu}{3}\frac{\partial}{\partial x}\operatorname{div}' + \mu \nabla^2 u'$$

$$\text{II}).\quad k(p_0 + \frac{1}{\rho_0}p' + \dots)(\cancel{\operatorname{div}_0} + \frac{1}{\rho_0}\operatorname{div}' + \dots) + (u_0 + \frac{1}{\rho_0}u' + \dots)\left(\frac{\partial p_0}{\partial x} + \frac{1}{\rho_0}\frac{\partial p'}{\partial x} + \dots\right) =$$
$$= (k-1)\left[\mu(\Phi_0 + \frac{1}{\rho_0}\Phi' + \dots) + \kappa(\Delta^2\theta_0 + \frac{1}{\rho_0}\Delta^2\theta')\right]$$

Albo:

$$\frac{kR}{k-1}(\rho_0 + \rho')\left[(u_0 + \frac{1}{\rho_0}u' + \dots)\left(\frac{\partial \theta_0}{\partial x} + \frac{1}{\rho_0}\frac{\partial \theta'}{\partial x} + \dots\right) + \dots\right] = \mu\,\Phi_0 + \frac{1}{\rho_0}\Phi' + \dots +$$
$$\mu\,(u_0 + \frac{1}{\rho_0}u' + \dots)(\nabla^2 u_0 + \frac{1}{\rho_0}\dots)$$
$$+ \kappa\,\Delta^2\theta_0 + \frac{1}{\rho_0}\Delta^2\theta' + \dots$$

$$\underbrace{k\frac{p_0}{\rho_0}}\operatorname{div}' + u_0\frac{\partial p_0}{\partial x} = (k-1)\left[\mu\,\Phi_0 + \kappa\,\Delta^2\theta_0\right]$$

$$kR\theta_0 \operatorname{div}' + u_0\frac{\partial p_0}{\partial x} = \nearrow$$

więc dla θ równanie formy:

$$\Delta^2\theta + \theta\,\varphi(x,y,z) = \psi(x,y,z)$$

P = ciśnienie w wielkiej odległości
= stała całkow.
θ

$$p = \cancel{\ldots}$$
$$= \cancel{\ldots}\; f\Big(\underbrace{\tfrac{1}{P},\ \cancel{\ }\ \Theta,\ x, y, z}_{}\Big)$$
$$= \cancel{\ldots}$$

zmieniając $\frac{1}{P}$: każdym razem inny rozkład p w

$\frac{1}{P}$ = parametr

Czy będzie można stawić:

$$p = \cancel{\ldots}\ \text{const} + p_0 + \frac{1}{P}\,p' + \frac{1}{P^2}\,p'' + \ldots \qquad ?$$

$$= f\left(0, \Theta_0, x_0, y_0, z\right) + \frac{1}{P}\,\frac{\partial f}{\partial \frac{1}{P}} \ldots$$

---

mogłaby drugiego najpierw III dla $=0$:
otrzyma się równanie dla θ a z tego zapomocą II dla

Eliminując dla z II III:

w miejsce mechanicznej $= 0{\cdot}000057 \cdot 42.10^6$

$$\text{III}.\ (1-k)\left(u\frac{\partial f}{\partial x} + \ldots\right) + k\,\frac{p}{\theta}\left(u\frac{\partial\theta}{\partial x} + \ldots\right) = (k-1)\left[\mu\,\Phi + \kappa\Delta^2\theta\right]$$

Obierając jako pierwsze przybliżenie rozkład adiabat.:

$$\mu\,\Phi + \kappa\Delta^2\theta = 0$$

z tego θ' a z tego zapomocą I dla

Obierając jako pierwsze przybliżenie rozkład izotermiczny $\rightarrow$ $\mu\,\Phi + \kappa\Delta^2\theta' = -\left(u_0\frac{\partial p_0}{\partial x} + \ldots\right)$

Która z wszystkich tych metod najracyonalniejsza?

Metoda:

Pierwsze przybliżenie $div = 0$

I). $\begin{cases} \frac{\partial p_0}{\partial x} = \mu \nabla^2 u_0 \\ div_0 = 0 \end{cases}$

~~Drugie~~ ~~III~~ III). $u_0 \frac{\partial p_0}{\partial x} + v_0 \frac{\partial p_0}{\partial y} + w_0 \frac{\partial p_0}{\partial z} = (k-1)\left[\mu \Phi_0 + \kappa \Delta^2 \theta_0\right]$

z tego θ_0

Drugie przybliżenie:

$\boxed{\text{IV). } \frac{d\rho}{\rho} = \frac{dp}{p} - \frac{d\theta}{\theta}}$

II). $div' = - \frac{u_0 \frac{\partial p_0}{\partial x} + v_0 \frac{\partial p_0}{\partial y} + w_0 \frac{\partial p_0}{\partial z}}{p_0} + \frac{u_0 \frac{\partial \theta_0}{\partial x} + \dots}{\theta_0}$

$\nabla^2 p' = 4/3 \nabla^2 div'$

I). $\frac{\partial p'}{\partial x} = \frac{\mu}{3} \frac{\partial\, div'}{\partial x} + \mu \nabla^2 u'$

$p' = 4/3\, div' + U$

Pytanie jak pogodzić równania

II). $div + \frac{u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} + }{p} - \frac{u \frac{\partial \theta}{\partial x} + v -}{\theta} = 0$

III). $k p\, div + (u \frac{\partial p}{\partial x} + \dots\quad) = (k-1)\left[\mu \Phi + \kappa \Delta^2 \theta\right]$

w drugim przybliżeniu

uwzględniając naprzód II).z $div = 0$ otrzymalibyśmy $\frac{dp}{p} = + \frac{d\theta}{\theta}$

$p \sim \theta$

z tego za pomocą III: $div = \dots\dots \quad \dots$

~~$(k-1)[\mu \Phi + \kappa \Delta^2 \theta] - \frac{\dots}{\dots}$~~

Odejmujemy II od III:

$$(k-1)\,p\,\mathrm{div} = -\frac{k p}{\theta}\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}\right)+(k-1)\left[\mu\Phi+\kappa\Delta^2\theta\right]$$

Dwie strony które charakteryzować będą [przy danem κ etc.] szczególny ruch:

P i Θ

Najprostszy jeśli P bardzo duże i Θ bardzo duże w porównaniu do $\frac{\Delta P}{\Delta\Theta}$

Wtedy I): $\mathrm{div}=0$

zatem ruch przez I i II zupełnie określony

θ jest wtedy obojętne ~~i takie~~

musi być obliczone z III z uwzględnieniem szczególnych wartości $P\ \Theta$:

$$(1-k)\left(u\frac{\partial p}{\partial x}+\dots\right)=-k\frac{P}{\Theta}\left(u\frac{\partial\theta}{\partial x}+\dots\right)+(k-1)\left(\mu\Phi+\kappa\Delta^2\theta\right)$$

Szczególne założenie jeszcze że

1. $\frac{1}{p}\frac{\partial p}{\partial x} \ll \frac{1}{\theta}\frac{\partial\theta}{\partial x}$

Wtedy: II przyjmuje kształt: $u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}=0$

$c=\mathrm{const}$ izoterm.

---

2. $\frac{1}{p}\frac{\partial p}{\partial x} \gg \frac{1}{\theta}\frac{\partial\theta}{\partial x}$

$$(1-k)\left(u\frac{\partial p}{\partial x}+\dots\right)=(k-1)\left[\mu\Phi+\kappa\Delta^2\theta\right]$$

$$u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}+\mu\Phi=-\kappa\Delta^2\theta$$

to samo byłoby drugiem przybliżeniem z (1)

Pytanie czy ścisłe równania III słuszne?

zatem oczywiście lepiej używać tego sposobu

z θ i φ w tr. współ. sferycznemi można wyjść z II). Ściśle dziw!

i z pomocą I rozkład poprawiony

Zawsze dodatnie bo $= \ldots \left\{ \frac{2a^4}{r^4} + \left(6 - \frac{12a^2}{r^2} + \frac{4a^4}{r^4}\right)\sin^2\theta \right\}$

minimum dla $\theta = 0$ $= 6\left[1 - \frac{a^2}{r^2}\right]^2 \geq 0$

$$\Phi = \frac{q}{8}\,\frac{c^2a^2}{r^4}\left\{ 6\frac{x^2}{r^2} + \frac{2a^4}{r^4} - \frac{12a^2x^2}{r^4} + \frac{4a^4x^2}{r^6} \right\}$$

$\Delta^2\left(\frac{x^2}{r^8}\right) = \frac{2}{r^8} + 24\frac{x^2}{r^{10}}$	α	$24\alpha = 4a^4$	$\alpha = \frac{a^4}{6}$
		$30\xi + 2\alpha = 2a^4$	$15\xi = a^4 - \alpha = \frac{5}{6}a^4$ $\quad \xi = \frac{a^4}{18}$
$\Delta^2\left(\frac{x^2}{r^6}\right) = \frac{2}{r^6} + 6\frac{x^2}{r^8}$	β	$6\beta = -12a^2$	$\beta = -\frac{a^2}{2}$
$\Delta^2\left(\frac{x^2}{r^4}\right) = \frac{2}{r^4} - 4\frac{x^2}{r^6}$	γ	$2\beta + 12\delta = 0$	$\delta = \frac{a^2}{12}$
$\Delta^2\left(\frac{1}{r^4}\right) = \frac{12}{r^6}$	δ	$-4\gamma = 6$	$\gamma = -\frac{3}{2}$
$\Delta^2\left(\frac{1}{r^2}\right) = \frac{2}{r^4}$	ε	$2\gamma + 2\varepsilon = 0$	$\varepsilon = \frac{3}{2}$
$\Delta^2\left(\frac{1}{r^6}\right) = \frac{30}{r^8}$	ξ		

$$\Phi_0 = \Delta^2\left\{ \frac{q}{8}c^2a^2\left[\frac{1}{6}\frac{a^4x^2}{r^8} \cancel{-\frac{1}{2}\frac{a^2x^2}{r^6}} + \frac{1}{18}\frac{a^4}{r^6} - \frac{3}{2}\frac{x^2}{r^4} + \frac{1}{12}\frac{a^2}{r^4} + \frac{3}{2}\frac{1}{r^2}\right]\right\}$$

$$u_0\frac{\partial p_0}{\partial x} + v_0\frac{\partial p_0}{\partial y} + w_0\frac{\partial p_0}{\partial z} = \Delta^2\left\{ -\mu c^2a^2\left[\frac{27}{32}\frac{1}{r^2} + \cancel{\ldots} - \frac{45}{32}\frac{x^2}{r^4} - \frac{3}{16}\frac{a^2x^2}{r^6}\right] - \mu c^2 a\cdot\frac{3}{4}\frac{x^2}{r^3}\right\}$$

$$\Delta^2\theta = \cancel{\ldots}\,\Delta^2\psi$$

$$\theta = \psi + U \qquad \Delta^2 U = 0$$

$r = a$ | $\theta_0 = \psi_0 + U_0$

$r = \infty$ $\quad U = 0$

$$\psi = \frac{\mu}{k}c^2a^2\left[-\frac{27}{32}\frac{1}{r^2} + \frac{9}{32}\frac{x^2}{r^4} - \frac{3}{32}\frac{a^2}{r^4} - \frac{1}{16}\frac{a^4}{r^6} + \frac{3}{8}\frac{a^2x^2}{r^6} - \frac{3}{16}\frac{a^4x^2}{r^8}\right] + \frac{\mu}{k}c^2a\frac{3}{4}\frac{x^2}{r^3}$$

$$U_0\Big|_{r=a} = -\frac{\mu}{K}\,c^2\theta_0\left[-\frac{27}{32}-\frac{3}{32}-\frac{3}{48}+\left(\frac{9}{32}+\frac{3}{8}-\frac{3}{16}\right)\frac{x^2}{a^2}+\frac{3}{4}\frac{x^2}{a^2}\right]$$

$$-\frac{3}{8}\left(\frac{9}{4}+\frac{1}{4}+\frac{1}{6}\right) \qquad \frac{9+12-6+24}{32} = \frac{39}{32}$$

$$-\frac{3}{8}\,\frac{64}{24} = -1$$

$$U_0\Big|_{r=a} = +\frac{\mu}{K}\,c^2\theta_0\left[1-\frac{39}{32}\frac{x^2}{a^2}\right]$$

$$U = \theta_0 + \frac{A}{r} + B\left(\frac{1}{r^3}-\frac{3x^2}{r^5}\right)$$

$$\frac{A}{a}+\frac{B}{a^3} = \frac{\mu c^2}{K} \quad \Big|\ 3$$

$$-\frac{3B}{a^3} = -\frac{39}{32}\frac{\mu c^2}{K}$$

$$3\frac{A}{a} = \left(2-\frac{7}{32}\right)\frac{\mu c^2}{K} = \frac{57}{32}\frac{\mu c^2}{K}$$

$$\frac{A}{a} = \frac{19}{32}\frac{\mu c^2}{K}$$

$$\frac{B}{a^3} = \frac{13}{32}\frac{\mu c^2}{K}$$

$$U = \theta_0 + \frac{\mu c^2}{32K}\left[19\frac{a}{r}+13\left(\frac{a^3}{r^3}-\frac{3x^2a^3}{r^5}\right)\right]$$

$$\theta = \theta_0 + \frac{\mu c^2}{K}\left\{\frac{3a^2}{32}\left[-\frac{9}{r^2}+\frac{3x^2}{r^4}-\frac{a^2}{r^4}-\frac{2}{3}\frac{a^4}{r^6}+4\frac{a^2x^2}{r^6}-2\frac{a^4x^2}{r^8}\right]+\right.$$
$$\left.\frac{a}{r}\left[\frac{19}{32}+\frac{13}{32}\frac{a^2}{r^2}+\frac{3}{4}\frac{x^2}{r^2}-\frac{39}{32}\frac{x^2a^2}{r^4}\right]\right\}$$

Gdyby się to utarło znowu do III

← po przyjęciu poprawki liczby rzędu $\frac{c\,p}{A\,\theta\,K} = \frac{c}{42.273}$

$$\frac{k}{k-1}\frac{p}{\theta}\left(u\frac{\partial\theta}{\partial x}+\cdots\right) = \frac{k}{k-1}\frac{p}{\theta}\frac{\mu c^3}{\kappa}\left[\cdots\right]$$

podczas gdy $\left(u\frac{\partial p}{\partial x}+\cdots\right) = \mu c^2\left[\cdots\right]$

zatem stosunek poprawki : $\frac{c\,p}{\kappa\,\theta} = \frac{c\,.\,10^6}{0.000057\cdot 42.10^6\cdot 273} = \frac{c}{0.6}$

zatem już większa niż tamta jeśli $c > 0.6\frac{cm}{sec}$!

$$\frac{\mu c^2}{\kappa} = \frac{0.00019\cdot c^2}{42.10^6\cdot 0.000057} = \frac{c^2}{15.10^6}$$ zatem to osiągnęłoby dopiero 1°

$= \frac{c^2}{15\cdot 10^6}$ $\frac{2}{3}$ gdy $c = 4.10^3\frac{cm}{sec}$!

Zatem ten sposób rozwinięcia zupełnie nieusprawiedliwiony

W równaniu III:

$$\underset{(1)}{\left(u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}\right)} - \frac{k}{k-1}\frac{P}{\Theta}\underset{(2)}{\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}\right)} + \underset{(3)}{\mu\Phi} + \underset{(4)}{\kappa\Delta^2\theta} = 0$$

1 i 3 są rzędu wielkości $\frac{\mu c^2}{a^2} = 0.00019\cdot\frac{c^2}{a^2}$

$(4) = 2400.\,\Delta^2\theta$

aby więc 4 stało się wielkością tego samego rzędu, musiałoby $\Delta^2\theta \neq \frac{\mu c^2}{\kappa a^2}$

$$\Delta^2\theta \neq \frac{0.00019}{42.10^6\cdot 0.000057}\frac{c^2}{a^2} = \frac{1}{15\cdot 10^6}\frac{c^2}{a^2}$$ co łatwo osiągnąć

podczas gdy (2) : $\frac{10^6}{273}c\frac{\partial\theta}{\partial x}$

μc^2

Przy wszystkim ze zaniedbaniem wyrazów drugiego stopnia

wynika że ρu^2 mało wobec $\mu\frac{\partial u}{\partial x}$

$\rho u \ll \frac{\mu}{x}$ $\qquad a\rho c \ll \mu$

(A). $\Delta^2\theta + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z} = \Psi$

Jeśli 2 rozwiązania θ_1, θ_2, to suma:

$$\Delta^2(\theta_1+\theta_2) + u\frac{\partial(\theta_1+\theta_2)}{\partial x} + v\frac{\partial(\theta_1+\theta_2)}{\partial y} + w\frac{\partial(\theta_1+\theta_2)}{\partial z} = 2\Psi$$

zatem jednio arytmetyczne takie zadość uczyni

różnica $\Delta^2(\theta_1-\theta_2) + u\frac{\partial(\theta_1-\theta_2)}{\partial x} + \cdots = 0$

Jeśli mamy rozwiązanie ~~zadania~~ równanie

(B). $\Delta^2\theta + u\frac{\partial\theta}{\partial x} + \cdots = 0$ || N.p. $\theta = e^{\varphi(x,y,z)}$

$$\frac{\partial\theta}{\partial x} = \frac{\partial\varphi}{\partial x}e^{\varphi}$$

$$\frac{\partial^2\theta}{\partial x^2} = \frac{\partial^2\varphi}{\partial x^2}e^{\varphi} + \left(\frac{\partial\varphi}{\partial x}\right)^2 e^{\varphi}$$

$$\Delta^2\varphi + \left[\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2 + \left(\frac{\partial\varphi}{\partial z}\right)^2\right] + u\frac{\partial\varphi}{\partial x} + v\frac{\partial\varphi}{\partial y} + w\frac{\partial\varphi}{\partial z} = 0$$

Jeśli warunek dla θ na powierzchni $\theta = \text{const}$, to wiadomo że $\theta = \text{const}$ jest jedynem rozwiązaniem dla skróconego równania $\Delta^2\theta + u\frac{\partial\theta}{\partial x} \cdots = 0$.

Z tego wynika że $\theta_1 - \theta_2 = \text{const}$, zatem tylko jedno rozwiązanie możliwe

Stawiając $\Psi = \Delta^2\chi$ w równaniu (A).

i podstawiając $\theta = \vartheta + \chi$ otrzymamy:

$$\Delta^2\vartheta + u\frac{\partial\vartheta}{\partial x} + \cdots = -\left(u\frac{\partial\chi}{\partial x} + v\frac{\partial\chi}{\partial y} + w\frac{\partial\chi}{\partial z}\right)$$

Podstawiając w B: $\theta = \vartheta + p$: $\boxed{\Delta^2 p = 0}$ $\boxed{\vartheta = \theta_0 - p}$

$$\Delta^2\vartheta + u\frac{\partial\vartheta}{\partial x} + \cdots + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + \cdots = 0$$

$$\to \Psi = \frac{Ax^2}{2^6} + \frac{B}{2^8} + \frac{Cx^2}{2^8} + \frac{Dx^2}{2^{10}}$$

$\cancel{\vartheta = \sum m_k \frac{x}{2^k}}$ Ponieważ Ψ parzyste potęgi u v

$\cancel{\Delta^2\theta = \sum}$ to θ musi zawierać mieszane parzyste i nieparzyste

---

Gdyby $\Phi = 0$ to niedobyśmy jako jedno możliwe rozwiązanie

$$\theta = \theta_0 + \alpha p$$

$$\left(u\frac{\partial p}{\partial x} + \cdot\quad\right)\left[1 - \frac{k}{k-1}\frac{P}{\Theta}\alpha\right] + \cancel{k\Delta^2 p} = 0$$

t. zn. wtedy $\qquad \alpha = \frac{k-1}{k}\frac{\Theta}{P}$

$\theta = \theta_0 + \frac{k-1}{k}\frac{\Theta}{P}p$, to nie byłoby jednak w zgodności z warunkiem dla $r = a$

zatem to nie mówi zastosowane

Gdyby $\kappa = 0$ to musiałoby być $\left.\frac{\partial v}{\partial r}\right|_{r=a} = 0$

Czyby to już wystarczyło do oznaczenia ruchu?

Znając rozwiązanie

$\Delta^2\theta + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + \ldots = \Phi$ otrzyma się III przez podstawienie $\theta = \vartheta + \alpha p$

$$u^2+v^2+w^2 = c^2\left\{\underbrace{\frac{9}{16}\frac{a^2}{2^4}\left(1-\frac{a^2}{r^2}\right)^2 x^2 - \frac{3}{2}\frac{a}{r^3}\left(1-\frac{a^2}{r^2}\right)\left(1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right)}x^2 + \left(1-\frac{3}{2}\frac{a}{r}\ldots\right)^2\right]$$

$$x^2\frac{\ }{\ }\left(1-\frac{a^2}{r^2}\right)\left[-\frac{3}{2}\frac{a}{r^3} + \underbrace{\frac{3}{2}\frac{a^2}{r^4}\left(\frac{3}{4}+\frac{1}{4}\frac{a^2}{r^2}\right) + \frac{9}{16}\frac{a^2}{r^4}\left(1-\frac{a^2}{r^2}\right)}_{\frac{a^2}{r^4}\left(\frac{27}{16}-\frac{3}{16}\frac{a^2}{r^2}\right)}\right] +$$

Weźmy osiowe zadanie dwuwymiarowe ; przy czym

$$u = -\frac{1}{\omega}\frac{\partial\psi}{\partial z} \qquad v = \frac{1}{\omega}\frac{\partial\psi}{\partial x}$$

$$2\frac{\partial^2(\theta r)}{\partial r^2} + \frac{1}{\sin\varphi}\frac{\partial}{\partial\varphi}\left(\sin\varphi\frac{\partial\theta}{\partial\varphi}\right) +$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$y = \omega \sin\varepsilon$$
$$z = \omega\cos\varepsilon$$

$$\frac{\partial V}{\partial y} = \frac{dV}{d\omega}\frac{\partial\omega}{\partial y} = \frac{\partial V}{\partial \omega}\frac{y}{\omega}$$

$$\frac{\partial^2 V}{\partial y^2} = \frac{\partial}{\partial\omega}\cdot\frac{y}{\omega} = \frac{\partial^2 V}{\partial\omega^2}\frac{y^2}{\omega^2} - \frac{\partial V}{\partial\omega}\frac{y^2}{\omega^3}$$

$$\frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{\partial^2 V}{\partial\omega^2} - \frac{1}{\omega}\frac{\partial V}{\partial\omega}$$

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial\omega^2} - \frac{1}{\omega}\frac{\partial\theta}{\partial\omega} - \frac{1}{\omega}\frac{\partial\psi}{\partial\omega}\frac{\partial\theta}{\partial x} + \frac{1}{\omega}\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial\omega} = \Phi$$

Gdyby się wprowadziło współrzędne

$$\frac{\partial\theta}{\partial x} = \frac{\partial\theta}{\partial r}\frac{\partial r}{\partial x} = \frac{x}{r}\frac{\partial\theta}{\partial r} = \frac{\sqrt{r^2-\omega^2}}{r}\frac{\partial\theta}{\partial r}$$

$$\frac{\partial^2\theta}{\partial x^2} = \frac{1}{r}\frac{\partial\theta}{\partial r} - \frac{x^2}{r^3}\frac{\partial\theta}{\partial r} + \frac{x^2}{r^2}\frac{\partial^2\theta}{\partial r^2}$$

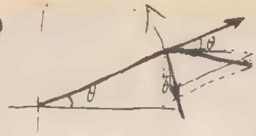
$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial\xi^2}\left(\frac{\partial\xi}{\partial x}\right)^2 + \frac{\partial^2 z}{\partial\eta^2}\left(\frac{\partial\eta}{\partial x}\right)^2 + 2\frac{\partial^2 z}{\partial\xi\partial\eta}\frac{\partial\xi}{\partial x}\frac{\partial\eta}{\partial x} + \frac{\partial z}{\partial\xi}\frac{\partial^2\xi}{\partial x^2} + \frac{\partial z}{\partial\eta}\frac{\partial^2\eta}{\partial x^2}$$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial\xi}\frac{\partial\xi}{\partial x} + \frac{\partial z}{\partial\eta}\frac{\partial\eta}{\partial x}$$

$$\frac{\partial^2\theta}{\partial x^2} = \frac{\partial^2\theta}{\partial r^2}\left(\frac{\partial r}{\partial x}\right)^2 + \frac{\partial^2\theta}{\partial}$$

$$\cancel{\frac{\partial\theta}{\partial r} = \frac{\partial\theta}{\partial x}\frac{x}{r} + \frac{\partial\theta}{\partial w}\frac{w}{r}}$$

$$\cancel{\frac{\partial\theta}{\partial w} = \frac{\partial\theta}{\partial r}\frac{w}{r}}$$



$$-u\sin\theta + \varphi\cos\theta =$$

$$= -u\frac{w}{r} + \varphi\frac{x}{r}$$

$$= +\frac{3}{2}\frac{ca}{r^3}\left(1-\frac{a^2}{r^2}\right)\frac{x^2w}{r} -$$

$$- c\frac{w}{r}\left(1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right)$$

$$\frac{dx}{u\Phi} = \frac{dy}{v\Phi} = \frac{dz}{w\Phi} = d\theta$$

$$r\frac{\partial\theta}{\partial r} + \theta \qquad \cos\varphi\frac{\partial\theta}{\partial\varphi} + \sin\varphi\frac{\partial^2\theta}{\partial\varphi^2}$$

$$r^2\frac{\partial^2\theta}{\partial r^2} + 2\frac{\partial\theta}{\partial r}\cdot r$$

$$r^2\frac{\partial^2\theta}{\partial r^2} + 2r\frac{\partial\theta}{\partial r} + \frac{\partial^2\theta}{\partial\varphi^2} + \frac{\cos\varphi}{\sin\varphi}\frac{\partial\theta}{\partial\varphi} + \cancel{f(r)\cos^2\varphi}$$ itd.

$$\to \frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial w^2} \underbrace{-\frac{1}{w}\frac{\partial\psi}{\partial w}}_{P=-\frac{1}{w}\frac{\partial\psi}{\partial w}}\frac{\partial\theta}{\partial x} + \underbrace{\frac{1}{w}\left(\frac{\partial\psi}{\partial x}-1\right)}_{\varphi}\frac{\partial\theta}{\partial w} = \Phi$$

$\downarrow$ u

$R=1$

$S=0$

$T=1$

$$k^2+1=0$$
$$k=\pm i$$

$$\frac{\partial\xi}{\partial x} = i\frac{\partial\xi}{\partial w} \qquad \frac{\partial\eta}{\partial x} = -i\frac{\partial\eta}{\partial w}$$

$$\xi = x+iw \qquad 4\frac{\partial\xi}{\partial y}\frac{\partial\eta}{\partial w} = +4$$

$$\eta = x-iw \qquad \boxed{r^2=\xi\eta \;\|\; x=\frac{\xi+\eta}{2}}$$

$$\frac{\partial^2\theta}{\partial\xi\partial\eta} \pm \underbrace{\frac{1}{4w}\frac{\partial\psi}{\partial w}}_{L}\frac{\partial\theta}{\partial\xi} \pm \underbrace{\frac{1}{4w}\left(1-\frac{\partial\psi}{\partial x}\right)}_{M}\frac{\partial\theta}{\partial\eta} = +\frac{\bar\Phi}{4}$$

Czy spełnione: $LM + \frac{\partial L}{\partial\xi} = 0$?

$\frac{\partial^2\psi}{\partial x^2}$

$$\psi = -\frac{c}{2}\left(1-\frac{3}{2}\frac{a}{r}+\frac{1}{2}\frac{a^3}{r^3}\right) r^2 \sin^2\vartheta$$

$$\frac{dr}{r\,d\varphi} = \operatorname{ctg}\alpha$$

$$\delta\psi = \frac{\partial\psi}{\partial r}\,\delta r + \frac{\partial\psi}{\partial\vartheta}\,\delta\vartheta$$

$$0 = \frac{\partial\psi}{\partial r}\,dr + \frac{\partial\psi}{\partial\vartheta}\,d\vartheta$$

$$\frac{1}{r}\frac{dr}{d\vartheta} = \frac{-\frac{\partial\psi}{\partial\vartheta}}{r\frac{\partial\psi}{\partial r}} = \operatorname{tg}\beta$$

$$\delta = \frac{\partial\psi}{\partial r}\,\delta r \cdot \cos\vartheta$$

$$\delta\psi\int_0^\infty \omega\frac{\partial\omega}{\partial\psi}\,\Phi(x,\omega)\,dx = \delta\psi\int \omega\frac{\Phi}{\frac{\partial\psi}{\partial\omega}}\,dx$$

$$= \delta\psi\int\frac{1}{2}\frac{\Phi(x,\omega)}{\frac{\partial\psi}{\partial(\omega^2)}}\,dx$$

$$\psi = -\frac{c}{2}\left(1-\frac{3}{2}\frac{a}{\sqrt{x^2+\omega^2}}+\frac{1}{2}\frac{a^3}{(x^2+\omega^2)^{3/2}}\right)\omega^2$$

$$\frac{\partial\psi}{\partial\omega^2} = -\frac{c}{2}\left[1-\frac{3}{2}\frac{a}{\sqrt{x^2+\omega^2}}+\frac{1}{2}\frac{a^3}{(x^2+\omega^2)^{3/2}}\right]$$

$$-\frac{c}{2}\omega^2\left[\frac{3}{4}\frac{a}{\sqrt{(x^2+\omega^2)}^3}-\frac{3}{4}\frac{a^3}{(x^2+\omega^2)^{5/2}}\right]$$

$$= -\frac{c}{2}\left[1-\frac{3}{2}\frac{a}{r}+\frac{1}{2}\frac{a^3}{r^3}+\frac{3}{4}\frac{a\omega^2}{r^3}-\frac{3}{4}\frac{a^3\omega^2}{r^5}\right]$$

$$\omega^2 = r^2 - x^2$$

$$= -\frac{c}{2}\Big[\underbrace{1-\frac{3}{2}\frac{a}{r}+\frac{1}{2}\frac{a^3}{r^3}+\frac{3}{4}\frac{a}{r}-\frac{3}{4}\frac{ax^2}{r^3}-\frac{3}{4}\frac{a^3}{r^3}+\frac{3}{4}\frac{a^3x^2}{r^5}}_{1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}-\frac{3}{4}\frac{ax^2}{r^3}+\frac{3}{4}\frac{a^3x^2}{r^5}}\Big]$$

$$= -\frac{u}{2}$$

$$\Phi = \frac{a}{8}\frac{c^2a^2}{r^2}\Big\{\underbrace{\frac{2a^4}{r^4}+6-\frac{6\omega^2}{r^2}-\frac{12a^2}{r^2}+\frac{12a^2\omega^2}{r^4}+4\frac{a^4}{r^4}-\frac{4a^4\omega^2}{r^6}}_{\left\{6-12\frac{a^2}{r^2}+6\frac{a^4}{r^4}-\left(6-\frac{12a^2}{r^2}+\frac{4a^4}{r^4}\right)\frac{\omega^2}{r^2}\right\}}\Big\}$$

$$dx = \frac{x}{r}\,dr = \frac{dr}{r}\sqrt{r^2-\omega^2}$$

$$\frac{a}{8}c^2a^2\frac{1}{2}\int_\rho^\infty dr\,\frac{\frac{1}{r^5}\left[6\left(1-\frac{2a^2}{r^2}+\frac{a^4}{r^4}\right)+\left(6-\frac{12a^2}{r^2}+\frac{4a^4}{r^4}\right)\frac{2\psi}{c\,r^2\left(1-\frac{3}{2}\frac{a}{r}+\frac{1}{2}\frac{a^3}{r^3}\right)}\right]}{-\frac{e}{2}\left[1-\frac{3}{2}\frac{a}{r}+\frac{1}{2}\frac{a^3}{r^3}+\frac{3}{4}\frac{a}{r^3}\left(1-\frac{a^2}{r^2}\right)\frac{2\psi}{c\left(1-\frac{3}{2}\frac{a}{r}+\frac{1}{2}\frac{a^3}{r^3}\right)}\right]}$$

$$\left(i\sqrt{r^2-\frac{2\psi}{c\left(1-\frac{3}{2}\frac{a}{r}+\frac{1}{2}\frac{a^3}{r^3}\right)}}\right.$$

$\psi = \mathrm{const}$

---

jeśli się uda znaleźć rozwiązanie

~~(B). $\Delta^2\theta + \left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}\right) = 0$~~

wtedy potrzeba tylko znaleźć taką funkcję φ : $\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2+\left(\frac{\partial\varphi}{\partial z}\right)^2 = 4$

stawiamy $\varphi = \log\vartheta$ ~~i stawiamy $\vartheta = e^\varphi$~~

$$\frac{\partial\varphi}{\partial x} = \frac{1}{\vartheta}\frac{\partial\vartheta}{\partial x} \qquad \vartheta = e^\varphi$$

$$\Delta^2\theta + u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z} = 0$$

~~$\theta = e^{\int \dots dr}$~~ ~~jeśli to możliwe~~

Jako partykularne rozwiązanie $\Delta^2\theta = 0$ $\quad u\frac{\partial\theta}{\partial x}+\dots = 0$

$$\theta = \frac{a}{r} + b\frac{\partial}{\partial x}\left(\frac{1}{r}\right) + c\frac{\partial^2}{\partial x^2}\left(\frac{1}{r}\right)+\dots$$

$$\theta = \frac{a}{r} + b\frac{\partial}{\partial x}\left(\frac{1}{r}\right) + c\frac{\partial}{\partial x^2}\left(\frac{1}{r}\right) + \text{---}$$

$$\frac{\partial\theta}{\partial x} = a\frac{\partial}{\partial x}\left(\frac{1}{r}\right) + b\frac{\partial^2}{\partial x^2}\left(\frac{1}{r}\right) + \cdots = \frac{x}{r}\Big[\overbrace{a + b(\;=\;) + \cdots}^{F_1}\Big] + F_2 \text{ --}$$

$$\frac{\partial\theta}{\partial y} = a\frac{\partial}{\partial y}\left(\frac{1}{r}\right) + b\frac{\partial}{\partial x}\frac{\partial}{\partial y}\left(\frac{1}{r}\right) + \cdots = \frac{y}{r}\left[\cdots\cdots = \right]$$

$$\frac{\partial\theta}{\partial z} = a\frac{\partial}{\partial z}\left(\frac{1}{r}\right) + \text{---} \qquad = \frac{z}{r}\left[\cdots\cdots = \right]$$

$$u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z} = -\cancel{\frac{3}{4}\frac{ca}{r^2}\left(1-\frac{a^2}{r^2}\right)xF_1 + G_1 + f_1(x,z) = 0}$$

$$= \frac{cx}{r}\left(1 - \frac{3}{2}\frac{a}{r} + \frac{1}{2}\frac{a^3}{r^3}\right)F_1 + F_2 u$$

$$\frac{\partial}{\partial x}\left(\frac{1}{r}\right) = -\frac{x}{r^3} \qquad \frac{\partial}{\partial y}\left(\frac{1}{r}\right) = -\frac{y}{r^3}$$

$$\frac{\partial^2}{\partial x^2}\left(\frac{1}{r}\right) = +\frac{3x^2}{r^5} - \frac{1}{r^3} \qquad \frac{\partial^2}{\partial y\,\partial x}\left(\frac{1}{r}\right) = +\frac{3xy}{r^5}$$

$$\frac{\partial^3}{\partial x^3} = -\frac{15x^3}{r^7} + \frac{3x}{r^5} + \frac{6x}{r^5} \qquad \frac{\partial^3}{\partial x^2\partial y} = -\frac{15x^2y}{r^7} + \frac{3y}{r^5}$$

$$F_1 = \left[-\frac{ax}{r^3} + b\left[\frac{3x^2}{r^5} - \frac{1}{r^3}\right] + c\left[-\frac{15x^3}{r^7} + \frac{3x}{r^5}\right] + \cdots\right]$$

$$F_2 = \qquad -\frac{b}{r^3} + \frac{6cx}{r^5}$$

$$\theta = \varphi\psi$$

$$\frac{\partial\theta}{\partial x} = \frac{\partial\varphi}{\partial x}\psi + \varphi\frac{\partial\psi}{\partial x}$$

$$\frac{\partial^2\theta}{\partial x^2} = \frac{\partial^2\varphi}{\partial x^2}\psi + 2\frac{\partial\varphi}{\partial x}\frac{\partial\psi}{\partial x} + \varphi\frac{\partial^2\psi}{\partial x^2}$$

$$\Delta^2\varphi.\,\psi + 2\left[\frac{\partial\varphi}{\partial x}\frac{\partial\psi}{\partial x} + \frac{\partial\varphi}{\partial y}\frac{\partial\psi}{\partial y} + \frac{\partial\varphi}{\partial z}\frac{\partial\psi}{\partial z}\right] + \varphi\Delta^2\psi + \psi\left[u\frac{\partial\varphi}{\partial x} + \cdots\right] + \varphi\left[u\frac{\partial\psi}{\partial x} + \cdots\right] = 0$$

Δ φ. ψ + 2[...] ... ψ spełnia (D) podczas gdy φ:

Będzie spełniać (A) jeżeli ψ spełnia (D) podczas gdy φ:

$$\Delta^2\varphi + 2\left[\frac{\partial\varphi}{\partial x}\frac{\partial\log\psi}{\partial x} + \frac{\partial\varphi}{\partial y}\frac{\partial\log\psi}{\partial y} + \frac{\partial\varphi}{\partial z}\frac{\partial\log\psi}{\partial z}\right] + u\frac{\partial\varphi}{\partial x} + \dots = 0$$

zatem jeśli się da znaleźć taki φ aby

$$2\,\frac{\partial\varphi}{\partial x}\frac{\partial\log\psi}{\partial x} + \dots = \Phi$$

$$\iiint \Delta^2\theta + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z} = 0 \quad dv$$

$$\iint \frac{\partial\theta}{\partial n}\,dS + \iint \underbrace{\left[u\theta\,dS\cos nx + v\theta\,dS\cos ny + \dots\right]} - \cancel{\iiint \theta\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)dv}$$

$$\iint\frac{\partial\theta}{\partial n}\,dS + \iint \theta\, v_n\, dS = 0$$

Zastosujmy to do rurek prądu cieczy

$$\Delta^2\theta + u\frac{\partial\theta}{\partial x} + \dots = 0$$

$$\Delta^2\theta' \pm u\frac{\partial\theta'}{\partial x} + \dots = 0$$

$$\Delta^2(\theta-\theta') + u\frac{\partial(\theta+\theta')}{\partial x} + v\frac{\partial(\theta+\theta')}{\partial y} + \dots = 0$$

$$\Delta^2(\theta+\theta') + u\frac{\partial(\theta-\theta')}{\partial x} + v\frac{\partial(\theta-\theta')}{\partial y} + \dots = 0$$

jednowymiarowe zadanie cieplne

$$\cancel{u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\mu}{3}\frac{\partial}{\partial x}\,\mathrm{div} + \mu\Delta^2}$$

I). $\rho u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = \frac{4\mu}{3}\frac{\partial^2 u}{\partial x^2}$

II). $\frac{\partial(\rho u)}{\partial x} = 0$ $\qquad\qquad \rho u = \text{const} = b$

III). $k p \frac{\partial u}{\partial x} + u \frac{\partial p}{\partial x} = (k-1)\,\mu\,\frac{4}{3}\left(\frac{\partial u}{\partial x}\right)^2 + \kappa\frac{\partial^2 \theta}{\partial x^2}$

I). $b\frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = \frac{4\mu}{3}\frac{\partial^2 u}{\partial x^2}$

$$b u + p = \frac{4\mu}{3}\frac{\partial u}{\partial x} + a$$

$$\frac{du}{dx} = \frac{bu + p - a}{\frac{4\mu}{3}}$$

III). $k p \cancel{\frac{\partial u}{\partial x}} + u\frac{dp}{du}\cancel{\frac{\partial u}{\partial x}} = (k-1)\frac{4\mu}{3}\left(\frac{\partial u}{\partial x}\right)^2$

$$\cancel{k p \frac{(bu+p-a)}{\frac{4\mu}{3}} + u\frac{dp}{du}}$$

$$k p + u\frac{\partial p}{\partial u} = (k-1)(bu + p - a)$$

$$u\frac{dp}{du} = \underbrace{(k-1)bu - p - (k-1)a}_{=z}$$

$$p = (k-1)bu - (k-1)a - z$$

$$\frac{dp}{du} = (k-1)b\frac{du}{du} - \frac{dz}{du}$$

$$u(k-1)b - u\frac{dz}{du} = z$$

$$\frac{dz}{du} + \frac{z}{u} = (k-1)b$$

$\frac{dp}{du} < 0 \qquad z < 0$

z = [illegible] $c < 0$

$$u\,dz + z\,du = (k-1)\,b\,du\,u$$

$$uz = (k-1)b\frac{u^2}{2} + c$$

$$z = (k-1)\frac{bu}{2} + \frac{c}{u}$$

$$p = (k-1)\,bu - (k-1)a - (k-1)\frac{bu}{2} - \frac{c}{u}$$

$$\cancel{u\frac{(k-1)b}{2} + \frac{c}{u} - (k-1)\frac{bu}{2} + \frac{c}{u} + (k-1)a}$$

$$p = \frac{(k-1)b}{2}\,u - \frac{c}{u} - (k-1)a$$

$$\frac{4\mu}{3}\,\frac{du}{dx} = bu - a + \frac{(k-1)b}{2}\,u - \frac{c}{u} - (k-1)a$$

$$= (k+1)\frac{b}{2}\,u - \frac{c}{u} - ka$$

$$\frac{\frac{4\mu}{3}\,u\,du}{(k+1)\frac{b}{2}u^2 - kau - c} = dx$$

wprowadzając zamiast a i c inne stałe:

$$A = \frac{ak}{(k+1)\frac{b}{2}} \qquad C = \frac{-c}{(k+1)\frac{b}{2}} \qquad a = \frac{(k+1)}{k}\frac{b}{2}A \qquad c = -(k+1)\frac{b}{2}C$$

$$\frac{u\,du}{u^2 - Au + C} = \frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}\,dx = \underbrace{\frac{(u - \frac{A}{2})\,du}{u^2 - Au + C}}_{\frac{1}{2}\log(u^2 - Au + C)} + \frac{\frac{A}{2}\,du}{u^2 - Au + C}$$

przypusz. $A > 0$

$C > 0$

$$\int \frac{du}{u^2 - Au + C} = \frac{2}{\sqrt{4C - A^2}}\,\mathrm{arctg}\,\frac{2u - A}{\sqrt{4C - A^2}} \qquad 4C > A^2$$

$$= \frac{1}{\sqrt{A^2 - 4C}}\,\log\left\{\frac{2u - A - \sqrt{}}{2u - A + \sqrt{}}\right\} \qquad A^2 > 4C$$

$$= \frac{1}{\sqrt{}}\,\log\frac{\left(u - \frac{A}{2} - \sqrt{(\frac{A}{2})^2 - C}\right)\left(u - \frac{A}{2} + \sqrt{}\right)}{\left(u - \frac{A}{2} + \sqrt{}\right)^2}$$

$$= \frac{1}{\sqrt{}}\left[\log\left(u^2 - uA + \cancel{\frac{A^2}{4}} - \cancel{\frac{A^2}{4}} + C\right) - 2\log\left(u - \frac{A}{2} + \sqrt{}\right)\right]$$

W przypadku logarytmicznym: $\quad u^2 - Au + C > 0 \quad \frac{A^2}{4} > C$

(1a)
$$\frac{(k+1)\,\frac{b}{2}}{\frac{4\mu}{3}}\,x + \text{const} = \left[\frac{1}{2} + \frac{\frac{A}{2}}{\sqrt{A^2-4C}}\right]\lg\left(u - \frac{A}{2} - \sqrt{\frac{A^2}{4} - C}\right) + \left[\frac{1}{2} - \frac{\frac{A}{2}}{\sqrt{A^2-4C}}\right]\lg\left(u - \frac{A}{2} + \sqrt{\frac{A^2}{4} - C}\right)$$

arctg: $\quad u^2 - Au + C > 0 \quad \frac{A^2}{4} < C$

(1b)
$$\frac{(k+1)\,\frac{b}{2}}{\frac{4\mu}{3}}\,x + \text{const} = \frac{1}{2}\lg\left(u^2 - Au + C\right) + \frac{A}{\sqrt{4C-A^2}}\operatorname{arctg}\frac{2u-A}{\sqrt{4C-A^2}}$$

(2) $p = \frac{b}{2}\left\{(k-1)u + (k+1)\frac{C}{u} - \frac{(k-1)(k+1)}{k}A\right\}$ ‖ zupełnie nie zawiera μ!

(3) $\theta = \frac{pu}{R\rho u} = \frac{pu}{Rb} = \frac{1}{2R}\left\{(k-1)u^2 - \frac{(k^2-1)}{k}Au + (k+1)C\right\}$ ‖ niezależnie od b!

$\frac{(1)}{(2)}$: $\frac{(k+1)}{\frac{4\mu}{3}}\,\frac{b}{2}\,x\,p = \left[f_k(u, A, C) - \text{const}\right]\frac{b}{2}\,\varphi_k(u, A, C)$

$\left(\frac{1}{2}\right)$ $\quad p\,x = \frac{4\mu}{3}\,\frac{1}{k+1}\left[f(u\,A\,C) - \text{const}\right]\varphi(u\,A\,C)$

Jeżeli np. θ dane przy dwóch przekrojach to można obliczyć $\underline{A}, \underline{C}$
oprócz tego wartość $p\,x$ przy pewnym u to wynika wartość const

N.p. dla $x=0$ $u=u_0$ $\theta=\theta_0$
dla $u=u_1$ $\theta=\theta_1$ } z tych zapomocą (3) otrzymuje się A i C

zapomocą (1): const

dalej: równanie $(\frac{1}{2})$: px = znana funkcja z $R\theta\rho x$

$$R\theta\rho x = \frac{4}{3}\mu \frac{1}{k+1}(f-\text{const})\varphi$$

zatem $$\rho = \frac{\frac{4}{3}\mu \frac{1}{k+1}(f-\text{const})\varphi}{R\theta x} = \frac{\frac{4}{3}\mu \frac{1}{k+1}(f-\text{const})}{\frac{1}{2}u\,x}$$

zatem $$b = \rho u = \frac{\frac{4}{3}\mu \frac{1}{k+1}}{\frac{1}{2}x}(f-\text{const})$$

$\frac{dp}{dx}$ musi w każdym razie być wszędzie ujemne, inaczej by ruch nie nastąpił

b!

$$\frac{dp}{dx} = \frac{dp}{du}\frac{du}{dx} = \frac{b}{2}\left\{(k-1) - \frac{(k+1)C}{u^2}\right\}\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}\;\frac{u^2-Au+C}{u}$$

$$= \frac{b^2}{4}\frac{k+1}{\frac{4\mu}{3}}\frac{\left[k-1-\frac{(k+1)C}{u^2}\right]\left[u^2-Au+C\right]}{u} \qquad <0$$

więc albo $k-1 < \frac{(k+1)C}{u^2}$
albo $u^2-Au+C<0$ } stanowi w każdym razie górną granicę dla u

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Próba: wstawić owe wyrażenia do I II III

$$\cancel{b\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}\frac{u^2-Au+C}{u} + \frac{b^2}{4}\frac{(k+1)}{\frac{4\mu}{3}}\frac{\left[(k-1)-\frac{(k+1)C}{u^2}\right]\left[u^2-Au+C\right]}{u}} =$$

$$\frac{4\mu}{3}\underbrace{\left\{\frac{2u-A}{u} - \frac{u^2-Au+C}{u^2}\right\}}_{=}\cancel{\frac{u^2-Au+C}{u}} \longrightarrow \frac{\cancel{2u^2}-\cancel{Au}-\cancel{u^2}+\cancel{Au}-C}{u^2}$$

$$\frac{\frac{b^2}{2}(k+1)}{\frac{4\mu}{3}} + \frac{\frac{b^2}{4}(k^2-1)}{\frac{4\mu}{3}} - \frac{\frac{b^2}{4}(k+1)^2}{\frac{4\mu}{3}}\frac{C}{u^2} = \frac{4\mu}{3}\left(1-\frac{C}{u^2}\right)$$

Spełnione tylko jeżeli:

$$\frac{b^2}{2}(k+1) + \frac{b^2}{4}(k^2-1) = \left(\frac{4\mu}{3}\right)^2 = \frac{b^2}{4}\left[k^2-1+2(k+1)\right] = \frac{b^2}{4}\left[k^2+2k+1\right] = \frac{b^2}{4}(k+1)^2$$

$$\frac{b^2}{4}(k+1)^2 = \left(\frac{4\mu}{3}\right)^2$$

Identyczne !!

$$\frac{b(k+1)}{2} = \frac{4\mu}{3}$$

$$b = \frac{8\mu}{3(k+1)}$$

Trzy równania przejmują kształt:

(1)a $\quad x + \text{const} = \left[\frac{1}{2} + \frac{1}{\sqrt{}}\right] \dots$

$=$

(2) $$p = \frac{4\mu}{3(k+1)}\left\{(k-1)u + (k+1)\frac{C}{u} - \frac{(k-1)(k+1)}{k}A\right\}$$

(3) $$\theta = \frac{1}{2R}\left\{(k-1)u^2 + (k+1)C - \frac{(k^2-1)}{k}Au\right\}$$

$$\rho = \frac{b}{u} = \frac{8\mu}{3(k+1)}\frac{1}{u}$$

$u = \frac{8\mu}{3(k+1)}\frac{1}{\rho}$ N.p. $\mu = 0{\cdot}00019$ $u = \frac{1}{10}$?

$\rho = 0{\cdot}0012$

Wstawiając 1a — 3 w III:

$$k\,\frac{b}{2}\left\{(k-1)u+(k+1)\frac{C}{u}-\frac{(k-1)(k+1)}{k}A\right\}+\cancel{u}\,\frac{b^2}{4}\,\frac{k+1}{\frac{4\mu}{3}}\,\frac{\left[k-1-\frac{(k+1)C}{u^2}\right]\left[u^2-Au+C\right]}{\cancel{u}}=$$

$$=(k-1)\frac{4\mu}{3}\left[\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}\;\frac{u^2-Au+C}{u}\right]^2$$

$$k\,\frac{4\mu}{3}\left\{(k-1)u+(k+1)\frac{C}{u}-\frac{(k-1)(k+1)}{k}A\right\}+\frac{b}{2}(k+1)\left\{k-1-\frac{(k+1)C}{u^2}\right\}\left\{u^2-Au+C\right\}=$$

$$=(k-1)(k+1)^2\,\frac{b}{2}\left\{u-A+\frac{C}{u}\right\}^2$$

$$\frac{4\mu}{3}k\left[(k-1)u-\frac{(k-1)(k+1)}{k}A+(k+1)\frac{C}{u}\right]=\frac{b}{2}(k+1)\left[u-A+\frac{C}{u}\right]\underbrace{\left\{(k+1)\left(u-A+\frac{C}{u}\right)-u(k-1)+\frac{(k+1)C}{u}\right\}}$$

$$=\frac{b}{2}(k+1)\left[u-A+\frac{C}{u}\right]\left[2u-(k+1)A+\frac{2(k+1)C}{u}\right]$$

Nie może być spełnione!

$$\text{I).}\quad \cancel{9u\frac{du}{dp}+1=\frac{4\mu}{3}\frac{d^2u}{dx\,dp}}$$

$$\text{II).}\quad kp\frac{du}{dp}+u=(k-1)\frac{4\mu}{3}\frac{du}{dx}\frac{du}{dp}$$

Spełnione

$$\text{I).}\quad b\frac{du}{dp}+1=\frac{4\mu}{3}\frac{d}{dp}\left(\frac{du}{dx}\right)$$

$$bu+p=\frac{4\mu}{3}\frac{du}{dx}+a$$

Spełnione

$$2\cancel{ka}+\frac{b}{2}\left\{(k+1)p+\cancel{(k+1)}\frac{C}{u}-\frac{(k-1)(k+1)}{k}A\right\}=\frac{4\mu}{3}\frac{(k+1)}{\frac{4\mu}{3}}\frac{u^2-Au+C}{u}\ldots\frac{k+1}{k}A)$$

II).

III.

$$\left\{k \frac{1}{4}\left\{\cancel{(k-1)u} + (k+1)\frac{C}{u} - \cancel{\frac{(k-1)(k+1)}{k}A}\right\} - (k-1)(k+1)\left[\cancel{u - A} + \frac{C}{u}\right]\right\}$$

$$\cancel{= u \frac{1}{2}\left\{(k-1) - (k+1)\frac{C}{u^2}\right\}}$$

Wstawiając w III:

$$k\frac{1}{4}\left\{(k-1)u + (k+1)\frac{C}{u} - \frac{(k-1)(k+1)}{k}A\right\} + u\,\cancel{\frac{(k+1)}{}}\left[k-1-\frac{(k+1)C}{u^2}\right]\cancel{\frac{(u^2-Au+C)}{u}}$$

$$=(k-1)\,(k+1)\left(\frac{u^2-Au+C}{u}\right)$$

$$\cancel{(k^2-k)u} + \cancel{(k-1)u} - \cancel{(k^2-1)u} + k\cancel{(k+1)}\frac{C}{u} - \cancel{(k+1)}\frac{C}{u} - \cancel{(k^2-1)}\frac{C}{u} +$$

$$\cancel{(k-1)(k+1)A} + \cancel{(k-1)(k+1)A} = 0$$

I). $$k\frac{(k+1)}{}\,2\,\frac{u^2-Au+C}{\cancel{u}} + \frac{k+1}{}\left[k-1-\frac{(k+1)C}{u^2}\right]\frac{[u^2-Au+C]}{\cancel{u}} =$$

$$=\left(\frac{(k+1)}{}\right)\frac{u^2-C}{u^2}\;\cancel{u^2-Au+C}$$

$$\underbrace{2+k-1}_{k+1} - \frac{(k+1)C}{u^2} = (k+1)\left(1-\frac{C}{u^2}\right)$$

zgadza się

~~Jeśli możliwe bardzo małe u to tam gdzie~~

~~$\lim u ..$~~

~~$p = \frac{b}{2}(k+1)\frac{C}{u}$~~

~~$\theta = \frac{(k+1)C}{2R}$~~

~~i zaczynając z tamtąd rozwiązać x:~~

~~(1a) nie możliwe~~

(1b): $\cancel{(k+1)\frac{A}{2}}$ $\quad wnt = \frac{1}{2}\lg C \mp \frac{A}{\sqrt{4C-A^2}} \operatorname{arctg} \frac{-A}{\sqrt{4C-A^2}}$

$\rho = \frac{p}{R\theta} = \frac{b}{u}$

Przy (1a) najmniejsza możliwa prędkość:

$$u > \frac{A}{2} + \sqrt{\frac{A^2}{4} - C}$$

zatem:

$$u - \frac{A}{2} > \sqrt{\frac{A^2}{4} - C}$$

$$u^2 - Au + \frac{A^2}{4} > \frac{A^2}{4} - C$$

$$u^2 - Au + C > 0$$

największa możliwa z powodu że $\frac{dy}{dx} < 0$:

$$u^2 < \frac{(k+1)C}{k-1}$$

$$Au - C < u^2 < \frac{k+1}{k-1}C$$

$$< \left(1 + \frac{2}{k-1}\right)C$$

$$Au < \underbrace{\left(\frac{k+1}{k-1} + 1\right)}C$$

$$Au < \frac{2k}{k-1}C$$

$$u < \left(\frac{2k}{k-1}\right)\frac{C}{A}$$

$$\frac{C}{A} < \frac{A}{4}$$

$$u < \frac{k}{2(k-1)}A$$

Ale także

$$u^2 < \frac{k+1}{k-1}\frac{A^2}{4}$$

$$\boxed{u < \frac{A}{2}\sqrt{\frac{k+1}{k-1}}}$$

$$\frac{k}{k-1} = \frac{1{\cdot}4}{0{\cdot}4} = \frac{7}{2} = 3{\cdot}5$$

$$\sqrt{\frac{k+1}{k-1}} = \sqrt{\frac{2{\cdot}4}{0{\cdot}4}} = \sqrt{6} = 2{\cdot}5$$

Przy (1a) warunek: $\frac{A^2}{4} > C$

$$\theta < \frac{1}{2R}\left\{(k-1)u^2 - \frac{(k^2-1)}{k}Au + (k+1)\frac{A^2}{4}\right\}$$

$$p < \frac{b}{2}\left\{(k-1)u + \frac{(k+1)}{u}\frac{A^2}{4} - \frac{(k-1)(k+1)}{k}A\right\}$$

$$p < \frac{b}{2}\left\{\frac{kA}{2} - \frac{k^2-1}{k}A + \frac{(k+1)A^2}{4(Au-C)}\right\}$$

$$\theta < \frac{1}{2R}\left\{(k-1)\frac{A^2}{4}\frac{k+1}{k-1} + (k+1)\frac{A^2}{4} - \frac{(k^2-1)}{k}Au\right\}$$

$$< \frac{1}{2R}(k+1)\left\{\frac{A^2}{2} - \frac{k-1}{k}Au\right\}$$

~~$u^2 - Au > C$~~

~~$\frac{A^2}{4} > C > Au - u^2$~~

~~$\frac{A^2}{4} > Au - u^2$~~

~~$u^2 - Au + \frac{A^2}{4} > 0 \quad \left(u - \frac{A}{2}\right)^2 > 0$~~

W (1a) i (1b) warunek Konieczny $u^2 - Au + C > 0$

zatem $\frac{du}{dx} > 0$ prędkość wzmagająca się z odległością x

zatem aby było spełnione $\frac{dp}{dx} < 0$

musi być $\frac{dp}{du} < 0$ czyli $u^2 < \frac{k+1}{k-1} C$

$$\text{const} = [\quad] \log(u_1 - \frac{A}{2} - \sqrt{}) + [\quad] -$$

$$\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}} x = \left[\frac{1}{2} + \frac{1}{\sqrt{A^2-4C}}\right] \log\left(\frac{u - \frac{A}{2} - \sqrt{\frac{A^2}{4} - C}}{u_1 - \frac{A}{2} - \sqrt{\frac{A^2}{4} - C}}\right) + [\ - \quad]$$

$$\frac{b}{2}(k+1) C = k p_0 \alpha \qquad C = \frac{k p_0 \alpha}{(k+1)\frac{b}{2}}$$

$$\frac{b}{2}\frac{(k+1)}{\alpha} A = p_0 \qquad A = \frac{k p_0}{(k+1)\frac{b}{2}}$$

$$\frac{1}{2}\log \frac{u - A}{u_1 - A} = \frac{1}{2}\log \frac{1 - \frac{u}{A}}{1 - \frac{u_1}{A}} = \frac{1}{2}\frac{u_1 - u}{A} = \frac{1}{2}\frac{u_1 - u}{k p_0}(k+1)\frac{b}{2}$$

$$\frac{1}{2}\log \frac{u + \frac{A}{2}\left[-1 + \left(1 - \frac{4C}{A^2}\right)^{\frac{1}{2}}\right]}{u_1 + \frac{A}{2}\left[-1 + \left(1 - \frac{4C}{A^2}\right)^{\frac{1}{2}}\right]} = \frac{1}{2}\log \frac{u + \frac{A}{2}\left[-\cancel{1} + \cancel{1} - \frac{2C}{A^2}\right]}{u_1 + \frac{A}{2}\left[\ - \frac{2C}{A^2}\right]} =$$

$$= \frac{1}{2}\log \frac{\frac{u}{A} - \frac{C}{A^2}}{\frac{u_1}{A} - \frac{C}{A^2}} = \frac{1}{2}\log \frac{u - \frac{C}{A}}{u_1 - \frac{C}{A}} = \frac{1}{2}\log \frac{u - \alpha}{u_1 - \alpha}$$

Zatem $\frac{(k+1)\frac{b}{2}x}{\frac{4\mu}{3}} = \frac{1}{2}\left[\frac{(u_1 - u)(k+1)}{k p_0}\frac{b}{2} + \log\frac{u - \alpha}{u_1 - \alpha}\right]$

Co jeśli $u^2 - Au + C < 0$ Jeśli $C > 0$, $A > u + \frac{C}{4}$

$$\int \frac{u\,du}{u^2 - Au + C} = \cancel{\frac{(u-\frac{A}{2})\,du}{-(u^2-Au+C)} + \frac{A}{2}\frac{du}{\dots}} = -\int \frac{u\,du}{Au - u^2 - C}$$

$$= \cancel{-\frac{1}{2}\lg\left[-u^2 + Au + C\right]}$$

$$= \int \frac{(\frac{A}{2} - u)\,du}{Au - u^2 - C} - \frac{A}{2}\underbrace{\frac{du}{Au - u^2 - C}}$$

$$= \lg(Au - u^2 - C) - \frac{A}{2}\underbrace{\int \frac{du}{\frac{A^2}{4} - C - (u - \frac{A}{2})^2}}$$

$$\frac{A}{2}\lg\frac{\sqrt{\frac{A^2}{4} - C} - u - \frac{A}{2}}{\sqrt{\quad} + \dots}$$

W granicznym przypadku, gdzie $C = \frac{A^2}{4}$

$$\frac{(k+1)\frac{b}{2}}{4\mu/3}\,x + \mathrm{const} = \lg\left(u - \frac{A}{2}\right) - \frac{A}{2(u - \frac{A}{2})}$$

dla $x=0$: u bardzo małe $= u_0$

$\theta = \theta_0$

$p = p_0$

$$\theta_0 = \frac{(k+1)C}{2R} \qquad \cancel{p_0 = \frac{(k+1)C\, b}{2 u_0}}$$

$$C = \frac{2R\theta_0}{k+1} \qquad b = \frac{p_0 u_0}{R\theta_0}$$

$$\text{const} = \frac{1}{2}\log C + \frac{A}{\sqrt{4C-A^2}} \operatorname{arctg} \frac{-A}{\sqrt{}}$$

Przy wzrastającym u:

arctg może tylko dojść do $\frac{\pi}{2}$ gdy u od 0 do ∞ · podczas gdy log. wzrasta stale z u

zatem dla wielkich u będzie już tylko:

$$\cancel{x = \frac{\frac{4\mu}{3}}{\frac{b}{2}(k+1)} \log u}$$

$$\cancel{u = e^{\frac{\frac{b}{2}(k+1)x}{\frac{4\mu}{3}}}}$$

$$\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}} x = \frac{1}{2}\log\frac{u^2 - Au + C}{C} + \frac{A}{\sqrt{4C-A^2}} \underbrace{\left[\operatorname{arctg}\frac{2u-A}{\sqrt{}} + \operatorname{arctg}\frac{A}{\sqrt{}}\right]}$$

$$\operatorname{tg}(\varphi+\psi) = \frac{\operatorname{tg}\varphi + \operatorname{tg}\psi}{1 - \operatorname{tg}\varphi \operatorname{tg}\psi}$$

$$\varphi + \psi = \operatorname{arctg}\frac{\operatorname{tg}\varphi + \operatorname{tg}\psi}{1 - \operatorname{tg}\varphi\operatorname{tg}\psi}$$

$$= \operatorname{arctg}\frac{\frac{2u}{\sqrt{}}}{1 - \frac{(2u-A)A}{4C-A^2}} = \operatorname{arctg}\frac{\frac{2u}{\sqrt{}}}{\frac{4C-2uA}{4C-A^2}}$$

$$= \operatorname{arctg}\frac{u\sqrt{4C-A^2}}{2C-uA}$$

Dla u bardzo dużego:

$$\frac{u}{\sqrt{C}} = e^{\cdots} \cdot e^{\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}x}$$

dla małych u potem przybliżenie:

$$\cancel{\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}} x = \frac{A}{\sqrt{4C-A^2}} \cdot \frac{\sqrt{4C-A^2}\cdot u}{2C} = \frac{Au}{2C}}$$

$$\cancel{\frac{(k+1)\, p_0 u_0}{\frac{4\mu}{3}\; 2R\theta_0}\, x = \frac{Au\,(k+1)}{2R\theta_0}}$$

$$\cancel{u = \frac{p_0 u_0}{\frac{4\mu}{3} A}\, x}$$

$$\frac{du}{dx} = \frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}\, \frac{u^2-Au+C}{u}\Big|_{u=u_0} = \frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}} \cdot \frac{C}{u_0} = \frac{(k+1)\,\frac{p_0 u_0}{R\theta_0}\,\frac{R\theta_0}{k+1}}{\frac{4\mu}{3}} = \frac{p_0 u_0}{\frac{4\mu}{3}}$$

więc tam będzie: $u = u_0 + \frac{p_0 u_0}{\frac{4\mu}{3}}\, x$

$$\frac{d\mu}{dx}\Big|_{u=u_0} = \cancel{\ldots}$$

$$= \frac{p_0^2}{\frac{4\mu}{3}\, u_0}$$

$$\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}\, x + \text{const}_1 = \frac{1}{2}\log\left(u^2-Au+C\right) + \frac{A}{2}\,\frac{1}{\sqrt{A^2-4C}}\,\log\left[\frac{A+\sqrt{A^2-4C}-2u}{A-\sqrt{A^2-4C}-2u}\right]$$

Gdy u od małych wartości początkowych | dla których: $\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}\, x + \text{const} = \frac{1}{2}\log C + \frac{A}{2}\frac{1}{\sqrt{\ }}\log\frac{A+\sqrt{\ }}{A-\sqrt{\ }}$

zbliża się do $u = \frac{A}{2} - \sqrt{\frac{A^2}{4}-C}$ to równanie owo przyjmie kształt:

$$\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}\, x + \text{const} = -\infty + \Delta$$

wtedy będzie $\frac{du}{dx} = 0$ równe dla u

gdy A, C wielkie:

$$u = \frac{A}{2}\left[1-\left(1-\frac{4C}{A^2}\right)^{\frac{1}{2}}\right] = \frac{A}{2}\left[1-1+\frac{2C}{A^2}\right] = \frac{C}{A} = \alpha$$

Obliczenie owej wartości $-\infty+\infty$.

$\cancel{\log(u^2-Au+C)}$

$$\lim_{u=\frac{A}{2}-\sqrt{\ }} \frac{1}{2}\left\{\log(u^2-Au+C)\left[\frac{\frac{A}{2}+\sqrt{\frac{A^2}{4}-C}-u}{\frac{A}{2}-\sqrt{\frac{A^2}{4}-C}-u}\right]^{\frac{A}{\sqrt{A^2-4C}}}\right\}$$

$$\downarrow$$

$$\underbrace{\left(\frac{A}{2}-\sqrt{\ }-u\right)}_{\delta}\left(\frac{A}{2}+\sqrt{\ }-u\right)$$

$$\frac{A}{2}+\sqrt{\frac{A^2}{4}-C}-u=\delta+\sqrt{A^2-4C}$$

$$=\frac{1}{2}\log\frac{\left[\delta+\sqrt{A^2-4C}\right]^{1+\frac{A}{\sqrt{A^2-4C}}}}{\delta^{\left(\frac{A}{\sqrt{A^2-4C}}-1\right)}}$$

więc będzie $=+\infty$ ~~jest~~ ponieważ $A>\sqrt{A^2-4C}$

zatem dla $x=\infty$ u osięga wartości maksymalnej $u=\frac{A}{2}-\sqrt{\frac{A^2}{4}-C}=\alpha$

$$u^2=\frac{A^2}{2}-C-A\sqrt{\ }$$

~~Oznaczenie stałych:~~

~~Dla $x=\infty$ $u=\alpha$~~

~~$x=0$ $u=u_0$~~

Teraz będzie:

$$\theta=\frac{1}{2R}\left\{(k-1)\left(\frac{A^2}{2}-C-A\sqrt{\ }\right)-\frac{(k^2-1)}{k}\left(\frac{A^2}{2}-A\sqrt{\ }\right)+(k+1)C\right\}$$

$$\theta_\infty=\frac{1}{2R}\left\{\frac{1-k}{k}\frac{A^2}{2}+2C+\frac{k-1}{k}A\sqrt{\frac{A^2}{4}-C}\right\}$$

$$\alpha^2-\alpha A+C=0$$

$$\boxed{A=\frac{\alpha^2+C}{\alpha}}$$

$$\theta_\infty=\frac{1}{2R}\left[-\frac{k-1}{k}A\left(\frac{A}{2}-\sqrt{\frac{A^2}{4}-C}\right)+2C\right]=\frac{1}{2R}\left[2C-\frac{(k-1)}{k}(\alpha^2+C)\right]$$

$$\theta_\infty=\frac{1}{2R}\left[C\left(2-\frac{k-1}{k}\right)-\frac{k-1}{k}\alpha^2\right]=\frac{1}{2kR}\left[(k+1)C-(k-1)\alpha^2\right]$$

$$p_\infty=\frac{Rb\,\theta_\infty}{\alpha}=\frac{b}{2k\alpha}\left[(k+1)C-(k-1)\alpha^2\right]$$

$$\sqrt{A^2-4C}=\sqrt{\left(\frac{\alpha^2+C}{\alpha}\right)^2-4C}=\sqrt{\frac{\alpha^4+2\alpha^2C+C^2-4\alpha^2C}{\alpha^2}}=\frac{\alpha^2-C}{\alpha}$$

$$\frac{p}{p_\infty} = \kappa\alpha \frac{\left[(\kappa-1)u + (\kappa+1)\frac{C}{u} - \frac{(\kappa^2-1)}{\kappa}\frac{\alpha^2+C}{\alpha}\right]}{(\kappa+1)C - (\kappa-1)\alpha^2}$$

$$\frac{(\kappa+1)\frac{b}{2}}{\frac{4\mu}{3}} x + \text{const} = \frac{1}{2}\log\left[u^2 - \frac{\alpha^2+C}{\alpha}u + C\right] + \frac{\alpha^2+C}{2\alpha}\frac{\alpha}{\alpha^2-C}\log\frac{\alpha + \frac{\alpha^2-C}{\alpha} - u}{\alpha - u}$$

$$= \frac{1}{2}\log\left[u^2 - \alpha u - \frac{C}{\alpha}u + C\right] + \frac{\alpha^2+C}{2(\alpha^2-C)}\log\left[\frac{2 - \frac{C}{\alpha^2} - \frac{u}{\alpha}}{1-\frac{u}{\alpha}}\right]$$

---

punkt gdzie $u = 0$: $x = 0$

$$\text{const} = \frac{1}{2}\log C + \frac{\alpha^2+C}{2(\alpha^2-C)}\log\left(2 - \frac{C}{\alpha^2}\right)$$

$$\frac{(\kappa+1)\frac{b}{2}}{\frac{4\mu}{3}} x = \frac{1}{2}\log\left[\frac{u^2-\alpha u}{C} - \frac{u}{\alpha} + 1\right] + \frac{\alpha^2+C}{2(\alpha^2-C)}\log\left[\frac{1 - \frac{u\alpha}{2\alpha^2-C}}{1-\frac{u}{\alpha}}\right]$$

$$p_0 = \frac{b}{2}(\kappa+1)\frac{C}{u_0} = \frac{(\kappa+1)C}{2}\rho_0$$

tam także $\frac{du}{dx} = \infty$ punkt osobliwy

Liniowe przybliżenie:

$$\vartheta = \vartheta_0 + \frac{\mu c^2}{32K}\left\{3a^2\left[-\frac{9}{r^2} + \frac{3x^2}{r^4} - \frac{a^2}{r^4} - \frac{2}{3}\frac{a^4}{r^6} + \frac{4a^2x^2}{r^6} - 2\frac{a^4x^2}{r^8}\right] + \right.$$

$$\left. + \frac{a}{r}\left[19 + 13\frac{a^2}{r^2} + 24\frac{x^2}{r^2} - 39\frac{x^2a^2}{r^4}\right]\right\}$$

dla $r = a$:

$$\{\ \} = 3\Big[\underbrace{-9 - 1 - \frac{2}{3}}_{-32} + 3\frac{x^2}{a^2} + 4\frac{x^2}{a^2} - 2\frac{x^2}{a^2}\Big] + \Big[19 + 13 - 15\frac{x^2}{a^2}\Big]$$

$$-32 + 15\frac{x^2}{a^2} + 32 - 15\frac{x^2}{a^2} = 0$$

dla $x = 0$:

$$\theta = \theta_0 + \frac{\mu c^2}{32K}\left\{-3a^2\left[\frac{9}{r^2} + \frac{a^2}{r^4} + \frac{2}{3}\frac{a^4}{r^6}\right] + \frac{a}{r}\left[19 + 13\frac{a^2}{r^2}\right]\right\}$$

$$\frac{\partial\theta}{\partial r} = \frac{\mu c^2}{32K}\left\{3a^2\left[\frac{18}{r^3} + \frac{4a^2}{r^5} + \frac{4a^4}{r^7}\right] - \frac{a}{r^2}\left[19 + 39\frac{a^2}{r^2}\right]\right\}\Big|_{r=a} =$$

$$= \frac{\mu c^2}{32Ka}\left\{78 - 58\right\} = \frac{5}{8}\frac{\mu c^2}{Ka}$$

$x = r$

$$\theta = \theta_0 + \frac{\mu c^2}{32K}\left\{3a^2\left[-\frac{9}{r^2} + \frac{3}{r^2} - \frac{a^2}{r^4} - \frac{2}{3}\frac{a^4}{r^6} + \frac{4a^2}{r^4} - \frac{2a^4}{r^6}\right] + \right.$$

$$\left. + \frac{a}{r}\left[19 + 13\frac{a^2}{r^2} + 24 - 39\frac{a^2}{r^2}\right]\right\}$$

$$\{\ldots\} = \left\{3a^2\left[-\frac{6}{r^2} + \frac{3a^2}{r^4} - \frac{8}{3}\frac{a^4}{r^6}\right] + \frac{a}{r}\left[43 - 20\frac{a^2}{r^2}\right]\right\}$$

$$\frac{\partial\theta}{\partial r} = 3a^2\left[\frac{12}{r^3} - \cancel{12}\frac{a^2}{r^5} + \frac{48}{3}\frac{a^4}{r^7}\right] - \frac{a}{r^2}\left[43 - \underset{35}{\cancel{78}}\frac{a^2}{r^2}\right]$$

$$\frac{\mu c^2}{aK}\left[\frac{48}{32} + \frac{35}{32}\right] = \frac{\mu c^2}{aK}\frac{83}{32} = \frac{\mu c^2}{aK}\left(2 + \frac{19}{32}\right)$$

$$\frac{\partial\theta}{\partial x} = \frac{\mu c^2}{32k}\left\{3a^2\left[+\frac{18x}{r^4} - \frac{12x^3}{r^6} + \frac{4a^2x}{r^6} + \cancel{\frac{4a^4x}{r^8}} - \frac{24a^2x^3}{r^8} + \frac{16a^4x^3}{r^{10}} + \right.\right.$$

$$\left.\left. + \underbrace{\frac{6x}{r^4} + \frac{8a^2x}{r^6} - \frac{4a^4x}{r^8}}\right] + \left[-19\frac{ax}{r^3} - 39\frac{a^3x}{r^5} - 72\frac{x^3a}{r^5} + 5.39\frac{x^3a^3}{r^7} + \underbrace{\frac{48xa}{r^3} - 2.39\frac{a^3x}{r^5}}\right]\right\}$$

$$= Mx + Nx$$

$$= \frac{\mu c^2}{32k}\left\{3a^2\left[\frac{24x}{r^4} + \frac{12a^2x}{r^6} - \frac{12x^3}{r^6} - \frac{24a^2x^3}{r^8} + \frac{16a^4x^3}{r^{10}}\right] + \right.$$

$$\left. + \left[29\frac{ax}{r^3} - 9.13\frac{a^3x}{r^5} - 72\frac{ax^3}{r^5} + 15.13\frac{a^3x^3}{r^7}\right]\right\}$$

$$\frac{\partial\theta}{\partial y} = \frac{\mu c^2}{32K}\left\{3a^2\left[\frac{18}{r^4} - \frac{12x^2}{r^6} + \frac{4a^2}{r^6} + \frac{4a^4}{r^8} - \frac{24a^2x^2}{r^8} + \frac{16a^4x^2}{r^{10}}\right]y\right.$$

$$\left. + \underbrace{\left[-\frac{19a}{r^3} - 39\frac{a^3}{r^5} - 72\frac{x^2a}{r^5} + 5.39\frac{a^3x^2}{r^7}\right]}_{\text{uu}} y\right\} \qquad = My$$

$$= Mz$$

$$\frac{\partial\theta}{\partial z} = z \cdots$$

$$u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z} = -\frac{3}{4}ca\left(1-\frac{a^2}{r^2}\right)\frac{x}{r}M + c\left(1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right)Mx$$

$$+ Nux$$

$$= Mcx\left[1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3} - \frac{3}{4}\frac{a}{r} + \frac{3}{4}\frac{a^3}{r^3}\right] + Nux$$

$$= Mcx\left[1 - \frac{3}{2}\frac{a}{r} + \frac{1}{2}\frac{a^3}{r^3}\right] + Nux$$

$$N = 3a^2\left[\frac{6}{r^4} + \frac{8a^2}{r^6} - \frac{4a^4}{r^8}\right] + \frac{48a}{r^3} - 2.39\frac{a^3}{r^5}$$

$$\left[\frac{1}{r^4} - \frac{12x^2}{r^6} + \frac{4a^2}{r^6} + \frac{4a^4}{r^8} - \frac{24a^2x^2}{r^8} + \frac{16a^4x^2}{r^{10}}\right] 3a^2 + \left[-\frac{14a}{r^3} - \frac{39a^3}{r^5} - 72\frac{x^2a}{r^5} + 5.39\frac{a^3x^2}{r^7}\right]$$

$$+3a^2\left[\frac{19}{2r^4} + 36\frac{x^2}{r^6} + \frac{39a^2}{2\cdot r^6} - \frac{5.39\,a^2x^2}{2\,r^8}\right] - \frac{81a^3}{r^5} + \frac{54x^2a^3}{r^7} - \frac{18a^5}{r^7} - \frac{18a^7}{r^9} + \frac{108a^5x^2}{r^9} - \frac{72a^7x^2}{r^{11}}$$

$$-\frac{19}{2}\frac{a^4}{r^6} - \frac{39}{2}\frac{a^6}{r^8} - 36\frac{x^2a^4}{r^8} + \frac{5.39}{2}\frac{a^6x^2}{r^{10}} + \frac{27a^5}{r^7} - \frac{18a^5x^2}{r^9} + \frac{6a^7}{r^9} + \frac{6a^9}{r^{11}} - \frac{36a^7x^2}{r^{11}} + \frac{24a^9x^2}{r^{13}}$$

$$-36\frac{a^2}{r^4} - 12\frac{a^4}{r^6} - 36\frac{a^2x^2}{r^6} + 36\frac{a^4x^2}{r^8} \qquad + \frac{48a}{r^3}$$

$$+18\frac{a^2}{r^4} \qquad -\frac{27}{2}\frac{a^3}{r^5} - \frac{9}{2}\frac{a^5}{r^7} - \frac{27}{2}\frac{a^3x^2}{r^7} + \frac{27}{2}\frac{a^5x^2}{r^9}$$

$$+24\frac{a^4}{r^6} \qquad -18\frac{a^5}{r^7} - 6\frac{a^7}{r^9} - 18\frac{a^5x^2}{r^9} + 18\frac{a^7x^2}{r^{11}}$$

$$-12\frac{a^6}{r^8} \qquad +9\frac{a^7}{r^9} + 3\frac{a^9}{r^{11}} + 9\frac{a^7x^2}{r^{11}} - 9\frac{a^9x^2}{r^{13}}$$

$$-6.13\frac{a^3}{r^5}$$

$$+\frac{9}{2}13\frac{a^4}{r^6} + \frac{3}{2}\cdot 13\frac{a^6}{r^8} + \frac{9}{2}\cdot 13\frac{a^4x^2}{r^8} - \frac{9}{2}\cdot 13\frac{a^6x^2}{r^{10}}$$

$$= \frac{a^2}{r^4}\left[3.18 + \frac{3.19}{2} - 36 + 18\right] - \frac{a^2x^2}{r^6}\left[3.12 - 3.36 + 36\right] + \frac{a^4}{r^6}\left[12 + \frac{3.39}{2} - \frac{19}{2} - 12 + 24 + \frac{9}{2}\cdot 13\right]$$

$$+\frac{a^6}{r^8}\left[12 - \frac{39}{2} - 12 + \frac{3}{2}\cdot 13\right] + \frac{a^4x^2}{r^8}\left[-72 - \frac{3.5.39}{2} - 36 + 36 + \frac{9}{2}\cdot 13\right] +$$

$$+\frac{a^6x^2}{r^{10}}\left[48 + \frac{5.39}{2} - \frac{9.13}{2}\right] + \frac{a}{r^3}\left[-14 + 48\right] + \frac{a^3}{r^5}\left[-39 - 81 - \frac{27}{2} - 6.13\right] +$$

$$+\frac{ax^2}{r^5}\left[-72\right] + \frac{a^3x^2}{r^7}\left[5.39 + 54 - \frac{27}{2}\right] + \frac{a^5}{r^7}\left[-18 + 27 - \frac{9}{2} - 18\right] +$$

$$+\frac{a^7}{r^9}\left[-18 + 6 - 6 + 9\right] + \frac{a^5x^2}{r^9}\left[108 - 18 + \frac{27}{2} - 18\right] + \frac{a^7x^2}{r^{11}}\left[-72 - 36 + 18 + 9\right]$$

$$+\frac{a^9}{r^{11}}\left[6 + 3\right] + \frac{a^7x^2}{r^{13}}\left[24 - 9\right]$$

$$= \frac{a^2}{r^4}\frac{129}{2} + \frac{a^2x^2}{r^6}\cdot 36 + \frac{a^4}{r^6}\;\frac{263}{2} + \frac{a^4x^2}{r^8}(-306) + \frac{a^6x^2}{r^{10}}\cdot 87$$

$$+ \frac{a}{r^3}\cdot 29 + \frac{a^3}{r^5}\left(-\frac{423}{2}\right) - \frac{ax^2}{r^5}\,72 + \frac{a^3x^2}{r^7}\left(+\frac{471}{2}\right)$$

$$+ \frac{a^5}{r^7}\left(-\frac{27}{2}\right) + \frac{a^7}{r^9}(-9) + \frac{a^5x^2}{r^9}\left(\frac{171}{2}\right) + \frac{a^7x^2}{r^{11}}(-81) + 9\frac{a^9}{r^{11}} + 15\frac{a^9x^2}{r^{13}}$$

$$u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z} = \frac{\mu c^3 x}{32 k}\Big[\frac{129}{2}\frac{a^2}{r^4} + 30\frac{a^2x^2}{r^6} + \frac{263}{2}\frac{a^4}{r^6} - 306\frac{a^4x^2}{r^8} + 87\frac{a^6x^2}{r^{10}}$$

$$+ 29\frac{a}{r^3} - \frac{423}{2}\frac{a^3}{r^5} - 72\frac{ax^2}{r^5} + \frac{471}{2}\frac{a^3x^2}{r^7} - \frac{27}{2}\frac{a^5}{r^7} - 9\frac{a^7}{r^9} + \frac{171}{2}\frac{a^5x^2}{r^9} - 81\frac{a^7x^2}{r^{11}}$$

$$+ 9\frac{a^9}{r^{11}} + 15\frac{a^9x^2}{r^{13}}\Big]$$

$$\cancel{\Big|_{r=a} = \frac{\mu c^3 x}{32 a^2}\Big[\frac{129}{2} + \frac{263}{2} - 306 + 29 - \frac{423}{2} - \frac{27}{2} - 9 + 9 + \frac{x^2}{a^2}\Big(30 + 87 - 72 - \frac{471}{2} + \frac{171}{2} - 81 + 15\Big)\Big]}$$

-225 190 6 -150

196, 29, 225 — 306, 225, 531, 225, -306 — 42, 15, +57 — 222, 52, -105

$\frac{129}{2}$, $\frac{263}{2}$, 29, $-\frac{423}{2}$, $-\frac{27}{2}$ — $\frac{392}{2} = 196$, 29, 225 — 450 . 225 — 144 — 48 — -43

Do $x = r$

$$u\frac{\partial\theta}{\partial x} = \frac{\mu c^2}{32 k}\Big[\overbrace{(29 - 72)}\frac{a}{r^2} + \frac{a^3}{r^4}\overbrace{\left(-\frac{423}{2} + \frac{471}{2}\right)}^{24}$$

$$+ \frac{a^5}{r^6}\underbrace{\left(-\frac{27}{2} + \frac{171}{2}\right)}_{+72} + \frac{a^7}{r^8}\underbrace{(-9 - 81)}_{-90} + 24\frac{a^9}{r^{10}}$$

$$\Big|_{r=a} = 0$$

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z} = v\cdot\nabla\theta\cdot\cos(\mathfrak{X}v.\nabla\theta)$$

$$= \frac{\mu c^2}{32\,\kappa}\left\{\frac{129}{2}\frac{a^2x}{r^4}+36\frac{a^2x^3}{r^6}+\frac{263}{2}\frac{a^4x}{r^6}-306\frac{a^4x^3}{r^8}+87\frac{a^6x^3}{r^{10}}\right.$$

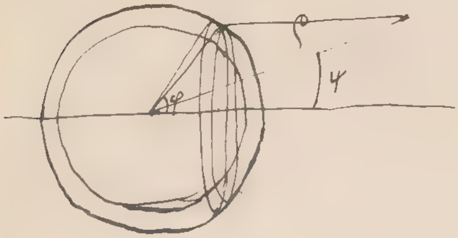
$$+29\frac{ax}{r^3}-\frac{423}{2}\frac{a^3x}{r^5}-72\frac{ax^3}{r^5}+\frac{471}{2}\frac{a^3x^3}{r^7}-\frac{27}{2}\frac{a^5x}{r^7}-9\frac{a^7x}{r^9}+\frac{171}{2}\frac{a^5x^3}{r^9}-$$

$$\left.-81\frac{a^7x^3}{r^{11}}+9\frac{a^9x}{r^{11}}+15\frac{a^9x^3}{r^{13}}\right\}$$

$\Delta^2\left(\frac{x}{r}\right) = -2\frac{x}{r^3}$	
$\Delta^2\left(\frac{x}{r^2}\right) = \frac{x}{r^4}$	
$\Delta^2\left(\frac{x}{r^3}\right) = 0$	$\Delta^2\left(\frac{x^3}{r^3}\right) = \frac{6x}{r^3} - 12\frac{x^3}{r^5}$
$\Delta^2\left(\frac{x}{r^4}\right) = 4\frac{x}{r^6}$	$\Delta^2\left(\frac{x^3}{r^4}\right) = \frac{6x}{r^4} - 12\frac{x^3}{r^6}$
$\Delta^2\left(\frac{x}{r^5}\right) = 10\frac{x}{r^7}$	$\Delta^2\left(\frac{x^3}{r^5}\right) = \frac{6x}{r^5} - 10\frac{x^3}{r^7}$
$\Delta^2\left(\frac{x}{r^6}\right) = 18\frac{x}{r^8}$	$\Delta^2\left(\frac{x^3}{r^6}\right) = \frac{6x}{r^6} - 6\frac{x^3}{r^8}$
$\Delta^2\left(\frac{x}{r^7}\right) = 28\frac{x}{r^9}$	$\Delta^2\left(\frac{x^3}{r^7}\right) = \frac{6x}{r^7}$!
$\Delta^2\left(\frac{x}{r^8}\right) = 40\frac{x}{r^{10}}$	$\Delta^2\left(\frac{x^3}{r^8}\right) = \frac{6x}{r^8} - 8\frac{x^3}{r^{10}}$
$\Delta^2\left(\frac{x}{r^9}\right) = 54\frac{x}{r^{11}}$	$\Delta^2\left(\frac{x^3}{r^9}\right) = \frac{6x}{r^9} - 18\frac{x^3}{r^{11}}$
	$\Delta^2\left(\frac{x^3}{r^{11}}\right) = \frac{6x}{r^{11}} - 44\frac{x^3}{r^{13}}$

$$\mathrm{pot}\left(\frac{x}{r^5}\right) = \mathrm{pot}\,\frac{\cos\varphi}{r^4} = \mathrm{pot}_{-\frac{1}{3}}\frac{\partial\left(\frac{1}{r^3}\right)}{\partial x}$$

$$\cos\chi = \cos\varphi\cos\psi + \sin\varphi\sin\psi\cos\varepsilon$$



$$\frac{1}{\rho} = \frac{1}{\sqrt{r^2 + R^2 - 2Rr\cos\chi}}$$

$$\int_{r=a}^{\infty}\int_{\varphi=0}^{\pi}\int_{\varepsilon=0}^{2\pi} \frac{dr\,d\varphi\,\sin\varphi\,d\varepsilon\,.\,\cos\varphi}{r^2\sqrt{r^2+R^2-2Rr\underbrace{(\cos\varphi\cos\psi+\sin\varphi\sin\psi\cos\varepsilon)}_{\cos\chi}}}$$

$a = R^2$
$b = -2R\cos\chi$
$c = 1$

$$\int_a^R \frac{dr}{r^2\sqrt{r^2+R^2-2Rr\cos\chi}} = -\frac{\sqrt{}}{R^2 r} + \frac{2R\cos\chi}{2R^2}\underbrace{\int\frac{dr}{r\sqrt{}}}\ \cancel{-\ \times}$$

$$= \frac{1}{R}\lg\frac{2R\sqrt{} - (2R^2 - 2Rr\cos\chi)}{r}$$

$$\mathrm{pot}\left(\frac{x}{r^n}\right) = -\frac{\mathrm{pot}}{n-2}\frac{\partial}{\partial x}\left(\frac{1}{r^{n-2}}\right) = -\frac{1}{n-2}\,\frac{\partial}{\partial x}\cdot\mathrm{pot}\left(\frac{1}{r^{n-2}}\right)$$

$$\int_a^R \frac{r^2\,dr}{r^{n-2}} = \int r^{4-n}\,dr = \frac{R^{4-n+1}}{(4-n+1)}\Big|_a^R = \frac{R^{4-n}}{4-n+1} - \frac{a}{R}\ \Big\|\ \frac{\partial}{\partial x} = \frac{4-n}{4-n+1}\,\frac{x}{R^{n-2}} = \mathrm{pot}\left(\frac{x}{r^n}\right)$$

$$\cancel{+\frac{a}{R^3}x}$$

$$\cancel{-\frac{1}{n-2}R^{4-n}\frac{x}{R} = -\frac{1}{n-2}\frac{x}{R^n}}$$

$$\frac{\partial}{\partial x}\left(\frac{1}{r}\right) = -\frac{x}{r^3}$$

$$\frac{\partial^2}{\partial x^2} = -\frac{1}{r^3} + \frac{3x^2}{r^5}$$

$$\frac{\partial^3}{\partial x^3} = -\frac{3x}{r^5} + \frac{6x}{r^5} - \frac{15x^3}{r^7} = \frac{3x}{r^5} - \frac{15x^3}{r^7}$$

$$\frac{\partial}{\partial x}\left(\frac{1}{r^3}\right) = \frac{-3x}{r^5}$$

$$\frac{\partial^2}{\partial x^2}(\;) = \frac{-3}{r^5} + \frac{15x^2}{r^7}$$

$$\frac{\partial^3}{\partial x^3} = +\frac{15x}{r^7} + \frac{30x}{r^7} - \frac{3.5.7.x^3}{r^9} = \frac{45x}{r^7} - \frac{105x^3}{r^9}$$

$$105\frac{x^3}{r^9} = -\frac{\partial^3}{\partial x^3}\left(\frac{1}{r^3}\right) + \frac{45}{5}\frac{\partial}{\partial x}\left(\frac{1}{r^5}\right)$$

$$\frac{x^3}{r^9} = -\frac{1}{105}\frac{\partial^3}{\partial x^3}\left(\frac{1}{r^3}\right) - \frac{3}{35}\frac{\partial}{\partial x}\left(\frac{1}{r^5}\right)$$

$$\mathrm{pot}\left(\frac{x^3}{r^9}\right) = -\frac{1}{105}\frac{\partial^3}{\partial x^3}\underbrace{\mathrm{pot}\left(\frac{1}{r^3}\right)}_{\frac{\lg r}{r}} - \frac{3}{35}\frac{\partial}{\partial x}\,\mathrm{pot}\underbrace{\frac{1}{(r^5)}}_{\frac{1}{2r^3}}$$

$$\int_0^R \frac{r^2\,dr}{r^3} = \lg R \qquad \int\frac{r^2\,dr}{r^5} = \int\frac{dr}{r^3} = -\frac{1}{2r^2}$$

$$\frac{\partial}{\partial x}\left(\frac{\lg r}{r}\right) = \left(\frac{1}{r^2} - \frac{\lg r}{r^2}\right)\frac{x}{r} \qquad \frac{x}{r^4}$$

$$\cancel{\frac{x}{r^3}} \quad \frac{x}{r^4} \quad \frac{x}{r^3}\lg r$$

$$\frac{\partial}{\partial x}\left(\frac{x}{r^3}\lg r\right) = \frac{1}{r^3}\lg r - \frac{3x^2}{r^5}\lg r + \frac{x^2}{r^5}$$

$$\frac{\partial^2}{\partial x^2}(\quad) = -\frac{3x}{r^5}\lg r + \frac{x}{r^5} - \frac{6x}{r^5}\lg r + 15\frac{x^3}{r^7}\lg r - \frac{3x^3}{r^7} + \frac{2x}{r^5} - \frac{5x^3}{r^7}$$

$$= \frac{3x}{r^5} - \frac{8x^3}{r^7} - \frac{9x}{r^5}\lg r + \frac{15x^3}{r^7}\lg r$$

$$\frac{\partial}{\partial y}\left(\frac{x}{r^3}\lg r\right) = -\frac{3xy}{r^5}\lg r + \frac{xy}{r^5}$$

$$\frac{\partial^2}{\partial y^2} \quad = -\frac{3x}{r^5}\lg r + \frac{15xy^2}{r^7}\lg r - \frac{3xy^2}{r^7} + \frac{x}{r^5} - \frac{5xy^2}{r^7}$$

$$= \frac{x}{r^5} - \frac{8xy^2}{r^7} - \frac{3x}{r^5}\lg r + \frac{15xy^2}{r^7}\lg r$$

$$\Delta^2\left(\frac{x}{r^3}\lg r\right) = \frac{5x}{r^5} - \frac{8x}{r^5} - \cancel{\frac{15x}{r^5}\lg r} + \cancel{\frac{15x}{r^5}\lg r} = -\frac{3x}{r^5}$$

$$\frac{\partial^3}{\partial x^3}\left(\frac{x}{r^3}\lg r\right) = \ldots\ldots + 45\frac{x^2}{r^7}\lg r - \frac{9}{r^5}\lg r + \frac{45x^2}{r^7}\lg r - 15\cdot 7\frac{x^4}{r^9}\lg r$$

$$\frac{\partial^2}{\partial x^2}\,\frac{x^2}{r^7}\lg r = \frac{2x}{r^7}\lg r - \frac{7x^3}{r^9}\lg r + \frac{x^3}{r^9} \qquad \frac{\partial}{\partial y} = -7\frac{x^2y}{r^9}\lg r + \frac{x^2y}{r^9}$$

$$\frac{2}{r^7}\lg r - \frac{35x^2}{r^9}\lg r - \cancel{\frac{7x}{r^9}\lg r} + \frac{63x^4}{r^{11}}\lg r + \frac{2x^2}{r^9} - \frac{7x^4}{r^{11}} + \frac{3x^2}{r^9} - \frac{9x^4}{r^{11}}$$

$$-\frac{7x^2}{r^9}\lg r + \frac{63x^2y^2}{r^{11}}\lg r \qquad -\frac{7x^2y^2}{r^{11}} + \frac{x^2}{r^9} - \frac{9x^2y^2}{r^{11}}$$

$$\Delta^2\left(\frac{x^2}{r^7}\lg r\right) = \frac{2}{r^7}\lg r - \frac{49x^2}{r^9}\lg r + \frac{63x^2}{r^9}\lg r + \frac{2x^2}{r^9} - \frac{7x^2}{r^9} + \frac{5x^2}{r^9} - \frac{9x^2}{r^9}$$

$$= \frac{2}{r^7}\lg r + \frac{14x^2}{r^9}\lg r - \frac{9x^2}{r^9}$$

$\frac{\partial}{\partial x}, \quad \frac{x}{2^5}\lg 2$

$\frac{1}{2^5}\lg 2 - \frac{5x^2}{2^7}\lg + \frac{x^2}{2^7}$

$-\frac{5xy}{2^7}\lg + \frac{xy}{2^7}$

$\frac{\partial^2}{\partial x^2}$

$-\frac{15x}{2^7}\lg + \frac{35x^3}{2^9}\lg + \frac{x}{2^7} - \frac{5x^3}{2^9} + \frac{2x}{2^7} - \frac{7x^3}{2^9}$

$-\frac{5x}{2^7}\lg + \frac{35xy^2}{2^9}\lg - \frac{5xy^2}{2^9} + \frac{x}{2^7} - \frac{7xy^2}{2^9}$

$$-\frac{25x}{2^7}\lg 2 + \frac{35x}{2^7}\lg 2 + \frac{5x}{2^7} - \frac{12x}{2^7}$$

$$= \frac{10x}{2^7}\lg 2 - \frac{7x}{2^7}$$

$\frac{x^3}{2^7}\lg 2$

$\frac{3x^2}{2^7}\lg - \frac{7x^4}{2^9}\lg + \frac{x^4}{2^9}$

$-\frac{7x^3y}{2^9}\lg + \frac{x^3y}{2^9}$

$\frac{\partial^2}{\partial x^2}$

$\frac{6x}{2^7}\lg - \frac{49x^3}{2^9}\lg + \frac{63x^5}{2^{11}}\lg + \frac{3x^3}{2^9} - \frac{7x^5}{2^{11}} + \frac{4x^3}{2^9} - \frac{9x^5}{2^{11}}$

$-\frac{7x^3}{2^9}\lg + \frac{63x^3y^2}{2^{11}}\lg \quad - \frac{7x^3y^2}{2^{11}} + \frac{x^3}{2^9} - \frac{9x^3y^2}{2^{11}}$

$$\Delta^2 = \frac{6x}{2^7}\lg 2 + \cancel{\frac{9x^3}{2^9}} - \cancel{\frac{9x^3}{2^9}} - \frac{7x^3}{2^9}$$

$$\frac{x}{2^7} = \Delta^2 \frac{x}{(10\,2^5)}$$

$\cancel{7}\frac{x^3}{2^9} = \frac{6x}{7\,2^7}\lg 2 - \Delta^2\left(\frac{x^3}{7\,2^7}\lg 2\right) \Big| \; 5$

$\cancel{7}\frac{x}{2^7} = \frac{1}{7}\frac{10x}{2^7}\lg 2 - \Delta^2\left(\frac{x}{7\,2^5}\lg 2\right) \; | \; 3$

$\frac{5x^3}{2^9} - \frac{3x}{2^7} = \Delta^2\left(\frac{3x}{7\,2^5}\lg 2 - \frac{5x^3}{7\,2^7}\lg 2\right)$

$$\frac{5x^3}{r^9} = \Delta^2\left(\frac{3}{10}\,\frac{x}{r^5} + \frac{3x}{7r^5}\lg r - \frac{5x^3}{7r^7}\lg r\right)$$

$$\frac{x^3}{r^9} = \Delta^2\left\{\frac{3}{50}\,\frac{x}{r^5} + \frac{3x}{35r^5}\lg r - \frac{x^3}{7r^7}\lg r\right\} = \Delta^2\left\{\frac{3x}{5r^5}\left[\frac{1}{10} + \frac{\lg r}{7}\right] - \frac{x^3}{7r^7}\lg r\right\}$$

$$\frac{x}{r^5} = -\frac{1}{3}\,\Delta^2\left(\frac{x}{r^3}\lg r\right)$$

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x}$$

$$\rho \frac{\partial v}{\partial t} = \cancel{-\frac{\partial p}{\partial x}} + \mu \frac{\partial^2 v}{\partial x^2}$$

$$\cancel{\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0} \rightarrow \frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} = 0$$

$$\frac{\partial p}{\partial t} + \kappa p \frac{\partial u}{\partial x} + u \frac{\partial p}{\partial x} = (k-1)\left[\mu \Phi + \kappa \frac{\partial^2 \theta}{\partial x^2}\right]$$

$$v = A e^{-\alpha x} \cos(\beta x + \gamma t)$$

$$\frac{\partial v}{\partial x} = A e^{-\alpha x}\left[-\alpha \cos - \beta \sin\right] \qquad \frac{\partial v}{\partial t} = -A e^{-\alpha x} \gamma \sin$$

$$\frac{\partial^2 v}{\partial x^2} = A e^{-\alpha x}\left[\alpha^2 \cos + \alpha\beta \sin - \beta^2 \cos\right]$$

$\alpha x \Gamma = d l \cdot g$

$\frac{du}{dt} = \frac{\partial v}{\partial t}$

$$-\rho \gamma = \mu \alpha \beta$$

$$\alpha = \beta$$

$$\alpha = \sqrt{\frac{-\rho\gamma}{\mu}}$$

~~Zatem~~ $v = A e^{-x\sqrt{\frac{\rho\gamma}{\mu}}} \cos\left(x\sqrt{\frac{\rho\gamma}{\mu}} - \gamma t\right)$

współczynnik zanikania $\sqrt{\frac{\mu}{\rho\gamma}}$

prędkość fali: $= \frac{\gamma}{\sqrt{\frac{\rho\gamma}{\mu}}} =$

tym większa im większe γ

Np. $\gamma = 2\pi n$ Np. $n = 140$

$\gamma = 1000$

pr. $= \frac{1000}{\sqrt{\frac{[illegible]}{0.00019}}} = 10 \frac{cm}{sec}$

$\sqrt{\frac{\mu}{\rho\gamma}} = 0.01$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + k p \frac{\partial u}{\partial x} = (k-1)\left[\mu\left(\frac{\partial v}{\partial x}\right)^2 + \kappa \frac{\partial^2 \theta}{\partial x^2}\right]$$

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x}$$

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} = 0$$

Gdyby u było $= 0$, $p = \text{const}$ Niemożliwe $\frac{K}{\mu}\frac{\partial^2 \theta}{\partial x^2} = \cancel{A e^{-2\alpha x}\left[\cos^2 + \sin(\alpha x - \gamma t)\right]}$

$$= \alpha^2 A^2 e^{-2\alpha x}\left[1 + 2 \sin \cos\right]$$

$$= \alpha^2 A^2 e^{-2\alpha x}\left[1 + \sin 2(\alpha x - \gamma t)\right]$$

$$\theta = B e^{-\alpha x}\left[1 + n \cos(bx + ct)\right]$$

$$\frac{\partial^2\theta}{\partial x^2} = B\, e^{-ax}\left[a^2 + \cancel{(a^2-b^2)m \cos} + ab\, m \sin\right] = \frac{\alpha^2 A^2}{\frac{k}{\mu}}\, e^{-2\alpha x}\left[1 + \cos 2(\alpha x - \gamma t)\right]$$

$a = b$ $\qquad bx + ct = 2(\alpha x - \gamma t)$

$a = 2\alpha$ $\qquad b = 2\alpha \qquad c = -2\gamma$

$$4\cancel{\alpha^2} B \frac{k}{\mu} = \cancel{\alpha^2} A^2 \qquad m = 1 \qquad B = \frac{A^2}{4\frac{k}{\mu}}$$

$$4\alpha^2 m B \frac{k}{\mu} = \alpha^2 A^2$$

$$\theta = M^{f(t)} + \frac{A^2}{4\frac{k}{\mu}}\, e^{-2\alpha x}\left[1 + \cos 2(\alpha x - \gamma t)\right] + \cancel{c\alpha}$$

$$\theta_0 = F(t) + \frac{A^2}{4\frac{k}{\mu}}\left[1 + \cos 2\gamma t\right]$$

$$F(t) = \theta_0 - \frac{A^2}{4\frac{k}{\mu}}\left[1 + \cos 2\gamma t\right]$$

$$\alpha = \sqrt{\frac{\rho}{\mu}\gamma}$$

$$\theta = \theta_0 + \frac{A^2}{4\frac{k}{\mu}}\left\{e^{-2\alpha x}\left[1 + \cos(2\alpha x - \gamma t)\right] - 1 - \cos 2\gamma t\right\}$$

$$\frac{1}{p}\frac{\partial p}{\partial t} = \frac{1}{\rho}\frac{\partial \rho}{\partial t} + \frac{1}{\theta}\frac{\partial\theta}{\partial t} \qquad \frac{\partial p}{\partial t} = R\left(\theta\frac{\partial\rho}{\partial t} + \rho\frac{\partial\theta}{\partial t}\right) = -p\frac{\partial u}{\partial x} + R\rho\frac{\partial\theta}{\partial t}$$

zaniedbując $\quad u\frac{\partial p}{\partial x} = -\rho u \frac{\partial u}{\partial t}$

$$(k-1)\, p\frac{\partial u}{\partial x} + R\rho\frac{\partial\theta}{\partial t} = (k-1)\left[\mu\left(\frac{\partial v}{\partial x}\right)^2 + k\frac{\partial^2\theta}{\partial x^2}\right]$$

~~Zaniedbując~~ $\cancel{u\frac{\partial u}{\partial t}}$ Gdyby $\frac{\partial u}{\partial x} = 0$ lub gdyby $k = 1$ $\qquad \alpha' = \sqrt{\frac{R\rho}{k}\gamma\mu}$

$$\theta = \frac{A^2}{4\frac{k}{\mu}}\left\{e^{-2\alpha x}\left[1 + \cos 2(\alpha x - \gamma t)\right] - e^{-2\alpha' x}\cos 2(\alpha' x - \gamma t)\right\} + \theta_0 \qquad \alpha = \sqrt{\frac{\rho}{\mu}\gamma}$$

$$\sqrt{\frac{R}{k}} = \sqrt{\frac{10^6\ \mu}{0.0013 \cdot 273 \cdot 0.00006 \cdot 4 \cdot 10^6}} = \frac{1}{0.00006 \cdot 1.5} \cdot 0.00018 = \frac{1}{0.0009} = \frac{1}{0.03}$$

zatem α' bezwzględnie większe od α

O ile wydzielanie ciepła następuje periodycznie?

$$\int_0^\infty \Phi\, dx = \alpha^2 A^2 \int_0^\infty e^{-2\alpha x}\left[1+\sin 2(\alpha x-\gamma t)\right] dx =$$

$$= \alpha^2 A^2 \int_0^\infty \left[e^{-2\alpha x} + e^{-2\alpha x}\sin 2\alpha x \cos\gamma t - e^{-2\alpha x}\cos 2\alpha x \sin\gamma t\right] dx$$

$$\int_0^\infty e^{-y}\sin y\, dy = -e^{-y}\cancel{\sin y}\Big|_0^\infty + \int_0^\infty e^{-y}\cos y\, dy = \frac{1}{2}$$

$$\int_0^\infty e^{-2\alpha x}\sin 2\alpha x\, dx = \frac{1}{2\alpha}\cdot\frac{1}{2} = \frac{1}{4\alpha}$$

$$\int_0^\infty e^{-2\alpha x}\, dx = \frac{1}{2\alpha}$$

$$\int\Phi = \alpha^2 A^2\left[\frac{1}{2\alpha} + \frac{1}{4\alpha}\cos\gamma t - \frac{1}{4\alpha}\sin\gamma t\right]$$

$$= \frac{\alpha A^2}{4}\left[2+\cos\gamma t - \sin\gamma t\right] = \cancel{\frac{\alpha A^2}{4}\left[2 + \cos\frac{\gamma t}{2}\sin\frac{\gamma t}{2} \ldots\right]}$$

Wyj. superpozycja period. [illegible]

Cząstki – dla małych ruchów cieczy ~~nieściśliwej~~:

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\mu}{3}\frac{\partial \,\mathrm{div}}{\partial x} + \mu \Delta^2 u$$

$$\rho \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \frac{\mu}{3}\frac{\partial \,\mathrm{div}}{\partial y} + \mu \Delta^2 v$$

$$\cdot \quad = \quad \cdot \;— \;- \;—$$

$$\frac{\partial \rho}{\partial t} + \rho\,\mathrm{div} + \cancel{u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z}} = 0$$

$$\frac{\partial^2 \rho}{\partial t^2} + \rho \frac{\partial}{\partial t}\mathrm{div} + \mathrm{div}\frac{\partial \rho}{\partial t} + \cancel{\frac{\partial u}{\partial t}\frac{\partial \rho}{\partial x} + u\frac{\partial^2 \rho}{\partial x \partial t}} = 0$$

$$\rho \frac{\partial}{\partial t}\mathrm{div} + \cancel{\left(\frac{\partial \rho}{\partial x}\frac{\partial u}{\partial t} + \frac{\partial \rho}{\partial y}\frac{\partial v}{\partial t} + \dots\right)} = -\nabla^2 p + \frac{\mu}{3}\nabla^2 \mathrm{div} + \mu \nabla^2 \mathrm{div}$$

$$= -\nabla^2 p + \frac{4}{3}\mu \nabla^2 \mathrm{div}$$

$$-\left[\frac{\partial^2 \rho}{\partial t^2} + \overbrace{\frac{\partial \rho}{\partial t}\mathrm{div}}^{2} + \cancel{u\frac{\partial^2 \rho}{\partial x \partial t} + \dots}\right] = \nearrow$$

$$\rho \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + \frac{\partial \rho}{\partial y}\frac{\partial u}{\partial t} - \frac{\partial \rho}{\partial x}\frac{\partial v}{\partial t} = \mu \Delta^2\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)$$

Podobieństwo dynam. dla powolnych ruchów:

$\cancel{\frac{b}{r} = \alpha \frac{m}{n}}$ z siłą zewn. $\sim \rho X$

$$b \equiv \alpha \frac{m}{n} \equiv \frac{bn}{r}$$

$$mb \equiv \equiv \beta \frac{r}{n}$$

$$\alpha \frac{m^2}{n} \equiv \beta \frac{r}{n} \qquad \frac{r}{n} \equiv 1$$

$$\beta = \alpha \frac{m^2}{n} \qquad \underline{\underline{r = n}} \qquad \underline{\underline{m}} = \sqrt{\frac{n\beta}{\alpha}}$$

$$b = \frac{\beta}{m} \qquad \underline{\underline{b}} = \frac{\beta}{m}$$

$$\nabla^2_\chi = -\varepsilon \cancel{\nabla^2 U} - \left[\frac{\partial \varepsilon}{\partial x}\frac{\partial U}{\partial x} + \frac{\partial \varepsilon}{\partial y}\frac{\partial U}{\partial y} + \quad\right] + \cancel{\mu \nabla^2 \ldots}$$

$$\iint \frac{\partial \chi}{\partial n} do = -\iint \qquad = \cancel{\int \varepsilon \nabla^2 U} - \int \varepsilon \frac{\partial U}{\partial n} do$$

$$= -\iint \varepsilon \frac{\partial U}{\partial n} do$$

ε zmienne tylko w kierunku n $\|$ $\quad \frac{\partial \varepsilon}{\partial x} = \frac{\partial \varepsilon}{\partial n}$ cos nx

U " " " " stycznym $\quad \frac{\partial \varepsilon}{\partial x}\frac{\partial U}{\partial x} + \ldots = 0$

jeśli ściana izolująca $\qquad \nabla^2_\chi = 0$

$$\left.\begin{aligned}
\frac{\partial p}{\partial x} &= X + \mu\nabla^2 u &&= -\varepsilon\frac{\partial U}{\partial x} + \mu\nabla^2 u\\
\frac{\partial p}{\partial y} &= Y + \mu\nabla^2 v &&= -\varepsilon\frac{\partial U}{\partial y} + \mu\nabla^2 v\\
\frac{\partial p}{\partial z} &= Z + \mu\nabla^2 w &&= -\varepsilon\frac{\partial U}{\partial z} + \mu\nabla^2 w
\end{aligned}\right\}\quad \nabla p = -\varepsilon\nabla U + \mu\nabla^2 v$$

U nie zmienia się znacznie w obrębie δ

$$\underbrace{\left[\frac{\partial p}{\partial x}\cos nx + \frac{\partial p}{\partial y}\cos ny + \dots\right]}_{\frac{\partial p}{\partial n}} = -\varepsilon\underbrace{\left[\frac{\partial U}{\partial x}\cos nx + \dots\right]}_{\frac{\partial U}{\partial n}} + \mu\left[\nabla^2 u\cdot\cos nx + \dots\right]$$

$$\cancel{u = u_0 + \xi u_1 + \frac{\xi^2}{2}u_2}$$

$$\cancel{v = v_0 + \eta v_1 + \frac{\eta^2}{2}v_2}$$

nie tylko w punkcie $0 = 0$ ale w płaszczyźnie dla niektórych

dla $\xi\cos nx + \eta\cos ny + \zeta\cos nz = 0$

będzie także $u = v = w = 0$

powierzchnia gdzie

u będzie $= 0$

$$u = \underbrace{u_0}_{=0} + \xi\left(\frac{\partial u}{\partial x}\right)_0 + \eta\left(\frac{\partial u}{\partial y}\right)_0 + \zeta\left(\frac{\partial u}{\partial z}\right)_0 + \frac{\xi^2}{2}\left(\frac{\partial^2 u}{\partial x^2}\right)_0 + \dots$$

$$\zeta\cos nz = -(\xi\cos nx + \eta\cos ny)$$

$$0 = \xi\underbrace{\left[\frac{\partial u}{\partial x}\cos nz - \frac{\partial u}{\partial z}\cos nx\right]}_{=0} + \eta\underbrace{\left[\frac{\partial u}{\partial y}\cos nz - \frac{\partial u}{\partial z}\cos ny\right]}_{=0}$$

zatem $\dfrac{\frac{\partial u}{\partial x}}{\cos nx} = \dfrac{\frac{\partial u}{\partial y}}{\cos ny} = \dfrac{\frac{\partial u}{\partial z}}{\cos nz} = \dfrac{\partial u}{\partial n}$

zatem: $u = \left(\dfrac{\partial u}{\partial n}\right)_0\left[\xi\cos nx + \eta\cos ny + \zeta\cos nz\right] + \dfrac{\xi^2}{2}\left(\dfrac{\partial^2 u}{\partial x^2}\right)_0 + \dots$

$v =$

$w =$

} zatem prop. odległości

$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$ Zatem: $\left(\frac{\partial u}{\partial n}\right)_0 \cos nx+\left(\frac{\partial v}{\partial n}\right)_0 \cos ny+\left(\frac{\partial w}{\partial n}\right)_0 \cos nz=0$

T. zn. że warstwy równoległe płyną do powierzchni

$\Delta^2 u=\left(\frac{\partial^2 u}{\partial x^2}\right)_0+\cdots$

$$\underbrace{\int\frac{\partial p}{\partial x}\,dv}_{p\cos nx\, do}=-\underbrace{\int\varepsilon\frac{\partial U}{\partial x}\,dv}+\mu\int\nabla^2 u\,dv$$

$$=\cancel{\frac{\partial U}{\partial x}\int\varepsilon\,dv}=\int\frac{\partial U}{\partial x}\,do\int\varepsilon\,d\delta+$$

---

s kierunek stycznej linji prądu w stałej odległości ν od powierzchni.

$$\frac{\partial p}{\partial s}=\cdot\varepsilon\frac{\partial U}{\partial s}+\mu\nabla^2 v_s$$

ponieważ w pierwszem przybliżeniu v równoległe do powierzchni, tylko różnym prędkości (laminarny)

zatem $\nabla^2 v_s=\frac{\partial^2 v_s}{\partial n^2}$ $\quad \frac{\partial U}{\partial s}=$ stałe w grubości ν

$$\varepsilon=\frac{1}{4\pi}\frac{\partial^2\varphi}{\partial\nu^2}$$

$$\underbrace{\int_0\frac{\partial p}{\partial s}\,dv}_{=\nu\left(\frac{\partial p}{\partial s}\right)_{n=\delta}}=\frac{1}{4\pi}\frac{\partial\varphi}{\partial\nu}\frac{\partial U}{\partial s}+\mu\frac{\partial v_s}{\partial\nu}+\text{const}$$

$=$ przybliżenie $=0$

zatem $\mu v_s=-\frac{1}{4\pi}(\varphi_i-\varphi_e)\frac{\partial U}{\partial s}$

ale gdzie tego istnieje v_s! ?

powodowi $\nabla^2 U = 0$

(proporcjonalnie)

zatem takie prędkości powierzchniowe jak gdyby U było potencyałem prędkości

w ruchu cieczy byłyby równania

$$\left.\begin{aligned} \frac{\partial p}{\partial x} &= \mu \nabla^2 u \\ \frac{\partial p}{\partial y} &= \mu \nabla^2 v \\ \frac{\partial p}{\partial z} &= \mu \nabla^2 w \end{aligned}\right\}$$

one byłyby zatem spełnione przez ~~$u \sim \frac{\partial U}{\partial x}$, $v \sim \frac{\partial U}{\partial y}$ etc., $w \sim \frac{\partial U}{\partial z}$, $p = \text{const.}$~~

warunki powierzchniowe spełnione, bo $\nabla^2 U = 0$

Nieprawda: bo jeszcze do tego przychodzą nie uwzględnione v_n

$u \sim \frac{\partial U}{\partial x}$ na powierzchni etc.?

układ ich byłby zatem ~~takie~~ potencyałem prędkości dla $u\ v\ w$

~~prędkości przez $\frac{1}{4\pi}\frac{\varphi_i - \varphi_e}{\mu}$~~

Otrzymujemy $U \cdot \frac{\varphi_i - \varphi_e}{4\pi\mu}$ jako potencyał prędkości istotnie otrzymuje się

prędkości odpowiednie czysto styczne! na powierzchni i spełniamy

równania hydr. kiedy $p = \text{const.}$ [ale p_{xx} etc ≥ 0!]

Aby otrzymać jednak nieruchomość dla $r = \infty$ trzeba by superponować

rozciąganie i ruch $-\frac{\varphi_i - \varphi_e}{4\pi\mu}\frac{\partial U}{\partial x}$ dla $r = \infty$ spoczynek dla $r = a$

t.j. (nieruchomej) kuli w cieczy poruszającej się z prędkością

zatem ciśnienie : $p = \frac{\varphi_i - \varphi_e}{4\pi\mu}\underbrace{\left(\frac{\partial U}{\partial x}\right)}_{= A}\frac{x}{r^3}$

Energia zużyta:

$$\Phi = \mu\left[2\left\{\left(\frac{\partial u}{\partial x}\right)^2 + \dots\right\} + \left\{\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \dots\right\}\right]$$

$$u = \alpha \frac{\partial U}{\partial x} \quad \text{etc.}$$

patrz Lamb p. 519
p. 51

$$\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = 2\frac{\partial^2 U}{\partial y \partial z}$$

$$\Phi = \mu\alpha^2\left[2\left\{\left(\frac{\partial^2 U}{\partial x^2}\right)^2 + \left(\frac{\partial^2 U}{\partial y^2}\right)^2 + \dots\right\} + 4\left\{\left(\frac{\partial^2 U}{\partial y \partial z}\right)^2 + \dots + \right\}\right] = -\mu\int \frac{\partial(v^2)}{\partial n}\, dS$$

$$p_{xx} = -p + 2\mu\frac{\partial u}{\partial x} = -p + 2\mu\alpha\frac{\partial^2 U}{\partial x^2}$$

$$p_{zy} = \mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) = 2\mu\alpha\frac{\partial^2 U}{\partial y \partial z}$$

$$p_x = -p\cos nx + 2\mu\alpha\left[\frac{\partial^2 U}{\partial x^2}\cos nx + \frac{\partial^2 U}{\partial x \partial y}\cos ny + \frac{\partial^2 U}{\partial x \partial z}\cos nz\right]$$

$$\int p_x = 2\mu\alpha\int \underbrace{\left[\frac{\partial^2 U}{\partial x^2}\cos nx + \dots\right]}\, do$$

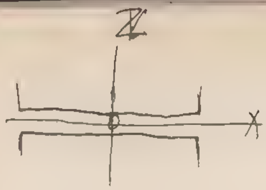
$$\int \frac{\partial}{\partial x}\cos nx + \frac{\partial}{\partial y}\cos ny + \dots = \int \frac{\partial}{\partial n}\frac{\partial U}{\partial x}\, do$$

wypadkowa ciśnień na kulę w kierunku $X = 2\mu\alpha\int \frac{\partial^2 U}{\partial x \partial n}\, do$

ponieważ jednak $\frac{\partial U}{\partial n} = 0$ wzdłuż całej powierzchni więc $F = 0$:

$$\cancel{\int U\nabla^2 U\, do = \int U\frac{\partial U}{\partial n}\, do - \int\left[\left(\frac{\partial U}{\partial x}\right)^2 + \dots\right] do}$$

Praca powierzchniowa $\int (p_x \cdot u + p_y \cdot v + p_z \cdot w)\, do$



~~Dwa krążki~~ ~~pr~~ zbliżające się w cieczy lepkiej

Zadanie dwuwymiarowe :

$$u = u_m \cdot \frac{4z(\delta - z)}{\delta^2}$$

$$\int_0^\delta u\, dz = u_m \cdot \frac{2}{3}\delta$$

Nieściśliwość : $$2x \frac{d\delta}{dt} = \int_0^\delta u\, dz = \frac{2}{3}\delta\, u_m$$

$$u_m = 3\frac{x}{\delta}\frac{d\delta}{dt} = \frac{3x}{\delta}\cdot c$$

$$u = 12 \cdot \frac{x\, z(\delta - z)}{\delta^3}\cdot c$$

$$\mu \nabla^2 u = \frac{\partial p}{\partial x} = -\frac{24 \cdot x c}{\delta^3}\mu$$

$$p = -12\frac{x^2 c\mu}{\delta^3} + \text{const}$$

$$p = -12(b^2 - x^2)\frac{c\mu}{\delta^3} + p_0$$

$$\int_{-b}^{+b} p\, dx = 12\frac{c\mu}{\delta^3}\cdot 2\left[b^3 - \frac{b^3}{3}\right] = 16 \cdot \frac{b^3 c\mu}{\delta^3}$$

$$\frac{\partial p}{\partial x} = -\varepsilon \frac{\partial U}{\partial x} + \mu \Delta^2 u$$

$$\frac{\partial p}{\partial y} = -\varepsilon \frac{\partial U}{\partial y} + \mu \Delta^2 v$$

$$\frac{\partial p}{\partial z} = -\varepsilon \frac{\partial U}{\partial z} + \mu \Delta^2 w$$

zaniedbując odrzucenie dodatkowe pochodzące z siły wywieranej przez na siebie same masy U wewn. jako stało w obszarze δ

$$\frac{\partial p}{\partial n} = -\varepsilon \frac{\partial U}{\partial n} + \mu \Delta^2 v_n$$

$$\frac{\partial p}{\partial s} = -\varepsilon \frac{\partial U}{\partial s} + \mu \Delta^2 v_s$$

$$\Delta^2 p = -\underbrace{\left[\frac{\partial \varepsilon}{\partial x}\frac{\partial U}{\partial x} + \frac{\partial \varepsilon}{\partial y}\frac{\partial U}{\partial y} + \right]}$$

[przyjmując że ε zmienne tylko w warunku ~~stycznym~~ normalnym i tylko w stycznym]

$$\cancel{= -\iint \varepsilon \frac{\partial U}{\partial n} do + \iiint \varepsilon \nabla^2 U \, dx\, dy\, dz}$$

$$= -\iint \varepsilon \frac{\partial U'}{\partial n} do + \iiint \varepsilon \nabla^2 U'$$

zatem będzie to tylko ważnem dopóki się pomija V i v w porównaniu do U t.j. dla Endosmozy elektr.

$$= \int \nabla \varepsilon \, \nabla U'$$

$$= 0$$

zatem p ~~może~~ może mieć Maxima i Minima tylko na powierzchni warstwy

w powierzchni $\Delta^2 p = 0$ takie tam gdzie $\varepsilon = 0$ więc wogóle Maxima Minima tylko w nieskończ. albo na powierzchni (metal) itd.

~~[illegible]~~

Gdy kula w wewnątrz —

$$p = p_0 - \frac{3}{2} \frac{\mu c a}{r^3} x = p_0 - \frac{3}{2} \frac{\mu c a}{r^2} \cos\vartheta$$

$$\Delta^2 v_x = \frac{3 c a}{r^3} \cos\vartheta$$

$$\frac{\partial p}{\partial x} = \mu \Delta^2 v_x = + \frac{3}{2} \mu \frac{c a}{r^3} \cos\vartheta$$

w płaszczyźnie: $\quad v = f(n)$

$$\frac{\partial p}{\partial s} = -\varepsilon \frac{\partial U}{\partial s} + \mu \Delta^2 v = -\varepsilon \frac{\partial U}{\partial s} + \mu \frac{\partial^2 v}{\partial n^2}$$

$$\frac{\partial p}{\partial n} = -\varepsilon \frac{\partial U}{\partial n}$$

$$\cancel{p_0 + \delta n\, \varepsilon_1 \frac{\partial U}{\partial n_1} + \mu \frac{\partial^2 v}{\partial n^2} = p_0 + \delta s\, \mu \frac{\partial^2 v}{\partial n^2 \partial t} + \varepsilon \frac{\partial U}{\partial s\, \partial t} + \delta n\, \varepsilon_1 \frac{\partial U}{\partial n_1}}$$

$$\frac{\partial p}{\partial s\, \partial n} = -\varepsilon \frac{\partial^2 U}{\partial n \partial s} = -\varepsilon \frac{\partial^2 U}{\partial n \partial s} + \mu \frac{\partial^3 v}{\partial n^3} - \underline{\frac{\partial \varepsilon}{\partial n} \frac{\partial U}{\partial s}}\,!$$

$$\frac{\partial^3 v}{\partial n^3} = 0 \qquad \frac{\partial^2 v}{\partial n^2} = \text{const}$$

$$\frac{dv}{dn} = cx + a$$
$$v = \frac{cx^2}{2} + ax$$
$$0 = c\frac{\delta^2}{2} + a\delta$$
$$a = -c\frac{\delta}{2}\,!$$
$$v = \frac{c}{2}(x^2 - x\delta)$$

$$\frac{\partial p}{\partial s} = -\varepsilon \frac{\partial U}{\partial s} + \underline{\underline{c}}\,\mu$$

$$\frac{\partial p}{\partial n} = -\varepsilon \frac{\partial U}{\partial n}$$

$$\cancel{p = -\int \varepsilon \frac{\partial U}{\partial n}\, dn + \int \varepsilon \frac{\partial U}{\partial s}\, ds + c s \mu}$$

$$\cancel{p = p_0 + \mu c x - \int \varepsilon \frac{\partial U}{\partial n}\, dn = p_0 + c x \mu - \frac{1}{4\pi} \frac{\partial \varphi}{\partial n} \frac{\partial U}{\partial n}}$$

I.

v wcale nie byłoby zmienione i nie określone

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0$$

$$\frac{\partial^2}{\partial x^2} \varphi_{\text{int}} = 0$$

$$p = \int \left(\varepsilon \frac{\partial U}{\partial x} dx + \underbrace{\frac{\partial U}{\partial z} dz}_{=0} \right) + \mu \int \frac{d^2 v}{dz^2}\, dx$$

$$= p_0 + \int \left(\varepsilon \frac{\partial U}{\partial x} + \mu \frac{d^2 v}{dz^2} \right) dx$$

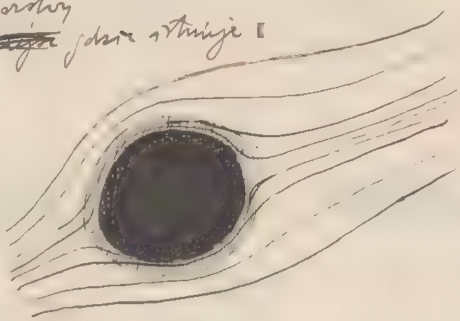
= Elektrische Endosmose

$$\frac{d^2 v}{dz^2} = -\varepsilon \frac{\partial U}{\partial x} = -\frac{1}{4\pi} \frac{\partial^2 \varphi}{\partial z^2} \frac{\partial U}{\partial x}$$

$$v = \frac{1}{4\pi} (\varphi_i - \varphi_a) \frac{\partial U}{\partial x} + \frac{c}{2}(z^2 - z\delta)$$

Równanie zasadnicze dla zjawiska odwrotnego: Strömungsströme

$\Delta^2 U = 0$ wszędzie z wyjątkiem ~~[illegible]~~ warstwy gdzie istnieje ε



Na tej powierzchni będzie

$$\lambda_1 \frac{\partial U}{\partial n_1} = \lambda_1 \frac{\partial U}{\partial n_2} + \varepsilon v_n$$

Ogólne równanie wszędzie ważne:

+ ΔU = [illegible]

$$\operatorname{div}\left[\lambda \nabla U + \varepsilon v\right] = 0$$

$$\lambda \nabla^2 U + \frac{\partial}{\partial x}(\varepsilon u) + \frac{\partial}{\partial y}(\varepsilon v) + \quad = 0$$

$$\boxed{\lambda . \nabla^2 U + u \frac{\partial \varepsilon}{\partial x} + v \frac{\partial \varepsilon}{\partial y} + w \frac{\partial \varepsilon}{\partial z} = 0}$$

$$= -4\pi\lambda\varepsilon + \underbrace{u \frac{\partial \varepsilon}{\partial x} + v \frac{\partial \varepsilon}{\partial y} + \ldots}_{} = 0$$

$$= \frac{D\varepsilon}{Dt} - \frac{\partial \varepsilon}{\partial t}$$

$$= \frac{D\varepsilon}{Dt} \text{ w rozkładzie stałym}$$

$$\frac{\partial p}{\partial x} = -\frac{1}{4\pi} \nabla^2 U \frac{\partial U}{\partial x} + \mu \Delta^2 u$$

$$\underbrace{\frac{dx}{u} = \frac{dy}{v}}_{\text{Strömungslinien}} = \frac{dz}{w} = \frac{d\varepsilon}{4\pi\lambda\varepsilon}$$

$$4\pi\lambda \log \varepsilon = \int \frac{dx}{u}$$

~~zatem ε funkcją tylko [illegible]~~

$$\varepsilon = \varepsilon_0 \, e^{4\pi\lambda \int \frac{dx}{u}}$$

$$x - \frac{x a^3}{2 r^3}$$

$$\left(r - \frac{a^3}{2r^2}\right) \cos\theta$$

$$1 - \frac{a^3}{r^3}$$

Więc jeśli [illegible] spoczynku ε' i temu odpowiedzająca U'

wtedy w stanie ruchu równanie:

$$\cancel{\lambda \nabla^2 U + u\frac{\partial\varepsilon}{\partial x} + v\frac{\partial\varepsilon}{\partial y} + w\frac{\partial\varepsilon}{\partial z}}$$

$$\operatorname{div}\left[\lambda \nabla (U - U') + \varepsilon v\right] = 0$$

$$\lambda \nabla^2 U + u\frac{\partial\varepsilon}{\partial x} + v\frac{\partial\varepsilon}{\partial y} + w\frac{\partial\varepsilon}{\partial z} = -4\pi\lambda\varepsilon$$

~~jeśli przyjmiemy nie [illegible] dużo wtedy inaczej stanie~~

zatem:

$$4\pi\lambda(\varepsilon' - \varepsilon) + u\frac{\partial\varepsilon}{\partial x} + v\frac{\partial\varepsilon}{\partial y} + w\frac{\partial\varepsilon}{\partial z} = 0$$

$$\varepsilon = \varepsilon' + \nu$$

$$4\pi\lambda\nu = u\frac{\partial\varepsilon'}{\partial x} + v\frac{\partial\varepsilon'}{\partial y} + w\frac{\partial\varepsilon'}{\partial z} + \underbrace{\left(u\frac{\partial\nu}{\partial x} + v\frac{\partial\nu}{\partial y} + w\frac{\partial\nu}{\partial z}\right)}_{\neq 0}$$

$$\text{pot}\,\varepsilon = \text{pot}\,\varepsilon' + \text{pot}\,\nu + \Phi \qquad \text{zatem: } U = U' + V - ?$$

$$\text{pot}\,\nu = \int \frac{\nu}{r}\,d\omega = \frac{1}{4\pi\lambda}\int \frac{1}{r}\Big[\underbrace{\underbrace{u\frac{\partial\varepsilon'}{\partial x} + \dots}_{v\,.\,\nabla\varepsilon'\ \ \cos(v.\nabla\varepsilon')}}_{v_n\,.\,\nabla\varepsilon'}\Big]\,d\omega$$

powiedzmy w płaskim przybliżeniu $v = \frac{z^2}{2}\left(\frac{\partial u}{\partial z^2}\right)$

$$= \frac{1}{4\pi\lambda}\int \frac{v_n \frac{\partial\varepsilon'}{\partial n}}{r}\,d\omega = \frac{1}{4\pi\lambda}\iint \frac{d\sigma}{r}\underbrace{\int \frac{\delta n^2}{2}\frac{\partial^2 v_n}{\partial n^2}\,d\delta n\,.\,\frac{\partial\varepsilon'}{\partial n}}$$

$$v_n = v_{n0} + \frac{\delta n^2}{2}\frac{\partial^2 v_n}{\partial n^2} = \frac{\delta n^2}{2}\frac{\partial^2 v_n}{\partial n^2}$$

$$\int \frac{z^2}{2}\frac{\partial\varepsilon'}{\partial z}\,dz$$

$$= \int \frac{\partial^3 U'}{\partial z^3}\frac{z^2}{2}\,dz$$

$$= \frac{\partial U}{\partial z^2}\,\cancel{\frac{z^2}{2}}\int \frac{\partial^2 U'}{\partial z^2}\,2z\,dz = U'$$

$\mathfrak{U} = \mathfrak{U}' + V + \Phi$

zaniedbujemy, co prawda tylko jeśli V ...

$$V = \frac{\varphi_i - \varphi_a}{4\pi\lambda} \iint \frac{\partial^2 v_n}{\partial n^2} \cdot \frac{do}{r}$$

$$\frac{\partial p}{\partial n} = -\varepsilon \frac{\partial \mathfrak{U}}{\partial n} + \mu \Delta^2 v_n$$

$$\frac{\partial^2 v_n}{\partial n^2} + \cancel{\frac{\partial^2 v_n}{\partial \sigma^2}} + \cancel{\frac{\partial^2 v_n}{\partial \tau^2}}$$

$$\cancel{\neq \frac{\varphi_i - \varphi_a}{4\pi\lambda\mu} \iint \frac{1}{r} \frac{\partial p}{\partial n} do}$$

~~zatem V taki jak~~ rozwiązanie (mieliśmy) będzie: $V = \frac{\varphi_i - \varphi_a}{4\pi\lambda\mu} p$

~~ostatnia transformacya tylko jeśli $\frac{\partial^2 v_n}{\partial n^2} = \frac{\partial p}{\partial n}$.~~

~~t. j. jeśli $\frac{\partial \mathfrak{U}}{\partial n} = 0$~~

~~Uwaga:~~ Na powierzchni jeśli ciało izolujące będzie:

zatem $\frac{\partial(V+\Phi)}{n} = 0$

$$\lambda \frac{\partial(\mathfrak{U} - \mathfrak{U}')}{\partial n} = 0$$

$$\frac{\partial \mathfrak{U}}{\partial n} = \frac{\partial \mathfrak{U}'}{\partial n}$$

przy założeniu że $\Delta^2 v_n = \frac{\partial^2 v_n}{\partial n^2}$!!!

$$\text{zatem: } \frac{\partial^2 v_n}{\partial n^2} = \frac{1}{\mu} \frac{\partial p}{\partial n} + \varepsilon \underbrace{\frac{\partial \mathfrak{U}'}{\partial n}}$$

zależne tylko od δn, nie od σ, t

$$V = \frac{\varphi_i - \varphi_a}{4\pi\lambda\mu} \iint \frac{1}{r} \frac{\partial p}{\partial n} do + \underbrace{\frac{\varphi_i - \varphi_a}{4\pi\lambda} \iint \frac{1}{r} \varepsilon \frac{\partial \mathfrak{U}'}{\partial n} do}_{f(r)}$$

rozwiązanie mieliśmy:

$$V = \frac{\varphi_i - \varphi_a}{4\pi\lambda\mu} p + f(r)$$

$$\cancel{\frac{\partial v_s}{\partial r} = -c \sin\theta \left[1 - \tfrac{3}{4}\tfrac{a}{r} \ldots\right]}$$

$$v_s = -u \frac{x}{r} + v \frac{y}{r} = -u \sin\theta + v \cos\theta$$

$$= -c \sin\theta \left[1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right]$$

$$\frac{\partial v_s}{\partial r} = -c \sin\theta \left[\frac{3}{4}\frac{a}{r^2} + \frac{3}{4}\frac{a^3}{r^4}\right]$$

$$\frac{\partial^2 v_s}{\partial r^2} = +c \sin\theta \left[\frac{3}{2}\frac{a}{r^3} + 3\frac{a^3}{r^5}\right]$$

W takim rozkładzie $U = U' + V + \Phi$

a zatem U' = pot. wewnętrzny prądu (w sobie)

Φ = zewnętrzny potencjał

V = zmiana wskutek ruchu mechan.

$$u = -\frac{3}{4}\frac{ca}{r}\left(1-\frac{a^2}{r^2}\right)\cos^2\theta + c\left(1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right)$$

$$v = -\frac{3}{4}\frac{ca}{r}\left(1-\frac{a^2}{r^2}\right)\sin\theta\cos\theta$$

$$v_r = u\frac{z}{r} + v\frac{x}{r} = u\cos\theta + v\sin\theta$$

$$= -\frac{3}{4}\frac{ca}{r}\left(1-\frac{a^2}{r^2}\right)\cos\theta + c\cos\theta\left(1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right)$$

$$= c\cos\theta\left[1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}-\frac{3}{4}\frac{a}{r}+\frac{3}{4}\frac{a^3}{r^3}\right]$$

$$v_r = c\cos\theta\left[1-\frac{3}{2}\frac{a}{r}+\frac{1}{2}\frac{a^3}{r^3}\right]$$

$$\frac{\partial v_r}{\partial r} = c\cos\theta\left[\frac{3}{2}\frac{a}{r^2}-\frac{3}{2}\frac{a^3}{r^4}\right] \qquad \left.\frac{\partial v_r}{\partial r}\right|_{r=a} = 0$$

$$\frac{\partial^2 v_r}{\partial r^2} = c\cos\theta\left[-\frac{3a}{r^3}+\frac{6a^3}{r^5}\right] \qquad \left.\frac{\partial^2 v_r}{\partial r^2}\right|_{r=a} = \frac{3c\cos\theta}{a^2}.$$

$$p = p_0 - \frac{3}{2}\frac{\mu ca}{r^2}\cos\theta$$

$$\frac{\partial p}{\partial r} = 3\mu\frac{ca}{r^3}\cos\theta \qquad \left.\frac{\partial p}{\partial r}\right|_{r=a} = 3\mu\frac{c}{a^2}\cos\theta$$

$$\therefore\quad \frac{\partial p}{\partial r} = \mu\frac{\partial^2 v_r}{\partial r^2}$$

$$\cancel{\Delta^2 v_s = \frac{1}{\mu}\frac{\partial p}{\partial s} = \dots \frac{3}{2}\frac{ca}{r^2}\sin\theta}$$

$$= \frac{1}{\mu}\frac{\partial p}{\partial \theta}\frac{\partial \theta}{\partial s} = \frac{3}{2}\frac{ca\sin\theta}{r^3} \qquad ds = r\,d\theta$$

Dwojaki ruch : jonowy
molarny

Równanie mechaniczne

W stanie spoczynku:

$$u = v = w = 0$$

$$U = U' + \Phi$$ ← czy to się zgadza z warunkiem $u = v = w = 0$?

$$\varepsilon = \varepsilon'$$

$$\frac{\partial p}{\partial x} = \varepsilon' \frac{\partial U'}{\partial x} = \varepsilon \frac{\partial U}{\partial x}$$

$$\frac{\partial p}{\partial x} = -\varepsilon \frac{\partial U}{\partial x} + \mu \Delta^2 u$$

$$\frac{\partial p}{\partial y} = -\varepsilon \frac{\partial U}{\partial y} + \mu \Delta^2 v$$

$$\frac{\partial p}{\partial z} = -\varepsilon \frac{\partial U}{\partial z} + \mu \Delta^2 w$$

$$\Delta^2 p = -\left[\frac{\partial \varepsilon}{\partial x}\frac{\partial U}{\partial x} + \frac{\partial \varepsilon}{\partial y}\frac{\partial U}{\partial y} + \ldots\right] - \varepsilon \Delta^2 U$$

$$\int \Delta^2 p \, dv = -\underbrace{\int \ldots}_{} - \int \varepsilon \Delta^2 U \, dv$$

$$= -\int \varepsilon \frac{\partial U}{\partial n} do + \int \varepsilon \Delta^2 U \, dv = -\int \varepsilon \frac{\partial U}{\partial n} do$$

$$= -\int \varepsilon \frac{\partial U'}{\partial n} do$$

Rozkład: $\cancel{p = p_1 + p_2}$

$$\cancel{\frac{\partial p_2}{\partial x} = -\varepsilon \frac{\partial V}{\partial x} + \mu \Delta^2 u \qquad \frac{\partial p_3}{\partial x} = -\varepsilon \frac{\partial \Phi}{\partial x} + \mu \Delta^2 u_3}$$

$$\cancel{\frac{\partial p_1}{\partial x} = -\varepsilon \frac{\partial U'}{\partial x}} \qquad \cancel{\frac{\partial p_2}{\partial x} = -\varepsilon \frac{\partial (V+\Phi)}{\partial x} + \mu \Delta^2 u}$$

$$\cancel{\frac{\partial p_1}{\partial y} = -\varepsilon \frac{\partial U'}{\partial y}} \qquad \cancel{\frac{\partial p_2}{\partial y} = -\varepsilon \frac{\partial}{\partial y} + \mu \Delta^2 v}$$

$$\cancel{\frac{\partial p_1}{\partial z} = -\varepsilon \frac{\partial U'}{\partial z}} \qquad \cancel{\frac{\partial p_2}{\partial z} = -\varepsilon \frac{\partial}{\partial z} + \ldots}$$

~~$\Delta^2 p_1 = -\varepsilon \Delta^2 U' - \left[\frac{\partial \varepsilon}{\partial x}\frac{\partial U'}{\partial x} + \dots\right]$~~

~~$\Delta^2 p_2 =$~~

$$\frac{\partial (V+\Phi)}{\partial n} = 0$$

$$\Delta^2 (V+\Phi) = u\frac{\partial \varepsilon}{\partial x} + v\frac{\partial \varepsilon}{\partial y} + w\frac{\partial \varepsilon}{\partial z}$$

~~pot~~

~~$p = \varepsilon$~~

$$\varepsilon\, \Delta^2 U + \frac{\partial \varepsilon}{\partial x}\frac{\partial U}{\partial x} + \frac{\partial \varepsilon}{\partial y}\frac{\partial U}{\partial y} + \frac{\partial \varepsilon}{\partial z}\frac{\partial U}{\partial z} = \Delta^2 p$$

z drugiej strony przedtem mieliśmy:

$$\varepsilon\,\lambda\, \Delta^2 U + \frac{\partial \varepsilon}{\partial x}\cdot u\varepsilon + \frac{\partial \varepsilon}{\partial y}\cdot v\varepsilon + \frac{\partial \varepsilon}{\partial z}\, w\varepsilon = -4\pi \lambda\, \varepsilon' \varepsilon$$

Czy nie wystarczyłoby starczyć:

~~$\frac{\partial U'}{\partial x} = \frac{1}{\lambda} u\varepsilon$~~ ~~$\Delta^2 p = -4\pi\lambda \varepsilon \varepsilon$~~

~~$\Delta^2 U = \frac{1}{\lambda}\left(u\frac{\partial \varepsilon}{\partial x} + v\frac{\partial \varepsilon}{\partial y} + w\frac{\partial \varepsilon}{\partial z}\right)$~~

$$p = p_1 + p_2 \qquad U = U' + V + \Phi$$

$$\frac{\partial p_1}{\partial x} = -\varepsilon\frac{\partial U'}{\partial x} \qquad \frac{\partial p_2}{\partial x} = -\varepsilon\frac{\partial (V+\Phi)}{\partial x} + \mu\,\Delta^2 u$$

$$\frac{\partial p_1}{\partial y} = \qquad \frac{\partial p_2}{\partial y} =$$

$$\Delta^2 p_2 = -\left[\frac{\partial \varepsilon}{\partial x}\frac{\partial (V+\Phi)}{\partial x} + \dots\right] - \varepsilon\, \Delta^2 (V+\Phi)$$

~~$p_2 = \int \frac{1}{r}\frac{\partial}{\partial n}\varepsilon$~~

$$\int \Delta^2 p_2\, do = -\int \varepsilon \frac{\partial (V+\Phi)}{\partial n}\, do + \cancel{\int \varepsilon\, \Delta^2 (V+\Phi) - \int \varepsilon\, \Delta^2 (V+\Phi)}$$

$$= 0$$

Jako jest wartość $\Delta^2 v_s$ przy powierzchni ? Czy równe $\frac{\partial^2 v_s}{\partial n^2}$?

$$\Delta^2 v_s = \left(\frac{\partial^2}{\partial n^2} + \frac{\partial^2}{\partial s^2} + \frac{\partial^2}{\partial t^2}\, v_s\right)$$

$$p = p_0 - \frac{3\mu c a}{r^2} \cos\theta \qquad v_r = c\left[1 - \frac{3}{2}\frac{a}{r} + \frac{1}{2}\frac{a^3}{r^3}\right]\cos\theta$$

$$v_{\theta} = -c\sin\theta\left[1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right]$$

$$\frac{\partial}{\partial r} v_{r\theta} = -c\sin\theta\left[\frac{3}{4}\frac{a}{r^2} + \frac{3}{4}\frac{a^3}{r^4}\right] \qquad \Big|^{r=a} = -\frac{3}{2}\frac{c\sin\theta}{a}$$

$$\frac{\partial^2}{\partial r^2} v_{r\theta} = +c\cos\theta\left[\frac{3}{2}\frac{a}{r^3} + 3\frac{a^3}{r^5}\right] \qquad = \frac{9}{2}\frac{c\sin\theta}{a^2}$$

$$v = \sqrt{v_r^2 + v_{r\theta}^2} = c\sqrt{\left(1 + \frac{9}{4}\frac{a^2}{r^2} + \frac{1}{4}\frac{a^6}{r^6} - \frac{3}{2}\frac{a}{r} - \frac{3}{4}\frac{a^4}{r^4} + \frac{1}{2}\frac{a^3}{r^3}\right)\cos^2\theta \quad \left(1 + \frac{9}{16}\frac{a^2}{r^2} + \frac{1}{16}\frac{a^6}{r^6} - \frac{3}{4}\frac{a}{r} - \frac{3}{16}\frac{a^4}{r^4} - \frac{1}{4}\frac{a^3}{r^3}\right)\sin^2\theta}$$

$$= c\sqrt{1}$$

$$\frac{\partial p}{\partial x\partial y} \;\Big\| \quad \mu\,\Delta^2\underbrace{\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)}_{} = \frac{\partial \varepsilon}{\partial y}\frac{\partial \mathcal{U}}{\partial x} - \frac{\partial \varepsilon}{\partial x}\frac{\partial \mathcal{U}}{\partial y}$$

$$\mu\Delta^2 \zeta = $$

$$\mu\Delta^2 \operatorname{curl} v = \begin{vmatrix} i & j & k \\ \frac{\partial \varepsilon}{\partial x} & \frac{\partial \varepsilon}{\partial y} & \frac{\partial \varepsilon}{\partial z} \\ \frac{\partial \mathcal{U}}{\partial x} & \frac{\partial \mathcal{U}}{\partial y} & \frac{\partial \mathcal{U}}{\partial z} \end{vmatrix}$$

~~Nieoznaczoności~~

Uproszczone zadanie:

w przestrzeni zewnętrznej u, v, w Tak jak ciecz idealna nieściśliwa

$$\varepsilon \frac{\partial U}{\partial x} = \mu \Delta^2 u$$

$$\varepsilon \frac{\partial U}{\partial y} = \mu \Delta^2 v$$

$$\varepsilon \frac{\partial U}{\partial z} = \mu \Delta^2 w$$

To nie może się zgadzać z równaniem $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ bo musiałoby wszędzie być:

$$\varepsilon \Delta^2 U + \left(\frac{\partial \varepsilon}{\partial x}\frac{\partial U}{\partial x} + \dots\right) = 0$$

$$u = \int \frac{\varepsilon \frac{\partial U}{\partial x}}{4\pi r} do + \psi$$

$$= \frac{1}{4\pi} \int_1 \frac{\partial U}{\partial x} do \int \frac{\varepsilon}{r} dn + \frac{1}{4\pi} \int_2 \frac{\partial U}{\partial x} do \int \frac{\varepsilon}{r} dn$$

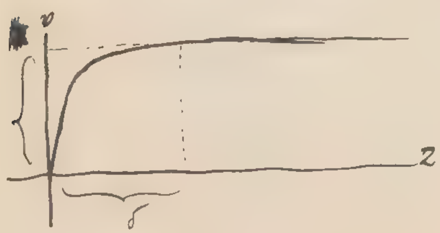
$$\varepsilon \frac{\partial U}{\partial x} = \mu \Delta^2 v_1 = \mu\left[\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial s^2} + \frac{\partial^2 v_1}{\partial z^2}\right]$$

$$\int dz\, z\, \varepsilon \frac{\partial U}{\partial x} = \frac{\partial U}{\partial x} \frac{1}{4\pi} \int \frac{\partial^2 \varphi}{\partial z^2} z\, dz = \cancel{\frac{1}{4\pi}\frac{\partial U}{\partial x}} = \mu\left[\int z \frac{\partial^2 v_1}{\partial z^2} dz + \int z \frac{\partial^2 v_1}{\partial s^2} dz + \int \right]$$

$$= \frac{1}{4\pi}\frac{\partial U}{\partial x}\left[\cancel{\frac{\partial \varphi}{\partial z} z} - \int \frac{\partial \varphi}{\partial z} dz\right] = \frac{1}{4\pi}\frac{\partial U}{\partial x}(\varphi_1 - \varphi_0) =$$

$$= \mu\left\{\left[\frac{\partial v}{\partial z} \cdot z\Big|^{\delta} - \int_0^{\delta} \frac{\partial v}{\partial z} dz\right] + \left(\frac{\partial^2 v_1}{\partial s^2}\right)_{\frac{\delta}{2}} \frac{\delta^2}{2} + \left(\frac{\partial^2 v_1}{\partial x^2}\right)_{\frac{\delta}{2}} \frac{\delta^2}{2}\right\}$$

$$\int_0^{\delta} \varphi f\, dx = f_M \cdot \int_0^{\delta} \varphi\, dx$$



$$\int_0^{\delta} z \frac{\partial^2 v}{\partial z^2} dz = -\left[v_\delta - v_0\right]$$

$$= \cancel{\delta \cdot \frac{\partial v}{\partial z}} = \delta \cdot M\left\{z \frac{\partial^2 v}{\partial z^2}\right\}$$

$$= \frac{\delta^2}{2} M\left\{\frac{\partial^2 v}{\partial z^2}\right\}$$

$$= \frac{\partial v}{\partial z}\Big|_0^{\delta} M(z) = \frac{\partial v}{\partial z}\Big|_0 M(z)$$

w części 1: $\frac{\partial U}{\partial x}$ może być uważane za stałe i powierzchnia za płaską

2: różnica w r tak mała w obrębie δ że $r = f_1(\rho, t)$

$$u = \frac{\partial U}{\partial x} \int_0^{\delta} d\sigma \int \frac{\varepsilon}{r}\, dr + \int_{\delta}^{\infty} \frac{1}{r} \frac{\partial U}{\partial x}\, d\sigma \int \varepsilon\, dr + \psi$$

~~Jeśli ... $\int \varepsilon\, d\rho = 0$~~

~~Jeśli ... $\int \frac{\varepsilon\, dr}{r} = \frac{x}{r^2} \frac{\partial r}{\partial n}$~~

$$\underbrace{\hphantom{xxxx}}_{\frac{\partial \varphi}{\partial n} \frac{1}{4\pi} \int_0^{\infty} \frac{1}{r} \frac{\partial U}{\partial x}\, d\sigma}^{\frac{1}{4\pi}\frac{\partial^2 \varphi}{\partial x^2}}$$

$$\int_0^R \frac{2\pi \rho\, d\rho}{\sqrt{x^2+\rho^2}} = 2\pi \sqrt{x^2+\rho^2}\Big|_0^R = 2\pi\sqrt{x^2+R^2} - 2\pi x$$

$$2\pi \int_0^x \left(\sqrt{R^2+x^2} - x\right) \frac{dx}{4\pi} \frac{\partial^2 \varphi}{\partial x^2}$$

$$= \frac{1}{2} \frac{\partial \varphi}{\partial x} \left[\sqrt{R^2+x^2} - x\right]_0^x - \frac{1}{2} \int \frac{\partial \varphi}{\partial x} \left[\frac{x}{\sqrt{R^2+x^2}} - 1\right] dx$$

$$= \frac{1}{2}\left[\frac{\partial \varphi}{\partial x_0}\left(\sqrt{R^2+x^2} - x\right) - R \frac{\partial \varphi}{\partial x_x}\right] -$$

$$+ \frac{\partial \varphi}{\partial x}$$

$$\sqrt{R^2+x^2} - x = R\left\{\left[1 + \left(\frac{x}{R}\right)^2\right]^{\frac{1}{2}} - \frac{x}{R}\right\} = R\left\{1 + \frac{1}{2}\frac{x^2}{R^2} - \frac{x}{R}\right\} = R - x + \frac{1}{2}\frac{x^2}{R}$$

W każdym razie ważne:

$$\cancel{\ldots}\ \varepsilon \frac{\partial U}{\partial n} = \mu \frac{\partial^2 v_n}{\partial n^2} = \frac{1}{4\pi} \frac{\partial^2 \varphi}{\partial n^2} \frac{\partial U}{\partial n}$$

$$\mu \frac{\partial v_n}{\partial n}\Big|^{\delta} = \frac{1}{4\pi} \frac{\partial \varphi}{\partial n}\Big|_0^{\delta} \frac{\partial U}{\partial n}$$

$$\left(\frac{\partial \varphi}{\partial n_\delta} - \frac{\partial \varphi}{\partial n_0}\right) dn$$

$$= \varphi_a - \varphi_i - \delta \cdot \mathcal{L}$$

$$\int n\, dn$$

$$\mu\, v_n\Big|^{\delta} = \frac{1}{4\pi} (\varphi_i - \varphi_a) \frac{\partial U}{\partial n}$$

to wystarczy do oznaczenia powierzchni w całej przestrzeni będzie

$$\cancel{\Delta^2 \varphi = 0} \quad \cancel{\frac{\partial \varphi}{\partial n}}$$

$$\begin{cases} \Delta^2 \psi = 0 \\ \dfrac{\partial \psi}{\partial n}\Big|^{\delta} = \dfrac{\varphi_i - \varphi_a}{4\pi\mu} \dfrac{\partial U}{\partial n} \end{cases}$$

$$u = \frac{\partial \psi}{\partial x} \qquad v = \qquad w =$$

Ale to ważne tylko jeżeli rzeczywiście takie zadanie jednoznacznie możliwe bo nie wiadomo jeszcze czy styczne składowe odpowiednie.

Czy nie można tego samego rozwiązania użyć ~~...~~ w ogólniejszym przykładzie

$$\Delta^2 v_n = \frac{\partial^2 v_n}{\partial z^2} + \left(\frac{1}{R} + \frac{1}{R'}\right) \frac{\partial v_n}{\partial z}$$

$$\frac{\partial p}{\partial n} = -\varepsilon \frac{\partial U}{\partial n} + \mu\, \Delta^2 v_n$$

$$\frac{\partial p}{\partial z} = -\frac{1}{4\pi} \frac{\partial^2 \varphi}{\partial z^2} \frac{\partial U}{\partial n} + \mu \frac{\partial^2 v_n}{\partial z^2}$$

$\int dz$

$$p = -\frac{1}{4\pi} \frac{\partial \varphi}{\partial z} \frac{\partial U}{\partial n} + \mu \frac{\partial v_n}{\partial z} + \text{const}$$

$\int dz$

$$\int p\, dz = -\frac{1}{4\pi}\, \varphi \cdot \frac{\partial U}{\partial n} + \mu v_n + z \cdot \text{const} + \text{const}'$$

$$\underbrace{\int_0^{\delta} p\, dz}_{= p_m \cdot \delta} = -\frac{\varphi_a - \varphi_i}{4\pi} \frac{\partial U}{\partial n} + \mu (v_{n\delta} - \underset{0}{v_{n0}}) + \delta \cdot \text{const}$$

To nie wystarczy jednak do oznaczenia ruchu bo teraz mamy $\Delta^2 u = -\frac{\partial p}{\partial x}$ etc.

$$\mu\, v_{n\delta} = \frac{\varphi_a - \varphi_i}{4\pi} \frac{\partial U}{\partial n}.$$

$$v_n = \frac{\varphi_a - \varphi_i}{4\pi\mu} \frac{\partial U}{\partial n}$$

Czy możliwy rozkład $u = u_1 + u_2$

$v = v_1 + v_2$

$w = w_1 + w_2$

Takie, że $p_1 = 0$ pozawarstwą

$$\frac{\partial p_1}{\partial x} + 2\frac{\partial U}{\partial x} = \mu \Delta^2 u_1 \qquad \frac{\partial p_2}{\partial x} = \mu \Delta^2 u_2$$

$$\frac{\partial p_1}{\partial y} + 2\frac{\partial U}{\partial y} = \mu \Delta^2 v_1 \qquad \frac{\partial p_2}{\partial y} = \mu \Delta^2 v_2$$

$$\frac{\partial p_1}{\partial z} + 2\frac{\partial U}{\partial z} = \mu \Delta^2 w_1 \qquad \frac{\partial p_2}{\partial z} = \mu \Delta^2 w_2$$

pozawarstwą musiałby to i małym być ruch potencyalny; byłby oznaczony jeżeli prędkości na powierzchni takie jakie

$\Delta^2 \varphi = 0 \qquad u = \frac{\partial \varphi}{\partial x}$; ~~[illegible]~~ odpowiadające zadaniu potencyalnemu.

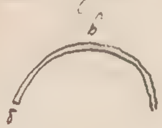
Co do prędkości normalnych ~~[illegible]~~ mamy

$$v_n = \frac{\partial \varphi}{\partial n} = \frac{\varphi_a - \varphi_i}{4\pi \mu} \frac{\partial U}{\partial n}$$

Gdyby założenie $\lambda \frac{\partial U}{\partial n} = 0$ to by było $v_n = 0$

To prawda bo v_n będzie do zaniedbania w porównaniu z prędkościami stycznemi jeżeli δ dostatecznie małe

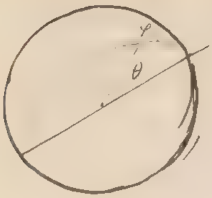
$$v_n = v_{n0} + z\left(\frac{\partial v_n}{\partial z}\right)_0 + \frac{z^2}{2}\left(\frac{\partial^2 v_n}{\partial z^2}\right)_0$$



v_n do v_s będzie w stosunku $\delta : \sigma$

porównanie równanie ciągłości : $v_n \cdot \sigma = v_s \cdot \delta$

$v_n : v_s = \delta : \sigma$



$$\Delta^2(u) = \frac{1}{r}\left[\frac{\partial^2(ru)}{\partial r^2} + \frac{1}{r\sin\theta}\,\frac{\partial\left[\sin\theta\,\frac{\partial u}{\partial\theta}\right]}{\partial\theta} + \frac{1}{r\sin^2\theta}\,\frac{\partial^2 u}{\partial\varphi^2}\right]$$

$$r = a + z$$

$$\Delta^2(v) = \frac{1}{a}\left[\frac{\partial^2}{\partial z^2}[(a+z)v] + \frac{1}{a\sin\theta}\,\underbrace{\frac{\partial}{\partial\theta}\left[\sin\theta\,\frac{\partial v}{\partial\theta}\right]}_{\left[\sin\theta\,\frac{\partial^2 v}{\partial\theta^2} + \cos\theta\,\frac{\partial v}{\partial\theta}\right]} + \cancel{\frac{1}{a\sin\theta}\,\underbrace{\frac{\partial^2 v}{\partial\varphi^2}}_{=0}}\right]$$

$$\frac{\partial}{\partial z}\left[(a+z)v\right]$$

$$= \frac{\partial v}{\partial z}(a+z) + v$$

$$\frac{\partial^2}{\partial z^2} = \frac{\partial^2 v}{\partial z^2}(a+z) + 2\frac{\partial v}{\partial z}$$

$$\Delta^2(v) = \left(1+\frac{z}{a}\right)\frac{\partial^2 v}{\partial z^2} + \frac{2}{a}\,\frac{\partial v}{\partial z} + \frac{1}{a}\,\frac{\partial^2 v}{\partial\theta^2} + \frac{1}{a}\,\frac{1}{\operatorname{tg}\theta}\,\frac{\partial v}{\partial\theta}$$

$$v_{r\theta} = -c\sin\theta\left[1 - \frac{3}{4}\,\frac{a}{r} - \frac{1}{4}\,\frac{a^3}{r^3}\right]$$

$$\frac{\partial}{\partial\theta}\quad v = \cos\theta\,[\ -\ \ —\ ]\qquad r=a\ \Big\}\ =0$$

$$\frac{\partial^2}{\partial\theta^2}\quad = -\sin\theta\,[\quad]$$

istotnie

$$\Delta^2 = \frac{3}{2}\,ca\,\frac{\sin\theta}{r^3}\Big|^{c} = \frac{3}{2}\,\frac{c}{a^2}\sin\theta$$

$$\left.\begin{aligned}\frac{\partial}{\partial z} &= -\frac{3}{2}\,\frac{c\sin\theta}{a}\\ \frac{\partial^2}{\partial z^2} &= \frac{9}{2}\,\frac{c\sin\theta}{a^2}\end{aligned}\right\}\quad \frac{\partial^2}{\partial z^2} + \frac{2}{a}\,\frac{\partial}{\partial z} = \frac{3}{2}\,c\,\frac{\sin\theta}{a^2}$$

$$\Delta^2(v)_{r\theta} = \frac{\partial^2 v_{r\theta}}{\partial z^2} + \frac{2}{a}\,\frac{\partial v_{r\theta}}{\partial z} = \cancel{\ \ }$$

tatem, przyj. więc r/z Kuli $\cdot$ $\Delta^2 v_{r\theta} = \dfrac{\partial^2 v_{r\theta}}{\partial z^2} + \dfrac{2}{a}\,\underset{\delta}{\dfrac{\partial v_{r\theta}}{\partial z}}$

$$\Delta^2\, z\,dz = \int_0^{\delta}\frac{\partial^2}{\partial z^2}\,z\,dz + \frac{2}{a}\int_0^{\delta}\frac{\partial v}{\partial z}\,z\,dz = v_0 - v_\delta + \frac{2}{a}\,\cancel{\ \ }\ \int_0 M(z)$$

$$\lim\,(v_\delta - v_0)\,\frac{\partial\delta}{a} = v$$

– tem istotnie:

$$\int_0^\delta \frac{\partial p}{\partial s}\, z\,dz = -\frac{1}{4\pi}\frac{\partial U}{\partial s}\int_0^\delta \frac{\partial^2\varphi}{\partial z^2}\, z\,dz + \mu\,(v_0 - v_\delta)$$

$$\underbrace{\qquad}_{\lim\ =0}$$

$$v_\delta = \frac{\varphi_1-\varphi_0}{4\pi\mu}\,\frac{\partial U}{\partial s}$$

ważne podzałożenia: 1). $\lim \frac{\delta}{a} = 0$ 2). $v_0 = 0$ 5). $\varepsilon = \frac{\partial^2\varphi}{\partial z^2}$

3). $\frac{\partial U}{\partial s}$ = const w obrębie δ 4). p skończone

---

rodzaj danego:

$U = U' + V + \Phi$ przy powierzchni: $\frac{\partial}{\partial n}(V+\Phi) = 0$

$$\frac{\partial U}{\partial n} = \frac{\partial U'}{\partial n}$$

$$\frac{\partial U'}{\partial s} = 0$$

$$\frac{\partial U}{\partial s} = \frac{\partial(V+\Phi)}{\partial s}$$

~~tutaj w przestrzeni zewnętrznej:~~ ~~$\frac{\varphi_1-\varphi_0}{4\pi\mu}\ p + \ldots$~~

$$\cancel{\begin{cases} \frac{\partial p}{\partial x} = \mu\,\Delta^2 u \\ \frac{\partial p}{\partial y} = \mu\,\Delta^2 v \\ \frac{\partial p}{\partial z} = \mu\,\Delta^2 w \end{cases}}$$

właściwie w poprzednym rachunku:

$$\varepsilon = \frac{1}{4\pi}\,\Delta^2 U = \frac{1}{4\pi}\left[\frac{\partial^2}{\partial z^2} \quad - \frac{\partial}{\partial s} \quad \right]$$

Ścisły warunek graniczny: $\frac{\partial^2 U}{\partial s\,\partial n} = 0$ co spełnione z powodu $\frac{\partial U'}{\partial s} = 0$ $\frac{\partial}{\partial n}(V+\Phi) = 0$

$$= \frac{\partial^2 U'}{\partial s\,\partial n} + \frac{\partial^2(V+\Phi)}{\partial s\,\partial n}$$

a przy całkowaniu $\int \varphi\, z\, dz$ i lim. odpowiadają inne wyrażenia

zatem ów rezultat będzie ściśle ważny.

$$v_s\delta = \frac{U_i - U_0}{4\pi\mu}\frac{\partial U}{\partial s} = \frac{U_i - U_0}{4\pi\mu}\frac{\partial}{\partial s}(V+\Phi)$$

w warstwie δ różnica V, Φ będzie jednak mała w porównaniu do $\varphi_i - \varphi_0$

$$v_s\delta = \frac{\varphi_i - \varphi_0}{4\pi\mu}\frac{\partial}{\partial s}(V+\Phi) = \frac{\varphi_i - \varphi_0}{4\pi\mu}\left[\frac{\varphi_i - \varphi_0}{4\pi\lambda\mu}\frac{\partial p}{\partial s} + \frac{\partial \Phi}{\partial s}\right]$$

$$V = \frac{\varphi_i - \varphi_0}{4\pi\lambda\mu}\, p + f(x)$$

Zatem zadanie : $\nabla^2\Phi = 0$ w wielkiej odległości $\frac{\partial\Phi}{\partial n} = 0$

$$\begin{cases} \frac{\partial p}{\partial x} = \mu\nabla^2 u \\ \frac{\partial p}{\partial y} = \mu\nabla^2 v \\ \frac{\partial p}{\partial z} = \mu\nabla^2 w \end{cases}$$

$$v_s\delta = \frac{\varphi_i - \varphi_0}{4\pi\mu}\left[\frac{\varphi_i - \varphi_0}{4\pi\lambda\mu}\frac{\partial p}{\partial s} + \frac{\partial\Phi}{\partial s}\right]$$

$$v_n\delta = 0$$

$$\therefore \nabla^2 p = 0$$

oznaczając $\frac{\varphi_i - \varphi_0}{4\pi\lambda\mu}\, p + \Phi$ przez Ψ $\qquad \Delta^2\Psi = 0$

jest warunek graniczny $v_{s0} = \frac{\varphi_i - \varphi_0}{4\pi\mu}\frac{\partial\Psi}{\partial s}$

dotąd $v_{n0} = 0$

---

zatem jedno rozwiązanie możliwe $v = \nabla\Psi \cdot \frac{\varphi_i - \varphi_0}{4\pi\mu}$

ogólniej : $v = \frac{\varphi_i - \varphi_0}{4\pi\mu}\nabla\Psi + v_0$ $\qquad v_{s0} = 0$, $v_{n0} = 0$

$$\begin{cases} \frac{\partial p}{\partial x} = \mu\nabla^2 u_0 \\ \ldots \end{cases} \qquad \Delta^2 p = 0$$

zwykłe tarcie

Zatem znając dla danego kształtu powierzchni etc. rozwiązanie

~~potencyalne~~ przewodnictwa elektrycznego $\tilde{\psi}$ i tarcia wewnętrznego v_0

można superponować:

$$v = \frac{\varphi_i - \varphi_0}{4\pi\mu} \nabla \tilde{\psi} + v_0$$

dane doświadczalne będą: ciśnienia p_1, p_2

potencyał obserwowany elektrometrycznie A_1 A_2

lub galwanometrycznie D_1 D_2

ponieważ ~~$\frac{\partial D}{\partial t}$ $\lambda \nabla D = i$~~ ~~zatem~~

$$i = \lambda \nabla_1 (\mathcal{U} - \mathcal{U}') + \varepsilon v$$

$$= \lambda \nabla (V + \Phi) + \varepsilon v$$

$$\int i \, d\omega = \lambda \int \frac{\partial}{\partial n}(V+\Phi)\, d\omega + \underbrace{\int \frac{1}{4\pi} \frac{\partial^2 \varphi}{\partial z^2} v \, d\omega}_{\tilde{q}}$$

$$= \int \frac{ds}{4\pi} \int \frac{\partial^2 \varphi}{\partial z^2} z \left(\frac{\partial v_0}{\partial z}\right) dz$$

$$= \int \frac{ds}{4\pi} (\varphi_i - \varphi_0) \left(\frac{\partial v_0}{\partial z}\right)$$

A oblicza się w drugim przybliżeniu w którym μ bardzo duże

$$\rho w \frac{\partial w}{\partial z} = \rho g - \frac{\partial p}{\partial z} + \frac{\mu}{3}\frac{\partial}{\partial z}\operatorname{div} + \mu \Delta^2 w$$

$$\frac{\partial(\rho w)}{\partial z} = 0$$

$$w\frac{\partial p}{\partial z} + kp\frac{\partial w}{\partial z} = (k-1)\mu\left[\frac{4}{3}\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right] + (k-1)\kappa\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}\right)$$

$$\frac{\partial w}{\partial x} = \frac{dw}{dr}\frac{x}{r}$$

$$\left.\begin{aligned}\frac{\partial^2 w}{\partial x^2} &= \frac{d^2 w}{dr^2}\frac{x^2}{r^2} + \frac{dw}{dr}\frac{1}{r} - \frac{dw}{dr}\frac{x^2}{r^3}\\ \frac{\partial^2 w}{\partial y^2} &= \frac{d^2 w}{dr^2}\frac{y^2}{r^2} + \frac{dw}{dr}\frac{1}{r} - \frac{dw}{dr}\frac{y^2}{r^3}\end{aligned}\right\} \Delta^2 = \frac{d^2 w}{dr^2} + \frac{1}{r}\frac{dw}{dr}$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)$$

$$\cancel{\frac{\partial}{\partial r}(rw) = w + r\frac{dw}{dr}}$$

$$\cancel{\frac{\partial^2}{\partial r^2} =}$$

$$\rho w \frac{\partial w}{\partial z} = \rho g - \frac{\partial p}{\partial z} + \frac{4\mu}{3}\frac{\partial^2 w}{\partial z^2} + \mu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right)$$

$$\frac{\partial}{\partial z}(\rho w) = 0$$

$$w\frac{\partial p}{\partial z} + kp\frac{\partial w}{\partial z} = (k-1)\mu\left[\frac{4}{3}\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial r}\right)^2\right] + (k-1)\kappa\left[\frac{\partial^2\theta}{\partial z^2} + \frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r}\right]$$

Pytanie, czy to może być rozwiązane pod założeniem

$\cancel{\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y}}$ $\frac{\partial p}{\partial r} = 0$ $\quad u = v = 0$

1) Pomijamy $\frac{\partial^2 w}{\partial z^2}$ i ~~[illegible]~~ Φ i $\frac{\partial^2\theta}{\partial z^2}$ i $w\frac{\partial w}{\partial z}$:

$$\rho w = \text{fc.}(r)$$

$$\left(\rho w \frac{\partial w}{\partial z}\right) = \rho g - \frac{dp}{dz} + \mu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right)$$

$$w\frac{\partial p}{\partial z} + kp\frac{\partial w}{\partial z} = (k-1)\kappa\left[\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r}\right]$$

Komin

W równaniu termicznem:

$$u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}+k\,p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=(k-1)\,\Phi+(k-1)\,\kappa\,\Delta^2\theta$$

co pochodzi z ciepła właściwego?

$$c\rho\left(u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}\right)=c\left[\frac{\partial}{\partial x}(\rho u\theta)+\frac{\partial}{\partial y}(\rho v\theta)+\dots\right]-c\,\theta\underbrace{\cancel{\left[\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}\right]}}_{=0}$$

$$=\frac{c}{R}\left[\frac{\partial}{\partial x}(pu)+\frac{\partial}{\partial y}(pv)+\frac{\partial}{\partial z}(pw)\right]$$

Biorąc c_v otrzymałoby się: $\frac{c_v}{R}=\frac{1}{k-1}$

zatem: $$u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}+p\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right]=(k-1)\,\kappa.\Delta^2\theta$$

Biorąc zaś c_p:

$$k\left(u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}\right)+k\,p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=(k-1)\,\kappa.\Delta^2\theta$$

Więc tutaj przypadek pośredni, między c_v i c_p.

$$\begin{cases}\dfrac{\partial p}{\partial x}=\dfrac{\mu}{3}\dfrac{\partial}{\partial x}\,\mathrm{div}+\mu\,\Delta^2 u \\ \dfrac{\partial p}{\partial y}= \\ \dfrac{\partial p}{\partial z}=\rho g+\dfrac{\mu}{3}\dfrac{\partial}{\partial z}\,\mathrm{div}+\mu\,\Delta^2 w\end{cases}\qquad \frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0$$

Jeżeli p znaczne to przy wszystkich prądach termokonwekcyjnych zmiany w ρ będą prawie wyłącznie pochodziły z zmiany w θ

można zatem stawić

$$\rho=\rho_0(1-\alpha\theta)$$

$$\cancel{p}\, \operatorname{div}\, \cancel{w} = \alpha \cancel{p_0} \left[u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z} \right]$$

$$\frac{\partial}{\partial x} \operatorname{div} = \alpha \left[\frac{\partial u}{\partial x} \frac{\partial \theta}{\partial x} + \dots + u \frac{\partial^2 \theta}{\partial x^2} + \dots \right]$$

Supponuję tymczasowo, że to może w obec $\Delta^2 u$ etc.

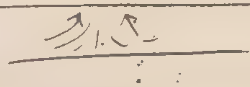
$$\left.\begin{aligned} \frac{\partial p}{\partial x} &= \mu \Delta^2 u \\ \frac{\partial p}{\partial y} &= \mu \Delta^2 v \\ \frac{\partial p}{\partial z} &= \rho g + \mu \Delta^2 w = f(z) + \mu \Delta^2 w \end{aligned}\right\} \quad \Delta^2 p = \frac{\partial f(z)}{\partial z} \cancel{+ \mu \Delta^2 \operatorname{div}}$$

Zupełne równanie wypadkowe:

$$\Delta^2 p = \frac{4}{3} \mu \Delta^2 \operatorname{div} + g \frac{\partial \rho}{\partial z} \qquad p = R \rho \theta$$

$$u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} + w \frac{\partial p}{\partial z} + k p \operatorname{div} = (k-1) \kappa \Delta^2 \theta$$

Dwuwymiarowe zadanie poziome:



$$u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} + k p \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = (k-1) \kappa \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right)$$

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = g \frac{\partial \rho}{\partial y}$$

$$\frac{\partial p}{\partial x} = \mu \Delta^2 u$$

$$\frac{\partial p}{\partial y} = \mu \Delta^2 v + \rho g$$

Ogólniej: czy może być rozwiązanie $p = \text{const}$ przy takich prędkościach?

$$\cancel{0} = \frac{\mu}{3}\frac{\partial}{\partial x}\operatorname{div} + \mu\Delta^2 u \quad + \rho X \qquad p_0 = R\rho\theta$$

$$\begin{cases} 0 = \frac{\mu}{3}\frac{\partial}{\partial y}\operatorname{div} + \mu\Delta^2 v & + \rho Y \\ 0 = & + \rho Z \end{cases}$$

$$\cancel{k}\, p_0 \operatorname{div} = (k-1)\,\Phi + (k-1)\kappa\,\Delta^2\theta$$

$$0 = \frac{4\mu}{3}\Delta^2\operatorname{div} + \frac{\partial}{\partial x}(\rho X) + \frac{\partial}{\partial y}(\rho Y) + \frac{\partial}{\partial z}(\rho Z)$$

~~Dla~~ Pomijając Φ: $\quad \operatorname{div} = \frac{k-1}{k p_0}\kappa \cdot \Delta^2\theta$

N. p. przy ciężkości:

$$0 = \frac{4\mu}{3}\Delta^2\operatorname{div} + g\frac{\partial\rho}{\partial z} \qquad \rho = \frac{p_0}{R\theta}$$

$$\cancel{\frac{4\mu}{3}\cdot\frac{k-1}{k p_0}\cdot\kappa\cdot\Delta^2\theta = -g\frac{p_0}{R}\frac{\partial}{\partial z}\frac{\partial\theta}{\partial z}}$$

$$\left.\begin{aligned} 0 &= \frac{\mu}{3}\frac{\partial}{\partial x}\cdot\operatorname{div} + \mu\Delta^2 u \\ 0 &= \frac{\mu}{3}\frac{\partial}{\partial y}\cdot\operatorname{div} + \mu\Delta^2 v \\ 0 &= \frac{\mu}{3}\frac{\partial}{\partial z}\cdot\operatorname{div} + \mu\Delta^2 w + \rho g \end{aligned}\right\} \quad \left.\begin{aligned} \frac{\partial\rho}{\partial z}g &= -\frac{4}{3}\mu\,\Delta^2\operatorname{div} \\ \frac{\partial\rho}{\partial y}g &= -\mu\,\Delta^2\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \\ \frac{\partial\rho}{\partial x}g &= -\mu\,\Delta^2\left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right) \end{aligned}\right\}$$

$$\cancel{\rho\operatorname{div} + u\frac{\partial\rho}{\partial x}} \qquad g\,\Delta^2\rho = -\mu\,\Delta^2\left[\Delta^2 w - \frac{\partial}{\partial z}\operatorname{div} + \frac{4}{3}\frac{\partial}{\partial z}\operatorname{div}\right]$$

$$\cancel{= -\mu\,\Delta^2\left[\Delta^2 w + \frac{1}{3}\frac{\partial}{\partial z}\operatorname{div}\right]}$$

natur dnie!

Czy jest jakiś ruch mający $v = w = 0$

$$\cancel{\rho u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = \frac{\mu}{3}\frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial^2 u}{\partial y^2} + \mu\frac{\partial^2 u}{\partial z^2}}$$

$$\rho u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = \frac{\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) + \mu\Delta^2 u$$

$$\frac{\partial p}{\partial y} = \frac{\mu}{3}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right)$$

$$\frac{\partial p}{\partial z} = \frac{\mu}{3}\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x}\right)$$

$$\frac{\partial(\rho u)}{\partial x} = 0$$

$$u\frac{\partial p}{\partial x} + k p\frac{\partial u}{\partial x} = (k-1)\left[+\frac{4}{3}\mu\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right] + (k-1)\kappa\,\Delta^2\vartheta$$

Czy jest możliwe np. $p = p_0 + \frac{\mu}{3}\frac{\partial u}{\partial x}$?

$$\rho u \frac{\partial u}{\partial x} = \mu\Delta^2 u \qquad \rho u\frac{\partial^2 u}{\partial x^2} = \mu\Delta^2\frac{\partial u}{\partial x}$$

$$u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z} = \frac{\mu}{3}\left[u\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) + \dots\right] + \mu\left[\underbrace{u\Delta^2 u + v\Delta^2 v + w\Delta^2 w}\right]$$

$$= \frac{\partial}{\partial x}\left(u\frac{\partial u}{\partial x}\right) - \left(\frac{\partial u}{\partial x}\right)^2 + \frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial y}\right) - \left(\frac{\partial u}{\partial y}\right)^2 + \dots$$

$$= \Delta^2\frac{(u^2+v^2+w^2)}{2} - \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \dots + \left(\frac{\partial v}{\partial x}\right)^2 + \dots\right]$$

$$\left.\begin{aligned}\frac{\partial p}{\partial x} &= \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)\\ \frac{\partial p}{\partial y} &= \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)\end{aligned}\right\} \qquad \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial p}{\partial x} = \frac{\mu}{3}\frac{\partial}{\partial x}\,\mathrm{div} + \mu\Delta^2 u$$

$$\Delta^2 p = \frac{4\mu}{3}\Delta^2 \mathrm{div} + g\frac{\partial \rho}{\partial z}$$

I. $\frac{\partial p}{\partial y} =$

$\frac{\partial p}{\partial z} = \rho g + \mu \Delta^2 w$

$p = R\rho\theta \qquad \rho = \frac{p}{R\theta}$

II. $\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$

III. $u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z} + k\,p\,\mathrm{div} = \underline{(k-1)}\,\Phi + (k-1)\,\kappa\,\Delta^2\theta$

$$\frac{\partial}{\partial x}\left(\frac{pu}{\theta}\right) + \qquad = 0$$

$$\frac{p}{\theta}\,\mathrm{div} + \frac{1}{\theta}\left(u\frac{\partial p}{\partial x} + \dots\right) - \frac{p}{\theta^2}\left(u\frac{\partial \theta}{\partial x} + \dots\right) = 0$$

$$p\,\mathrm{div} + \left(u\frac{\partial p}{\partial x} + \dots\right) = \frac{p}{\theta}\left(u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z}\right)$$

III. a) $p\,\mathrm{div} = \frac{-1}{k-1}\frac{p}{\theta}\left(u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z}\right) + \kappa\Delta^2\theta$

b) $u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z} = \frac{-k}{k-1}\frac{p}{\theta}\left(u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z}\right) + \kappa\Delta^2\theta$

W każdym razie: $\frac{1}{p}\frac{\partial p}{\partial x}$ bardzo mała w porównaniu do $\frac{1}{\theta}\frac{\partial \theta}{\partial x}$ i tem mniejsza czem większe p_0, zatem z wielkiem przybliżeniem:

$$p\,\mathrm{div} = \frac{p}{\theta}\left(u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z}\right)$$

II. $\mathrm{div} = \frac{1}{\theta}\left(u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z}\right) = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$

$$u\frac{\partial \theta}{\partial x} - \theta\frac{\partial u}{\partial x} + v\frac{\partial \theta}{\partial y} - \theta\frac{\partial v}{\partial y} + w\frac{\partial \theta}{\partial z} - \theta\frac{\partial w}{\partial z} = 0$$

$$\frac{\partial}{\partial x}\left(\frac{u}{\theta}\right) + \frac{\partial}{\partial y}\left(\frac{v}{\theta}\right) + \frac{\partial}{\partial z}\left(\frac{w}{\theta}\right) = 0$$

III b): $\frac{k}{k-1}\,\frac{p}{\theta}\left[u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}\right] = \kappa.\Delta^2\theta = \frac{k}{k-1}\,p\,\mathrm{div}$

I). $\Delta^2 p = \frac{4\mu}{3}\Delta^2\mathrm{div} + \frac{g}{R\theta}\frac{\partial p}{\partial z} - \frac{gp}{R\theta^2}\frac{\partial\theta}{\partial z} \mp \frac{4\mu}{3}\Delta^2\mathrm{div} - \frac{gp}{R\theta^2}\frac{\partial\theta}{\partial z}$

małe w porównaniu do ↑

$p = \frac{4\mu}{3}\,\mathrm{div} - \mathrm{pot}\,\frac{gp}{R\theta^2}\frac{\partial\theta}{\partial z} + A \qquad \nabla^2 A = 0$

$\cancel{\mu\frac{\partial}{\partial x}\mathrm{div} - \frac{\partial}{\partial x}\ldots + \frac{\partial A}{\partial x} = \ldots}$

I). $\frac{\partial p}{\partial x} = \frac{\mu}{3}\frac{\partial}{\partial x}\mathrm{div} + \mu\Delta^2 u$

$\frac{\partial p}{\partial y} = \frac{\mu}{3}\frac{\partial}{\partial y}\mathrm{div} + \mu\Delta^2 v$

$\frac{\partial p}{\partial z} = \frac{\mu}{3}\frac{\partial}{\partial z}\mathrm{div} + \mu\Delta^2 w - \frac{gp}{R\theta^2}\frac{\partial\theta}{\partial z}$

$\mu\Delta^2\xi = -\frac{gp}{R\theta^2}\frac{\partial\theta}{\partial x}$

$\mu\Delta^2\eta = -\frac{gp}{R\theta^2}\frac{\partial\theta}{\partial y}$

$\Delta^2\zeta = 0$

Graniczny przypadek

1). $k = \infty$

II). $\Delta^2\theta = 0 \qquad \theta = f\ \text{kwa}\ldots$

$\left\{\begin{array}{l} \text{II} \\ \text{I} \end{array}\right.$ Przybliżenie:

$\mathrm{div} = 0$

$p = A - \mathrm{pot}\left(\frac{gp}{R\theta^2}\frac{\partial\theta}{\partial z}\right) \quad \Delta A = 0$

$\frac{\partial p}{\partial x} = \mu\Delta^2 u$

$\vdots$

$\frac{\partial p}{\partial z} = \mu\Delta^2 w - \frac{gp}{R\theta^2}\frac{\partial\theta}{\partial z}$

$\mathrm{div} = -\frac{1}{\theta}\left(u\frac{\partial\theta}{\partial x} + \ldots\right.$

$\frac{\partial}{\partial x} = \ldots$

$\frac{\partial}{\partial y} = \ldots$

Lepiej tak:

I Przybliżenie: div = 0

$$\Delta^2\theta = 0$$

$$\left.\begin{aligned} \frac{\partial p}{\partial x} &= \mu\Delta^2 u \\ \frac{\partial p}{\partial y} &= \mu\Delta^2 v \\ \frac{\partial p}{\partial z} &= \mu\Delta^2 w - \frac{gp}{R\theta} \end{aligned}\right\} \quad \Delta^2 p = - \frac{gp}{R\theta^2}\frac{\partial\theta}{\partial z}$$

$u, v, w = 0 \ldots$

II. Przybliżenie:

$$\mathrm{div} = \frac{1}{\theta}\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right)$$

$$\kappa\Delta^2\theta = \frac{k}{k-1}\,p\,\mathrm{div}$$

$$\frac{\partial p}{\partial x} = \ldots$$

$$\frac{\partial p}{\partial y} = \ldots$$

$$\frac{\partial p}{\partial z} =$$

$$\Delta^2(x^2+y^2+z^2) = 6$$

$$\Delta^2(r^2) = 6$$

$$\cancel{\frac{\partial r}{\partial x} = \frac{x}{r}}$$

$$\cancel{\frac{\partial^2 r}{\partial x^2} = \frac{1}{r} - \frac{x^2}{r^3}}$$

$$\vartheta_0 = T\frac{r_1}{r} \qquad \alpha = -\beta T r_1$$

$$\Delta p_1 = -\beta T r_1 \frac{z}{r^3} \qquad p_1 = \frac{\alpha z}{2r}$$

$$\cancel{p_1 = \alpha z\left[A + \frac{1}{2r} + \frac{B}{r^3}\right]}$$

$$\cancel{\Delta^2 p_1 =} \qquad p_1 = \alpha z\left[A + \frac{1}{2r} + \frac{B}{r^3}\right] + \frac{C}{r}$$

$$\Delta^2 u_1 = -\alpha 2x\left[\frac{1}{2r^3} + \frac{3B}{r^5}\right] - \frac{Cx}{r^3}$$

$$\Delta^2 v_1 = -\alpha 2y\left[\qquad\qquad\right] + \frac{Cy}{r^3}$$

$$\Delta^2 w_1 = +\alpha\left[A + \frac{1}{2r} + \frac{B}{r^3}\right] - C\frac{z}{r^3} - \alpha z^2\left[\frac{1}{2r^3} + \frac{3B}{r^5}\right] - \alpha\left[\frac{1}{r}\right]$$

$$= \alpha\left[A - \frac{1}{2r} + \frac{B}{r^3}\right] - C\frac{z}{r^3} - \alpha z^2\left[\frac{1}{2r^3} + \frac{3B}{r^5}\right]$$

$$u_1 = \alpha x z\left[\frac{1}{8r} + \frac{B}{2r^3}\right]$$

$$v_1 = \alpha y z\left[\frac{1}{8r} + \frac{B}{2r^3}\right]$$

$$w_1 = \alpha\left[\frac{A}{10}(2r^2 - z^2) + \frac{Bz^2}{2r^3} + \frac{1}{8}\left(\frac{z^2}{r} - 3r\right)\right]$$

To byłoby dla $r = \infty$:

$A = 0$

$$\left.\begin{array}{l} u_\infty = \dfrac{\alpha}{8}\, r.\cos\alpha\cos\varphi \\ v_\infty = \dfrac{\alpha}{8}\, r.\cos\beta\cos\varphi \end{array}\right\} \; q = \frac{\alpha}{8}\, r\cos\varphi$$

$$w_\infty = \frac{\alpha}{8}\, r(\cos^2\varphi - 3)$$

Ten sposób przybliżania dałby zawsze w nieskończoności: $u = v = w = \infty$

Polega to na opuszczeniu w II wyrazu: $u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$

Chodzi o równanie kształtu : $\Delta^2\theta = a\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right)$

Stawiając $\Delta^2\varphi = 0 = \cancel{u\Delta^2 v + v\Delta^2 u + 2\left(\frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial y}\right)}$

$$\varphi = \psi\cdot\chi$$

$$\frac{\partial\varphi}{\partial x} = u\frac{\partial v}{\partial x} + v\frac{\partial u}{\partial x}$$

$$\frac{\partial^2\varphi}{\partial x^2} = u\frac{\partial^2 v}{\partial x^2} + 2\frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + v\frac{\partial^2 u}{\partial x^2}$$

$$\Delta^2\varphi = \psi\Delta^2\chi + \chi\Delta^2\psi + 2\left[\frac{\partial\psi}{\partial x}\frac{\partial\chi}{\partial x} + \frac{\partial\psi}{\partial y}\frac{\partial\chi}{\partial y} + \frac{\partial\psi}{\partial z}\frac{\partial\chi}{\partial z}\right]$$

Przyjmujemy zatem :

$$\chi = \theta \qquad u = \frac{1}{\psi}\frac{\partial\psi}{\partial x} \qquad v = \frac{1}{\psi}\frac{\partial\psi}{\partial y} \qquad w = \frac{1}{\psi}\frac{\partial\psi}{\partial z} \qquad \Delta^2\psi = 0$$

$$u = \frac{\partial}{\partial x}(\log\psi) \qquad v = \frac{\partial}{\partial y}(\log\psi) \qquad w = \frac{\partial}{\partial z}(\log\psi)$$

$$\operatorname{div} = \cancel{\frac{1}{\psi}\Delta^2\psi} - \frac{1}{\psi^2}\left[\left(\frac{\partial\psi}{\partial x}\right)^2 + \left(\frac{\partial\psi}{\partial y}\right)^2 + \left(\frac{\partial\psi}{\partial z}\right)^2\right]$$

Jeśli zatem u, v, w mogą być wyrażone przez taką funkcję ψ że

$$-\frac{a}{2}u = \frac{\partial}{\partial x}(\lg \psi); \quad -\frac{a}{2}v = \frac{\partial}{\partial y}(\lg \psi); \quad -\frac{a}{2}w = \frac{\partial}{\partial z}(\lg \psi) \qquad \Delta^2\psi = 0$$

to stawiając $\theta = \frac{\varphi}{\psi}$, gdzie $\Delta^2\varphi = 0$

będziemy mieli rozwiązanie równania: $\Delta^2\theta - a\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right) = 0$

~~Zatem ewaporacja i tych potencjałów~~

$$div = \frac{1}{\theta}\left[u\frac{\partial\theta}{\partial x} + \cdots\right]$$

Przy powierzchni $u = v = w = 0$ $\quad div = 0$

$$\Delta^2\theta = 0 \qquad \frac{\partial\theta}{\partial s} = 0 \qquad \frac{\partial\theta}{\partial n} = \cdots$$

$$\theta = \theta_0 + n\left(\frac{\partial\theta}{\partial n}\right)_0 + \frac{n^2}{2}\left(\frac{\partial^2\theta}{\partial n^2}\right)_0 + \frac{\xi^2}{2}\frac{\partial^2\theta}{\partial\xi^2} + \frac{\eta^2}{2}\frac{\partial^2\theta}{\partial\eta^2} + \cancel{\eta\xi\frac{\partial^2\theta}{\partial\xi\partial\eta}} + n\xi\frac{\partial^2\theta}{\partial n\partial\xi} + \cdots$$

$$\Delta^2\theta = a \int v . \nabla\theta$$

Specyalny przykład: ~~$v = w = 0$~~ $\quad w = u = 0$

$$div = \frac{1}{\theta}\left[v\frac{\partial\theta}{\partial y}\right] = \frac{\partial v}{\partial y} = \cdots$$

$$\frac{1}{\theta}\frac{\partial\theta}{\partial y} = \frac{1}{v}\frac{\partial v}{\partial y}$$

~~$\frac{\partial p}{\partial y} = \partial p$~~ ~~$\frac{\partial p}{\partial z} = \frac{\partial p}{\partial z} = 0$~~ $p = f_{co}$

$$\frac{\partial p}{\partial x} = \frac{\mu}{3}\frac{\partial}{\partial x} div \qquad = \frac{\mu}{3}\frac{\partial^2 v}{\partial x\partial y}$$

$$\frac{\partial p}{\partial z} = \qquad = \frac{\mu}{3}\frac{\partial^2 v}{\partial z\partial y}$$

$$\frac{\partial p}{\partial y} = \frac{\mu}{3}\frac{\partial}{\partial y} div + \mu\nabla^2 v + g\rho \qquad = \frac{\mu}{3}\frac{\partial^2 v}{\partial y^2} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + \rho g$$

$$\Delta^2 p = \frac{4\mu}{3}\Delta^2\frac{\partial v}{\partial y} + g\frac{\rho_0}{\theta_0}\frac{\partial\theta}{\partial y}$$

Przybliżenie pierwsze $\frac{\partial\theta}{\partial y} = \frac{\partial v}{\partial y} = \underset{0}{\text{nada}}$ $\quad p = f_c.(y)$ $\quad \frac{\partial^2\theta}{\partial x^2} = 0$ $\quad \frac{\partial\theta}{\partial x} = \frac{\theta_1 - \theta_2}{\delta}$

$$\frac{dp}{dy} = \mu\frac{\partial^2 v}{\partial x^2} + g\rho_0\left[1 + \frac{\theta}{\theta_0}\right] \qquad \frac{\partial}{\partial x}: \quad \frac{\partial^3 v}{\partial x^3} = \rho_0 g\frac{\theta_1 - \theta_2}{\delta}$$

$$v = \left[\rho_0 g\frac{\theta_1 - \theta_2}{\delta}\frac{x^3}{6} + a x^2 + b x + c\right]$$

$$\frac{dp}{dy} = \mu\left(\rho_0\frac{\theta_1 - \theta_2}{\delta}x + 2a\right) + g\rho_0\left[1 - \cdots\right]$$

Przypuśćmy ruch „lamellarny"

Można wyobrazić że tylko warstwy w bezpośredniej styczności ścian zmienią znacznie temperaturę

$$\Delta^2\theta = a\mathcal{S}(v.\nabla\theta) \qquad \nabla\theta \text{ przyśrednio } \perp s \qquad v \parallel s \qquad \left.\right\} \begin{matrix}\Delta^2\theta = 0 \\ \mathrm{div} = 0\end{matrix}$$

$$\mathrm{div} = \alpha\left(\mu \frac{\partial\theta}{\partial x} + \right)$$

$$\frac{\partial p}{\partial x} = 0$$

$$\frac{\partial p}{\partial s} = \mu \frac{\partial v}{\partial n^2} + g\rho_0\left[1 - \frac{\theta}{\theta_0}\right] \frac{\partial y}{\partial s} \qquad \mu \frac{\partial^2 v}{\partial n^2} = \frac{\rho g}{\theta_0}\theta$$

$$\mu \frac{\partial^3 v}{\partial n^3} = \frac{g\rho}{\theta_0}\frac{\partial\theta}{\partial n}$$

Zaniedbujemy wpływ różnicy p w kierunku poziomym:

$$\frac{\partial p}{\partial z} + \rho g = \frac{\theta}{\theta_0}\rho_0 g$$

$$0 = \mu \cancel{\frac{\partial}{\partial x}\mathrm{div}} + \mu\Delta^2 u$$

$$0 = \cancel{\frac{\partial}{\partial y}\mathrm{div}} + \mu\Delta^2 v$$

$$0 = \mu \cancel{\frac{\partial}{\partial z}\mathrm{div}} + \mu\Delta^2 w + \frac{\theta}{\theta_0}\rho_0 g$$

$$a\left[u\Delta^2\frac{\partial w}{\partial x} + v\Delta^2\frac{\partial w}{\partial y} + w\Delta^2\frac{\partial w}{\partial z}\right] = \Delta^2\Delta^2 w$$

$$\vartheta = -\frac{\eta}{\rho}\frac{T}{g}\frac{\partial^2 w}{\partial x^2}$$

$$\cancel{\frac{\partial^2 w}{\partial x^2}} = k\left(\frac{\partial^4 w}{\partial x^4}\right) \qquad 0 = g\frac{\vartheta}{T} + \frac{\eta}{\rho}\frac{\partial^2 w}{\partial x^2}$$

$$\vartheta w = k\frac{\partial\theta}{\partial x^2}$$

$$w\frac{\partial^2 w}{\partial x^2} = K\frac{\partial^4 w}{\partial x^4}$$

$$\frac{\partial w}{\partial x^2} = \frac{K}{w}\frac{\partial^4 w}{\partial x^4}$$

$$w = \frac{a}{x^2}$$

$$\frac{6a}{x^4} = \frac{K}{a}x^2 \cdot \frac{2.3.4.5}{x^6}$$

$$a = 20K$$

$$\cancel{\frac{\partial\vartheta}{\partial t} + \theta\frac{\partial w}{\partial x} = k\frac{\partial\theta}{\partial x} = -k\frac{\partial\theta}{\partial x}\frac{\partial\theta}{\partial x} + \theta\frac{\partial w}{\partial x}}$$

$$v = \frac{K}{\theta}\frac{\partial^2\theta}{\partial x^2}$$

$$\frac{\partial v}{\partial x} = -\frac{K}{\theta^2}\frac{\partial\theta}{\partial x}\frac{\partial^2\theta}{\partial x^2} + \frac{K}{\theta}\frac{\partial^3\theta}{\partial x^3}$$

$$\frac{\partial^2 v}{\partial x^2} = +\frac{2K}{\theta^3}\left(\frac{\partial\theta}{\partial x}\right)^2\frac{\partial^2\theta}{\partial x^2} - \frac{K}{\theta^2}\left(\frac{\partial^2\theta}{\partial x^2}\right)^2 - 2\frac{K}{\theta^2}\frac{\partial\theta}{\partial x}\frac{\partial^3\theta}{\partial x^3} + \frac{K}{\theta}\frac{\partial^4\theta}{\partial x^4} = -\frac{g}{f}\frac{f}{3}\theta$$

$$\frac{dv}{dx} = p \qquad \frac{d^2v}{dx^2} = p\frac{dp}{dv} \qquad \cancel{\frac{d^3v}{dx^3} = \frac{dp}{dx}}$$

$$\frac{d^3v}{dx^3} = p\left(\frac{dp}{dv}\right)^2 + p^2\frac{d^2p}{dv^2}$$

$$\frac{d^4v}{dx^4} = p\left[\left(\frac{dp}{dv}\right)^3 + 4p\frac{dp}{dv}\frac{d^2p}{dv^2} + p^2\frac{d^3p}{dv^3}\right]$$

$$v\frac{dp}{dv} = K\left[p^2\frac{d^3p}{dv^3} + 4p\frac{dp}{dv}\frac{d^2p}{dv^2} + \left(\frac{dp}{dv}\right)^3\right]$$

$u = \frac{\partial}{\partial x}(\gamma\psi) + u'$

$v = \qquad + v'$

$w = \qquad + w'$

Zatem: $\psi = \frac{c}{2}\frac{a^3 z}{r^3} + cz + b$

$\left.\frac{\partial\psi}{\partial z}\right| = -\frac{3}{2}\sin^2\varphi$

$\left.\frac{\partial\psi}{\partial\omega}\right|_{r=a} = -\frac{3}{2}\cos\varphi\sin\varphi$

$\Delta^2\varphi = 0 \qquad \varphi = \theta.\psi$

$\psi = e$

$\lg\varphi = \psi$

$\varphi = e^{\psi}$

$r=a \qquad \varphi = \theta_1\, e^{b+\frac{3c}{2}a\cos\varphi} = \theta_2\, e^{\frac{3ca}{2}\cos\varphi} = \theta_2\, e^{\frac{3c}{2}x}$

$r=\infty \qquad \varphi = \theta_0\, e^{c z_\infty} = \theta_0\, e^{c r_\infty \cos\varphi} = \theta_0\, e^{cx}$

W każdym razie: rozwiązanie równania $\Delta\theta + a\,\cos\frac{\partial\theta}{\partial x} + \ldots\ ; = 0$

można wykonać jeśli ...

$$\varphi = A_0 + A_1 P_1 r + A_2 P_2 r^2 + \ldots$$
$$+ B_0 \frac{1}{r} + \frac{B_1 P_1}{r^2} + \frac{B_2 P_2}{r^3} + \ldots$$

$$\int_{-1}^{+1} e^{\alpha x}\cdot P_{(x)}\, dx$$

$\theta = u = 0$ $\quad \frac{\partial p}{\partial x} = 0$

I Ipotesi: $\quad \frac{\partial \theta}{\partial y} = \frac{\partial v}{\partial y} = 0 \quad \to \quad p = f_1(y)$

$\text{div} = 0 \qquad \cancel{u\frac{\partial \theta}{\partial x}} + v\frac{\partial \theta}{\partial y} = 0$

$v = f_2(x)$

$\theta = f_2(x)$

$$\frac{d^2\theta}{dx^2} = 0$$

$$\theta = \theta_1 + (\theta_2 - \theta_1)\frac{x}{\delta}$$

$$\frac{dp}{dy} = \mu \frac{d^2 v}{dx^2} + g\rho_0 \,[redacted]\, \frac{\theta}{\theta_1}\Big] = \mu \frac{d^2 v}{dx^2} + g\rho_0 \frac{\theta_2 - \theta_1}{\theta_1}\frac{x}{\delta} + g\rho_0$$

$$0 = \mu \frac{d^3 v}{dx^3} + g\rho_0 \frac{\theta_2-\theta_1}{\theta_1}\frac{1}{\delta}$$

$$\frac{d^2 p}{dy^2} = 0$$

$$p = p_0 + y\frac{p_1 - p_0}{h} \qquad \frac{dp}{dy} = \frac{p_1 - p_0}{h} =$$

$$\left(\frac{p_1 - p_0}{h} - g\rho_0\right)x = \mu \frac{dv}{dx} + g\rho_0 \frac{\theta_2 - \theta_1}{\theta_1}\frac{x^2}{2\delta} + b$$

$$\frac{p_1 - p_0}{h}\frac{x^2}{2} = \mu v + \rho_0 g \frac{\theta_2 - \theta_1}{\theta_1}\frac{x^3}{6\delta} + bx + c$$

$$\left.\begin{array}{l} x = 0 \\ x = \delta \end{array}\right\} v = 0 \qquad c = 0 \qquad \frac{p_1 - p_0}{h}\frac{\delta^2}{2} = \rho_0 g \frac{\theta_2-\theta_1}{\theta_1}\frac{\delta^2}{6} + b\delta$$

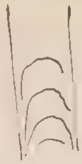
$$\left[g\rho_0 - \frac{p_1 - p_0}{h}\right]\frac{x^2 - \delta x}{2} = \mu v + \rho_0 g \frac{\theta_2 - \theta_1}{\theta_1}\frac{x^3 - x\delta^2}{6}$$

II). $u \neq 0$ $\qquad v_0 = ax + bx^2 + cx^3$

$$\alpha^2 \Delta^2 \theta = (ax + bx^2 + cx^3)\frac{\partial \theta}{\partial y} = \alpha^2\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) = \cancel{\frac{\partial u}{\partial x}} + \frac{\partial v}{\partial y}$$

$p = f_2(y)$

$$\frac{dp}{dy} = \mu\left(\frac{\partial^2 v}{\partial x^2} + \cancel{\frac{\partial^2 v}{\partial y^2}}\right) + g\rho_0 \frac{\theta}{\theta_0}$$



$$\frac{\partial p}{\partial x} = \cancel{\frac{\mu}{3}\frac{\partial}{\partial x}\,\mathrm{div}} + \cancel{\mu\Delta^2 u} = 0$$

$$\frac{\partial p}{\partial y} = \cancel{\frac{\mu}{3}\frac{\partial}{\partial y}\,\mathrm{div}} + \mu\Delta^2 v + g\rho_0\frac{\theta}{\theta_0}$$

$p = f(y)$ $\qquad$ $\theta = f_1(x,y)$

$$\alpha^2\Delta^2\theta = u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + \theta\,\mathrm{div}$$

$u=0$

$$\frac{dp}{dy} = \mu\frac{\partial^2 v}{\partial x^2} + \rho_0 g\frac{\theta}{\theta_0} \qquad 0 = \mu\frac{\partial^3 v}{\partial x^3} + \frac{\rho_0 g}{\theta_0}\frac{\partial\theta}{\partial x}$$

$$\alpha^2\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right) = v\frac{\partial\theta}{\partial y} = \theta\frac{\partial v}{\partial y} \qquad \theta = \frac{\mu\theta_0}{g\rho_0}\frac{\partial^2 v}{\partial x^2} + \varphi(y)$$

$$\alpha^2\left[\underbrace{\frac{\mu\theta_0}{\rho_0 g}\left(\frac{\partial^4 v}{\partial x^4}+\frac{\partial^4 v}{\partial x^2\partial y^2}\right)}_{} + \frac{d^2\varphi}{dy^2}\right] = \frac{\mu\theta_0}{g\rho_0}v\frac{\partial^3 v}{\partial x^2\partial y} + v\frac{d\varphi}{dy}$$

$$\frac{\partial}{\partial y}\left(\frac{v}{\theta}\right) = 0 \qquad \frac{v}{\theta} = \psi(x)$$

$$v = \theta\cdot\psi(x)$$

$$\alpha^2\left(\frac{\partial^2\theta}{\partial x^2}+\cancel{\frac{\partial^2\theta}{\partial y^2}}\right) = \theta\frac{\partial\theta}{\partial y}\psi(x)$$

$$\theta = \frac{\mu\theta_0}{g\rho_0}\left[\frac{\partial^2\theta}{\partial x^2}\psi + 2\frac{\partial\theta}{\partial x}\frac{d\psi}{dx} + \theta\frac{d^2\psi}{dx^2}\right] + \varphi(y)$$

Dla wielkiego y w każdym razie $\theta=\mathrm{const}$

zatem $\frac{d^2\psi}{dx^2} = a$

$$\psi = a\frac{x^2}{2} + bx + c$$

Wzdłuż ściany $\frac{\partial\theta}{\partial y} = \frac{\partial^2\theta}{\partial x^2} = 0$

$\varphi(y) = \mathrm{const}$

$$\theta = \mu \frac{\vartheta_0}{g \rho_0} \frac{\partial^2 v}{\partial x^2} + \text{const}$$

$$\theta = \text{const}.$$

w linii środkowej: $\frac{\partial \theta}{\partial x} = 0 \qquad \frac{d^2 v}{dx^2} = a$

$$\theta \left[1 - \mu \frac{\vartheta_0}{g \rho_0} a \right] = \mu \frac{\vartheta_0}{g \rho_0} \left(a \frac{x^2}{2} + bx + c \right) \frac{\partial^2 \theta}{\partial x^2} + \text{const} \quad \Big| \; \alpha^2$$

$$\theta \frac{\partial \theta}{\partial y} \, 4 \qquad = \alpha^2 \frac{\partial^2 \theta}{\partial x^2} \quad \Big| \; 4 \mu \frac{\vartheta_0}{g \rho_0}$$

$$\alpha^2 \theta \left[1 - \mu \frac{\vartheta_0}{g \rho_0} a \right] - \mu \frac{\vartheta_0}{g \rho_0} \theta \frac{d\theta}{dy} \, \Big| 4 \Big|_{\frac{1}{2}}^{2} = \alpha^2 \cdot \text{const}$$

$$\theta = m + n y \qquad \theta \frac{d\theta}{dy} + A \theta = B$$

$$\frac{\theta \, d\theta}{B - A\theta} = dy$$

$$y =$$

I stacjonarny ruch (dla cewy)

Ruch który jest stały:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\mu\nabla^2 u \qquad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

$$=$$

$$=$$

$$\frac{\partial U}{\partial t}+\dots = -\frac{1}{\rho}\frac{\partial(p+P)}{\partial x}+\mu\nabla^2(u+U)$$

$$\frac{\partial U}{\partial t}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\mu\nabla^2 U-\left[u\frac{\partial U}{\partial x}+U\frac{\partial u}{\partial x}+v\frac{\partial U}{\partial y}+V\frac{\partial u}{\partial y}+w\frac{\partial U}{\partial z}+W\frac{\partial u}{\partial z}\right]$$

$$\frac{\partial v}{\partial t}+(v\nabla\cdot v)=\frac{1}{2}\nabla(v^2)+Vv\,\mathrm{curl}\,v=\frac{1}{\rho}\nabla P+\mu\nabla^2 v$$

$$\frac{\partial\,\mathrm{curl}\,v}{\partial t}+\mathrm{curl}\,Vv\,\mathrm{curl}\,v=\mu\nabla^2\mathrm{curl}\,v$$

$$\frac{\partial\,\mathrm{curl}(v+V)}{\partial t}+\mathrm{curl}\,V(v+V)\,\mathrm{curl}(v+V)=\mu\nabla^2\mathrm{curl}(v+V)$$

$$\frac{\partial\,\mathrm{curl}\,V}{\partial t}+\mathrm{curl}\,V\left[V\mathrm{curl}\,v+v\,\mathrm{curl}\,V\right]=\mu\nabla^2\mathrm{curl}\,V$$

$$\frac{\partial p_{xx}}{\partial x}+\frac{\partial p_{xy}}{\partial y}+\frac{\partial p_{xz}}{\partial z}=-\frac{\partial p}{\partial x}+\lambda\frac{\partial}{\partial x}\,\mathrm{div}+2\mu\left(\frac{\partial a}{\partial x}+\frac{\partial f}{\partial y}+\frac{\partial e}{\partial z}\right)$$

$$=\mu\left[2\frac{\partial^2 u}{\partial x^2}+\left(\frac{\partial^2 v}{\partial y\partial x}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 w}{\partial x\partial z}\right)\right]$$

$$=-\frac{\partial p}{\partial x}+(\lambda+\mu)\frac{\partial}{\partial x}\,\mathrm{div}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$\lambda=-\tfrac{2}{3}\mu$$

$$=-\frac{\partial p}{\partial x}+\frac{1}{3}\frac{\partial}{\partial x}\,\mathrm{div}+$$

$$-p+(2\mu+\lambda)\frac{\partial u}{\partial x}$$

$$2\mu+\lambda=3\mu \quad \text{Stokes}$$

$$\lambda=\mu$$

$$p=R\rho\theta \qquad \rho\,\mathrm{div}+u\frac{\partial\rho}{\partial x}+\dots=0$$

$$p=p_0+\alpha\,\mathrm{div}$$

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \nu\frac{\partial}{\partial x}\operatorname{div} + \mu\,\Delta u = -\frac{\partial p}{\partial x} + (\nu+\mu)\frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial\rho}{\partial t} = 0$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + kp\frac{\partial u}{\partial x} = (k-1)\,\Phi$$

$$\frac{\partial\rho}{\partial t} + \rho\frac{\partial u}{\partial x} = 0$$

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + (\nu+\mu)\frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial p}{\partial t} + kp\frac{\partial u}{\partial x} = (k-1)\,\Phi$$

Który wzgląd zmniejszy się dynamiczny czy termiczny?

I). $\cancel{\frac{\partial^2\rho}{\partial t^2} = -\frac{\partial\rho}{\partial t}\frac{\partial u}{\partial x} - \rho\frac{\partial^2 u}{\partial x\,\partial t}}$

$\cancel{\frac{\partial\rho}{\partial x}\frac{\partial u}{\partial t} + \rho\frac{\partial^2 u}{\partial x\,\partial t} = -\frac{\partial^2 p}{\partial x^2} + (\nu+\mu)\frac{\partial^3 u}{\partial x^3}}$

$\cancel{p \sim \rho^k}$

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^k$$

Φ

2). $\cancel{\frac{\partial\rho}{\partial t}\frac{\partial u}{\partial t}} + \rho\frac{\partial^2 u}{\partial t^2} \qquad = -\frac{\partial^2 p}{\partial x\,\partial t} + (\nu+\mu)\frac{\partial^3 u}{\partial x^2\,\partial t}$

$$= k\,\cancel{\frac{\partial p}{\partial x}\frac{\partial u}{\partial x}} + kp\frac{\partial^2 u}{\partial x^2} + (\nu+\mu)\frac{\partial^3 u}{\partial x^2\,\partial t}$$

3).

$$\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2} + (\nu+\mu)\frac{\partial^3 u}{\partial x^2\,\partial t}$$

$$u = \sin(\alpha x + \beta t)\,e^{-\gamma t}$$

$$\frac{\partial u}{\partial t} = \beta\cos\,e^{-\gamma t} - \gamma\sin\,e^{-\gamma t}$$

$$\frac{\partial^2 u}{\partial t^2} = -\beta^2\sin\,e^{-\gamma t} - 2\beta\gamma\cos\,e^{-\gamma t} + \gamma^2\sin\,e^{-\gamma t} =$$

$$(\gamma^2-\beta^2)\sin - 2\beta\gamma\cos = -\alpha^2 a^2\sin + (\nu+\mu)\left[\alpha^2\gamma\sin - \alpha^2\beta\cos\right]$$

$$\gamma^2-\beta^2 = -\alpha^2 a^2 + (\nu+\mu)\,\alpha^2\gamma$$

$$-2\beta\gamma = -\alpha^2\beta(\nu+\mu)$$

$$\gamma = \frac{\alpha^2}{2}(\nu+\mu)$$

$$\beta^2 = \alpha^2 a^2 + \frac{\alpha^4(\nu+\mu)^2}{4} - \frac{\alpha^2 a^2}{2}(\nu+\mu)^2$$

$$\beta = a\alpha\sqrt{1 - \frac{(\nu+\mu)^2}{2} + \frac{(\nu+\mu)^2\alpha^2}{4a^2}}$$

II). $\rho \frac{\partial^2 u}{\partial t^2} = -\frac{\partial^2 p}{\partial x \partial t}$

$$\frac{\partial p}{\partial t} = -kp\frac{\partial u}{\partial x} + (k-1)\frac{4}{3}\mu\left(\frac{\partial u}{\partial x}\right)^2$$

$$\frac{\partial^2 p}{\partial x\partial t} = -kp\frac{\partial^2 u}{\partial x^2} - k\cancel{\frac{\partial p}{\partial x}\frac{\partial u}{\partial x}} + \frac{8\mu}{3}(k-1)\underbrace{\frac{\partial u}{\partial x}\cdot\frac{\partial^2 u}{\partial x^2}}_{\text{mało wyższego rzędu}}$$

Kula ściągająca się

jeśli zaniedbujemy inercję to odkształcenie

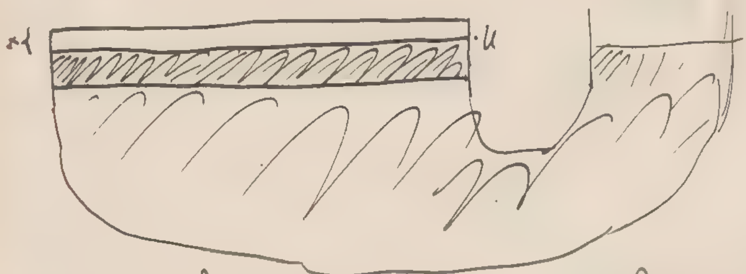
~~przyśpieszenie~~ będzie jednorodne

$\cancel{\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = \frac{\partial w}{\partial z}}$ $x : x_0 = a : a_0$

$$\frac{dx}{dt} = u = \frac{x_0}{a_0}\frac{da}{dt} \qquad u = x\,\frac{1}{a}\frac{da}{dt}$$

$$\frac{\partial u}{\partial x} = \frac{1}{a}\frac{da}{dt} = c \qquad \frac{\partial u}{\partial y} = 0$$

$\Phi = 0$!



$$u = U\frac{\xi}{x}$$

$$\frac{\partial u}{\partial x} = \frac{U}{x}$$

$$p_{xx} = -p + 2\mu\frac{\partial u}{\partial x} - \frac{2\mu}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = -p + \frac{4}{3}\mu\frac{\partial u}{\partial x} = -p + \frac{4}{3}\mu\frac{U}{x}$$

$$M\frac{d^2x}{dt^2} = -2\varphi\, x\, s g\, A - p_0 + p - \frac{4}{3}\frac{\mu}{x}\frac{dx}{dt} \qquad p = p_0\frac{a}{x}$$

$$M\frac{d^2x}{dt^2} = -p_0 - 2\varphi s g . x + \frac{p_0 a}{x} - \frac{4}{3}\frac{\mu}{x}\frac{dx}{dt}$$

$$\frac{dx}{dt} = z \qquad \frac{d^2x}{dt^2} = \frac{dz}{dt} = \frac{dz}{dx}\cdot z$$

$$M z\frac{dz}{dx} + \frac{4}{3}\mu\,\frac{z}{x} - \frac{p_0 a}{x} + 2\varphi s_g x + p_0 = 0$$

$$dm\frac{D}{Dt}\left[\frac{c}{A}\theta + \frac{1}{2}(u^2+v^2+w^2) + U\right] = \Big[\frac{\partial}{\partial x}(u . p_{xx}) + \frac{\partial}{\partial y}(v . p_{yy}) + \frac{\partial}{\partial z}(w . p_{zz})$$

$$+\underbrace{\frac{\partial}{\partial y}(u . p_{xy}) + \frac{\partial}{\partial x}(v . p_{yx})} + \ldots + \ldots\Big] dx\,dy\,dz$$

$$\cancel{\frac{D}{Dt}}\Sigma = \cancel{p_{xx}\frac{\partial u}{\partial x} + p_{yy}\frac{\partial v}{\partial y} + \ldots + p_{xy}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)} + \ldots$$

$$\cancel{+ u\frac{\partial p_{xx}}{\partial x} + v\frac{\partial p_{yy}}{\partial y} + u\frac{\partial p_{xy}}{\partial y} + v\frac{\partial p_{xy}}{\partial x} + \ldots} + \ldots$$

$$= u\left(\rho\frac{Du}{Dt} - \rho X\right) + v\left(\rho\frac{Dv}{Dt} - \rho Y\right) + w(\quad) - p\,\mathrm{div} + \Phi$$

$$\cancel{\frac{D(u^2)}{Dt} = 2u\frac{Du}{Dt}}$$

$$\cancel{2u\frac{\partial u}{\partial t} + 2u^2\frac{\partial u}{\partial x} + 2vu\frac{\partial u}{\partial y} + 2wu\frac{\partial u}{\partial z}}$$

$$\cancel{= 2u\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\ldots} \ldots\right]}$$

$$\rho\frac{c}{A}\frac{D\theta}{Dt} = -p\,\mathrm{div} + \Phi$$

$\cancel{\rho u = \frac{\partial \varphi}{\partial x}}$ $\cancel{\rho v = \frac{\partial \varphi}{\partial y}}$ $\cancel{\rho w = \frac{\partial \varphi}{\partial z}}$

$u = f(x, t)$ $\quad w = 0 \quad$ $\operatorname{div} = \frac{\partial u}{\partial x}$

$v = \varphi(x, t)$

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right] = -\frac{\partial p}{\partial x} + \frac{4\mu}{3}\frac{\partial^2 u}{\partial x^2}$$

$$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x}\right] = \mu\frac{\partial^2 v}{\partial x^2}$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + (k)\, p\frac{\partial u}{\partial x} = \Phi$$

$$= \frac{4\mu}{3}\left(\frac{\partial u}{\partial x}\right)^2 + \mu\left(\frac{\partial v}{\partial x}\right)^2 + \kappa\Delta^2 t$$

$$\frac{\partial v}{\partial t} = \frac{\mu}{\rho_0}\frac{\partial^2 v}{\partial x^2}$$

$$\frac{\partial p}{\partial t} + (k)\, p_0\frac{\partial u}{\partial x} = \mu\left(\frac{\partial v}{\partial x}\right)^2 + \kappa\frac{\partial^2\theta}{\partial x^2} \quad \Big|\ \frac{\partial}{\partial x}$$

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} \quad \Big|\ \frac{\partial}{\partial t}$$

$$\rho_0\frac{\partial^2 u}{\partial t^2} - k p_0\frac{\partial^2 u}{\partial x^2} = \mu\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}\right)^2 + \kappa\frac{\partial^3\theta}{\partial x^3}$$

$$\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2} + \frac{\mu}{\rho_0}\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}\right)^2 + \kappa\frac{\partial^3\theta}{\partial x^3}$$

$v = A e^{-\alpha x}\cos(\beta x + \gamma t)$

$\frac{\partial v}{\partial x} = A e^{-\alpha x}\left[-\alpha\cos + \beta\sin\right]$

$\left(\frac{\partial v}{\partial x}\right)^2 = A^2 e^{-2\alpha x}\left[\alpha^2\cos^2 + \beta^2\sin^2 \mp 2\alpha\beta\cos\sin\right]$

$\gamma = 2\pi n$

$-\sin = \left(\frac{\mu}{\rho_0}\right)\left[(\alpha^2 - \beta^2)\cos \pm 2\alpha\beta\sin\right]$

$\alpha = \beta$

$1 = 2\frac{\mu}{\rho_0}\alpha^2$

$\alpha = \sqrt{\frac{\rho_0}{2\mu}}$

$$v = A e^{-x\sqrt{\frac{\rho_0}{2\mu}}}\cos\left(2\pi n t - x\sqrt{\frac{\rho_0}{2\mu}}\right)$$

$$\left(\frac{\partial v}{\partial x}\right)^2 = A^2 e^{-2\alpha x}\cdot\alpha^2\underbrace{\left[1 - 2\cos\sin\right]}_{\left[1 - \sin 2(\gamma t - \beta x)\right]}$$

$\gamma\sqrt{\frac{2\mu}{\rho_0}}$

$2n\pi\sqrt{\frac{2\mu}{\rho_0}}$

$$\rho \frac{\partial v}{\partial t} \cancel{} = \mu \frac{\partial v}{\partial x^2}$$

$$\frac{\partial p}{\partial t} = \mu\left(\frac{\partial v}{\partial x}\right)^2 + \cancel{}\, \kappa \frac{\partial^2 \theta}{\partial x^2}$$

$$\frac{\partial \rho}{\partial t} + \cancel{} = 0 \qquad \rho = \text{const} = \frac{p}{R\theta}$$

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} = 0$$

$$R\rho_0 \frac{\partial \theta}{\partial t} = \mu\left(\frac{\partial v}{\partial x}\right)^2 + \kappa \frac{\partial^2 \theta}{\partial x^2}$$

$$\rho_0 \frac{\partial v}{\partial t} = \mu \frac{\partial^2 v}{\partial x^2}$$

$$\cancel{\frac{\partial p}{\partial t}} + u \cancel{\frac{\partial p}{\partial x}} + kp \frac{\partial u}{\partial x} = \left[\frac{4\mu}{3}\left(\frac{\partial u}{\partial x}\right)^2 + \kappa \frac{\partial^2 \theta}{\partial x^2}\right](k-1)$$

$$\cancel{p \frac{\partial u}{\partial t}} \qquad u = f(t)\,x = x f$$

$$p_{xx} = p - \frac{4}{3}\mu \frac{\partial u}{\partial x} = a$$

$$p = a + \frac{4}{3}\mu \frac{\partial u}{\partial x}$$

$$\frac{4\mu}{3}\frac{\partial f}{\partial t} + kf\left[a + \frac{4}{3}\mu f\right] = \frac{4\mu}{3} f^2 (k-1)$$

$$\frac{4\mu}{3}\frac{\partial f}{\partial t} + akf + \frac{4\mu}{3} f^2 = 0$$

$$-\frac{1}{f^2}\frac{\partial f}{\partial t} + \frac{3ak}{4\mu}\frac{1}{f} + 1 = 0$$

$$-\frac{\partial}{\partial t}\left(\frac{1}{f}\right) + \frac{3ak}{4\mu}\left(\frac{1}{f}\right) + 1 = 0$$

$$\frac{1}{f} = A e^{\alpha t} + B$$

$$-A\alpha e^{\alpha t} + \frac{3ak}{4\mu}\left[A e^{\alpha t} + B\right] + 1 = 0$$

$$\alpha = +\frac{3ak}{4\mu}$$

$$B = -\frac{4\mu}{3ak}$$

$$u = \frac{x}{A e^{+\frac{3ak}{4\mu}t} - \frac{4\mu}{3ak}}$$

$$\frac{1}{A - \frac{4\mu}{3ak}} = \frac{1}{(a - p_0)\frac{3}{4\mu}}$$

$$A = \frac{4\mu}{3ak} + \frac{4\mu}{\frac{3}{\cancel{}}(p_0 - a)}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}$$

$$\rho\left(u\frac{\partial v}{\partial x}+\qquad\right)=-\frac{\partial p}{\partial y}$$

$$\rho\left(u\frac{\partial w}{\partial x}+\qquad\right)=-\frac{\partial p}{\partial z}$$

$$\frac{\partial \rho u}{\partial x}+\dots=0$$

$$\frac{\rho}{\rho_0}=\left(\frac{p}{p_0}\right)^{\frac{1}{k}}$$

$$\frac{1}{\rho}\frac{\partial \rho}{\partial x}=\frac{1}{k}\frac{1}{p}\frac{\partial p}{\partial x}$$

$$p_1^2=\cancel{a}$$

$$p_2^2=\sqrt{a-cl}$$

$$p_1^2-p_2^2=cl$$

$$p=\sqrt{a-cx}=\sqrt{p_1^2-\frac{x}{l}(p_1^2-p_2^2)}$$

$$u=\frac{(y\delta-y^2)\,c}{4\mu\sqrt{a-cx}}$$

$$\frac{\partial u}{\partial x}=+\frac{(y\delta-y^2)\,c^2}{8\mu\sqrt{(\quad)}^3}$$

$$\frac{\partial^2 u}{\partial x^2}=\frac{3(y\delta-y^2)\,c^3}{16\mu\sqrt{(\quad)}^5}$$

$$\frac{\partial u}{\partial y}=\frac{(\delta-2y)\,c}{4\mu\sqrt{\quad}}$$

$$\frac{\partial^2 u}{\partial y^2}=\frac{-2c}{4\mu\sqrt{\quad}}$$

$$\frac{\partial^2 u}{\partial x\partial y}=\frac{(\delta-2y)\,c^2}{8\mu\sqrt{(\quad)}^3}$$

$$\frac{\frac{\partial^2 u}{\partial x^2}}{\frac{\partial^2 u}{\partial y^2}}\sim\frac{c^2\delta^2}{(a-cx)^2}\sim\frac{\delta^2}{l^2}$$

$$\left.\begin{aligned}\frac{\partial p}{\partial x}&=\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{\mu}{3}\frac{\partial^2 u}{\partial x^2}\\ \frac{\partial p}{\partial y}&=\qquad\frac{\mu}{3}\frac{\partial^2 u}{\partial x\partial y}\end{aligned}\right\}\quad 0=\mu\frac{\partial}{\partial y}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\Delta^2 p=\mu\frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{1}{3}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\right)$$

$$=\frac{4\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)=\frac{4\mu}{3}\frac{df}{dx}$$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f(x)=f(x)$$

$$\frac{\partial}{\partial x}(pu)=0$$

$$pu=f_1(y)=\varphi(y)$$

$$p = \frac{4\mu}{3}\frac{\partial u}{\partial x} + \psi(x,y) \quad \Big\| \quad \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0$$

$$\frac{\partial p}{\partial y} = \frac{4\mu}{3}\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial \psi}{\partial y}$$

$$\boxed{\frac{\partial \psi}{\partial y} = -\mu \frac{\partial^2 u}{\partial x \partial y} = -3\frac{\partial p}{\partial y} \qquad \psi(x,y) = \chi(x) - 3p}$$

$$\cancel{\frac{\partial p}{\partial x} = \frac{\mu}{3}\frac{\partial^2 u}{\partial x^2} + \frac{1}{4}\frac{d\chi}{dx} = \frac{\mu}{3}\frac{\partial^2 u}{\partial x^2} + \mu f(x)}$$

$$p = \frac{\mu}{3}\frac{\partial u}{\partial x} + \frac{1}{4}\chi(x)$$

$$\Delta^2 u = \frac{2}{4\mu}\frac{d\chi}{dx} \quad \Big\| \quad pu = \varphi(y)$$

$$\cancel{\frac{\partial p}{\partial x} = \frac{\mu}{3}\frac{\partial^2}{\partial x^2} + \frac{1}{4}\frac{d}{dx}\chi(x) = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + \frac{\mu}{3}\frac{\partial^2}{\partial x^2}}$$

$$\cancel{\Delta^2 p = \frac{\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) = \frac{\mu}{3}\frac{d^2\chi(x)}{dx^2} = \frac{1}{12}}$$

$$z = \frac{1}{y} \qquad \frac{dz}{dx} = -\frac{1}{y^2}\frac{dy}{dx}$$

$$\cancel{pu = \frac{\mu}{3}u\frac{\partial u}{\partial x} + \frac{1}{4}u\chi(x) = \varphi(y)}$$

$$\cancel{= \frac{\mu}{6}\frac{\partial(u^2)}{\partial x} + \frac{1}{4}u\chi(x)}$$

$$z\frac{dz}{dx} + z X = a$$

$$-\frac{1}{y^3}\frac{dy}{dx} + \frac{1}{y}X = a$$

$$\cancel{\frac{\mu}{3}\left(\frac{\partial u}{\partial x}\right)^2 + \frac{\mu}{3}u\frac{\partial^2 u}{\partial x^2} + \frac{1}{4}\frac{\partial u}{\partial x}\chi(x) + \frac{u}{4}\frac{d\chi}{dx} = 0}$$

$$\frac{dy}{dx} + ay^3 - Xy^2 = 0$$

$$\cancel{\chi\frac{\partial u}{\partial x} + u\frac{\partial \chi}{\partial x} = 0}$$

$$\frac{\varphi(y)}{u} = \frac{\mu}{3}\frac{\partial u}{\partial x} + \frac{1}{4}\chi(x)$$

$$\cancel{\frac{\mu}{3}\frac{u\,du}{\varphi(y) - \frac{u}{4}\chi(x)} = dx}$$

$$\cancel{\frac{\mu}{3}\left(\frac{\partial u}{\partial x}\right)^2}$$

$$\frac{\mu}{3}u\frac{\partial u}{\partial x} + \frac{1}{4}u\chi(x) = \varphi(y)$$

$$\frac{1}{4}\frac{d\chi}{dx} = -\frac{\mu}{3}\frac{\partial^2 u}{\partial x^2} - \frac{\varphi(y)}{u^2}\frac{\partial u}{\partial x} = \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]$$

$$\frac{\partial^2 u}{\partial y^2} = -\frac{4}{3}\frac{\partial^2 u}{\partial x^2} - \frac{\varphi(y)}{u^2\mu}\frac{\partial u}{\partial x} = -\frac{\partial}{\partial x}\left[\frac{4}{3}\frac{\partial u}{\partial x} - \frac{\varphi(y)}{\mu u}\right]$$

$$= \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial u}{\partial r^2}$$

$$\frac{\partial p}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = \frac{dp}{dx} \quad \Big\| \quad \frac{\partial p}{\partial r} = 0$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = r\frac{dp}{dx}$$

$$r\frac{\partial u}{\partial r} = \frac{r^2}{2}\frac{dp}{dx} + \varphi(x)$$

$$\frac{\partial u}{\partial r} = \frac{r}{2}\frac{dp}{dx} + \frac{1}{r}\varphi(x)$$

$$\rho = \frac{p}{R\theta}$$

$$\theta = \theta_0 -$$

$$u = \frac{r^2}{4}\frac{dp}{dx} + \cancel{\varphi(x)\lg(r)} + \psi(x)$$

$$\rho u = fc.(r)$$

$$u = \frac{r^2-\delta^2}{4}\frac{dp}{dx}$$

$$\frac{p}{R\left\{\theta_0 - \frac{(\delta^2-r^2)^2}{128\mu k}\left[\frac{p_1^2-p_2^2}{l}\right]^2\frac{1}{p^2}\right\}}\frac{r^2-\delta^2}{4}\frac{dp}{dx} = fc.(r)$$

$$\frac{(r^2-\delta^2)\quad p^2\, d(p^2)}{8R\left[p^2\theta_0 - \frac{(\delta^2-r^2)^2}{128\mu k}\left[\frac{p_1^2-p_2^2}{l}\right]^2\right]} = fc(r).dx$$

$$\int\frac{z\,dz}{az-b} = \frac{1}{a}\int\left\{dz + \frac{b\,dz}{az-b}\right\} = \frac{1}{a}\left[z + \frac{b}{a}\lg(az-b)\right]$$

$$= \frac{1}{a}\left[p^2 + \frac{b}{a}\lg(ap^2-b)\right]$$

$$\frac{1}{a}\left[p^2 + \frac{b}{a}\lg(ap^2-b)\right] = \frac{8R}{r^2-\delta^2}fc(r).x + \text{const}$$

$$\frac{1}{a}\left[p_1^2 + \frac{b}{a}\lg(ap_1^2-b)\right] = \text{const}$$

$$\frac{1}{a}\left[p_2^2 + \frac{b}{a}\lg(ap_2^2-b)\right] = \frac{8R}{r^2-\delta^2}fc(r)\,l + \text{const}$$

$$\frac{8R}{r^2-\delta^2}fc(r) = \frac{1}{a}\left[p_2^2-p_1^2 + \frac{b}{a}\lg\left(\frac{ap_2^2-b}{ap_1^2-b}\right)\right]\frac{1}{l}$$

Supozycja: $\frac{\partial v_n}{\partial r} \odot \delta$

$$\frac{\partial v_s}{\partial \theta} + v_s \operatorname{ctg}\theta = 0$$

$$\log v_s = -\int \operatorname{ctg}\theta\, d\theta$$

$$= -\log \sin\theta$$

$$v_s \sin\theta = f_c(r)$$

$$V = -cx\left(1+\frac{a^3}{2r^3}\right)$$

$$= -cr\cos\theta\left[1+\frac{a^3}{2r^3}\right]$$

$$\frac{\partial V}{\partial r} = -c\cos\theta\left[1+\frac{a^3}{2r^3}\right] + \frac{3c\cos\theta\, a^3}{2r^3}$$

$$\Delta^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\cos\theta}{r^2\sin\theta}\frac{\partial}{\partial\theta}$$

$$\lambda\frac{\partial}{\partial r}(V+\Phi) = 0$$

$$\Delta^2 V = \frac{\partial^2 V}{\partial r^2} + \left[\frac{1}{a^2}\frac{\partial^2 V}{\partial\theta^2} + \frac{\cos\theta}{a^2\sin\theta}\frac{\partial V}{\partial\theta}\right] = v_n\frac{\partial\varepsilon'}{\partial n} + v_s\frac{\partial\varepsilon'}{\partial\theta}\left[+v_n\frac{\partial v}{\partial n} + v_s\frac{\partial v}{\partial s}\right.$$

$$\Delta^2 V = v \odot \frac{1}{\delta} \qquad \varepsilon' \odot \frac{1}{\delta^2}$$

$$\begin{cases} \dfrac{\partial^2 V}{\partial r^2} = v_n\dfrac{\partial\varepsilon'}{\partial n} + v_s\dfrac{\partial v}{\partial s} \\[2ex] \dfrac{\partial p}{\partial n} = \varepsilon'\dfrac{\partial V}{\partial n} + \left[\mu\dfrac{\partial^2 v_n}{\partial r^2}\right. \\[2ex] \dfrac{\partial p}{\partial s} = \varepsilon'\dfrac{\partial V}{\partial s} + \mu\left(\dfrac{\partial^2 v_s}{\partial r^2} + \dfrac{2}{a}\dfrac{\partial v_s}{\partial r}\right) - \cancel{\dfrac{1}{a}\dfrac{\partial^2 v_n}{\partial r\,\partial\theta}} \end{cases}$$

$$\left.\begin{matrix}\frac{\partial}{\partial s}\\ \frac{\partial}{\partial r}\end{matrix}\right\} 0 = \frac{\partial\varepsilon'}{\partial r}\frac{\partial V}{\partial s} + \frac{\partial}{\partial r}\mu\left(\frac{\partial^2 v_s}{\partial r^2} + \frac{2}{a}\frac{\partial v_s}{\partial r}\right)$$

$$\mu v_s = \frac{\varphi_1 - \varphi_0}{4\pi}\frac{\partial V}{\partial s}$$

$$\mu\frac{\partial^2 v_s}{\partial r^2} = -\varepsilon'\frac{\partial V}{\partial s} = \frac{1}{4\pi}\frac{\partial^2 U'}{\partial r^2}\frac{\partial V}{\partial s}$$

$$\frac{\partial v_s}{\partial r} = \frac{1}{4\pi}\frac{\partial U'}{\partial r}\frac{\partial V}{\partial s} + f(s)$$

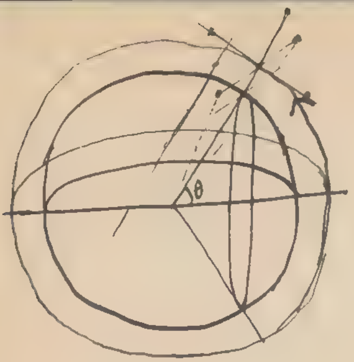
$$v_s = \frac{\varphi_1 - U'}{4\pi}\frac{\partial V}{\partial s} + \int f(s) = \frac{f_c(r)}{\sin\theta}$$

$$\frac{\partial v_n}{\partial r} + \frac{\partial v_s}{\partial s} + \frac{v_s \operatorname{ctg}\theta}{a} = 0 \int dr$$

$$v_n +$$

$$\frac{\partial V}{\partial s} = \frac{f_c(r)}{\sin\theta} + \delta$$

$$V = f(r)\int\frac{d\theta}{\sin\theta} + \delta\ldots$$

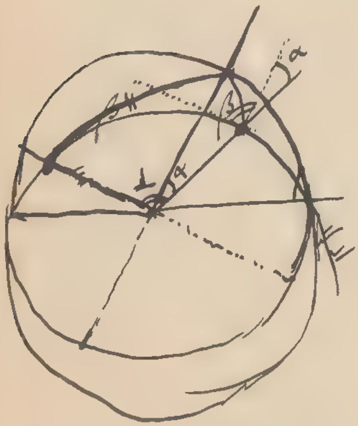


v_n $\quad$ v_s $\qquad\qquad$ $\cos\theta\cos\theta_1 - \sin\theta\sin\theta_1 = \cos(\theta+\theta_1)$

$\cancel{v_\xi = v_n \cos\alpha}$

$$v_\xi = v_n\left[\sin\theta_1\cos\theta + \cos\theta_1\sin\theta\cos\varphi\right] + v_s\left[\sin\theta_1\sin\theta + \cos\theta_1\cos\theta\cos\varphi\right]$$

$$v_\xi = v_n\cos\alpha + v_s\cos\beta$$



$$\cos\alpha = \cos\theta\cos\theta_1 + \sin\theta\sin\theta_1\cos\varphi$$

$$\cancel{\cos\beta = \cos\alpha \ldots + \sin\alpha \ldots \quad \frac{\cos\theta\cos\alpha - \cos\theta_1}{\sin\theta\sin\alpha}}$$

$$\cancel{= \frac{\cos^2\theta\cos\theta_1 + \sin\theta\cos\theta\sin\theta_1\cos\varphi - \cos\theta_1}{\sin\theta}}$$

$$\cos\beta = -\sin\theta\cos\theta_1 + \sin\theta_1\cos\theta\cos\varphi$$

$$v_\xi = v_n(\cos\theta\cos\theta_1 + \sin\theta\sin\theta_1\cos\varphi) + v_s(-\sin\theta\cos\theta_1 + \sin\theta_1\cos\theta\cos\varphi)$$

$$v_\xi = \cos\theta_1(v_n\cos\theta - v_s\sin\theta) + \sin\theta_1(v_n\sin\theta + v_s\cos\theta)\cos\varphi$$

$$\Delta^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\cos\theta}{r^2\sin\theta}\frac{\partial}{\partial\theta}$$

$$\Delta^2 v_\xi = \cos\theta_1\left[\frac{\partial^2 v_n}{\partial r^2}\cos\theta - \frac{\partial^2 v_s}{\partial r^2}\sin\theta\right] + \sin\theta_1\left[\frac{\partial^2 v_n}{\partial r^2}\sin\theta + \frac{\partial^2 v_s}{\partial r^2}\cos\theta\right]\cos\varphi +$$

$$+ \frac{2}{a}\cos\theta_1\left[\frac{\partial v_n}{\partial r}\cos\theta - \frac{\partial v_s}{\partial r}\sin\theta\right] + \frac{2}{a}\sin\theta_1\left[\frac{\partial v_n}{\partial r}\sin\theta + \frac{\partial v_s}{\partial r}\cos\theta\right]\cos\varphi \; +$$

$$-\frac{1}{a^2\sin^2\theta}\sin\theta_1\left[v_n\sin\theta + v_s\cos\theta\right]\cos\varphi +$$

$$\cancel{+\frac{1}{a^2}\ldots\left[v_n\cos\theta \ldots\right.}$$

$$v_r = -\cos\theta\left[1-\frac{a^3}{r^3}\right] \qquad v_\theta = +\frac{3}{2}\sin\theta$$

$$\frac{\partial v_r}{\partial r} = -\cos\theta\ \frac{3a^3}{r^4} \qquad \frac{\partial v_r}{\partial\theta} = +\sin\theta\left[1-\frac{a^3}{r^3}\right] \qquad \frac{\partial v_\theta}{\partial\theta} = \frac{3}{2}\cos\theta$$

$$\frac{\partial^2 v_r}{\partial r^2} = +\cos\theta\ \frac{12a^3}{r^5} \qquad \frac{\partial^2 v_r}{\partial\theta^2} = +\cos\theta\left[1-\frac{a^3}{r^3}\right]$$

$$\Delta^2 v_\xi = +12\frac{\cos\theta}{a^2} \pm 6\frac{\cos\theta}{a^2} - \frac{3\cos\theta}{a^2} - \frac{3}{a^2}\cos\theta = 0$$

Wogóle ~~...~~ ~~...~~ na powierzchni (gdzie $v=0$):

$$\Delta^2 v\xi = \frac{\partial^2 v_r}{\partial r^2} + \frac{2}{a}\frac{\partial v_r}{\partial r} \qquad \Delta^2 v\xi = \frac{\partial^2 v_\theta}{\partial r^2} + \frac{2}{a}\frac{\partial v_\theta}{\partial r}$$

$$v_\theta = \sin\theta\left[\frac{r}{a} + \frac{a^2}{2r^2}\right]$$

$$\frac{\partial v_\theta}{\partial r} = \sin\theta\left[\frac{1}{a} - \frac{a^2}{r^3}\right]\Big| = 0 \qquad \frac{\partial v_\theta}{\partial\theta} = \cos\theta\left[\quad\right]$$

$$\frac{\partial^2 v_\theta}{\partial r^2} = \sin\theta\left[\frac{3a^2}{r^4}\right] \qquad \frac{\partial^2 v_\theta}{\partial\theta^2} = -\sin\theta\left[\quad\right]$$

$$\Delta^2 v\xi = \frac{3}{a^2}\sin\theta - \frac{3}{2a^2}\frac{\sin\theta}{\sin^2\theta} - \frac{\sin\theta}{a^2}\frac{3}{2} + \frac{1}{a^2}\frac{3}{2}\frac{\cos^2}{\sin}$$

$$= \frac{\sin\theta}{a^2}\left[\frac{3}{2} - \frac{3}{2\sin^2} + \frac{3}{2}\frac{\cos^2}{\sin^2}\right] = \cancel{\frac{3}{2a^2\sin}}\left[0\right] = 0$$

$$\cos\theta_1\left[\frac{\partial v_n}{\partial\theta}\cos\theta-\frac{\partial v_s}{\partial\theta}\sin\theta - v_n\sin\theta - v_s\cos\theta\right] + \sin\theta_1\left[\frac{\partial v_n}{\partial\theta}\sin\theta+\frac{\partial v_s}{\partial\theta}\cos\theta + v_n\cos\theta \pm v_s\sin\theta\right]\cos\varphi$$

$$\frac{\cos\theta_1}{a^2\sin\theta_1}\left[\frac{\partial v_n}{\partial\theta}(\cos^2\theta_1+\sin^2\theta_1\cos\varphi) - \cancel{\frac{\partial v_s}{\partial\theta}(1-\cos\varphi)\sin\theta_1\cos\theta_1} \pm v_n\,\cancel{\sin\theta_1\cos\theta_1(1-\cos\varphi)} \pm v_s(\cos^2\theta_1+\sin^2\theta_1\cos\varphi)\right]$$

$$\cos\theta_1\left[\frac{\partial^2 v_n}{\partial\theta^2}\cos\theta-\frac{\partial^2 v_s}{\partial\theta^2}\sin\theta - 2\frac{\partial v_n}{\partial\theta}\sin\theta - 2\frac{\partial v_s}{\partial\theta}\cos\theta - v_n\cos\theta + v_s\sin\theta\right] + \sin\theta_1\cos\varphi\left[\frac{\partial^2 v_n}{\partial\theta^2}\sin\theta+\frac{\partial^2 v_s}{\partial\theta^2}\cos\theta + 2\frac{\partial v_n}{\partial\theta}\cos\theta - 2\frac{\partial v_s}{\partial\theta}\sin\theta - v_n\sin\theta - v_s\cos\theta\right]$$

$$\frac{1}{a^2}\left\{\frac{\partial^2 v_n}{\partial\theta^2}(\cos^2\theta_1+\sin^2\theta_1\cos\varphi) - \cancel{\frac{\partial^2 v_s}{\partial\theta^2}(1-\cos\varphi)\sin\theta_1\cos\theta_1} - 2\,\cancel{\frac{\partial v_n}{\partial\theta}\sin\theta_1\cos\theta_1(1-\cos\varphi)} - 2\frac{\partial v_s}{\partial\theta}(\cos^2\theta_1+\sin^2\theta_1\cos\varphi) - v_n(\cos^2\theta_1+\sin^2\theta_1\cos\varphi) + \cancel{v_s\sin\theta_1\cos\theta_1(1-\cos\varphi)}\right\}$$

$\varphi=0 \quad \cos\varphi = 1$

$\cos^2\theta_1+\sin^2\theta_1\cos\varphi =$

$$\Delta^2 v_\xi = \frac{\partial^2 v_n}{\partial r^2} + \frac{2}{a}\frac{\partial v_n}{\partial r} - \frac{v_n}{a^2} - \frac{v_s}{a^2}\operatorname{ctg}\theta_1 + \frac{1}{a^2}\left(\frac{\partial^2 v_n}{\partial\theta^2} - 2\frac{\partial v_s}{\partial\theta} - v_n\right) + \frac{1}{a^2}\operatorname{ctg}\theta_1\left[\frac{\partial v_n}{\partial\theta} - v_s\right]$$

$$\Delta^2 v_\xi = \frac{\partial^2 v_n}{\partial r^2} + \frac{2}{a}\frac{\partial v_n}{\partial r} - \frac{2v_n}{a^2} + \frac{1}{a^2}\frac{\partial^2 v_n}{\partial\theta^2} + \frac{1}{a^2}\operatorname{ctg}\theta_1\frac{\partial v_n}{\partial\theta} - \frac{2v_s}{a^2}\operatorname{ctg}\theta_1 - \frac{2}{a^2}\frac{\partial v_s}{\partial\theta}$$

Jestli $v_n = 0 \quad \frac{\partial v_n}{\partial r} \odot \delta \quad \frac{\partial^2 v_n}{\partial r^2} \odot 1$

$$\Delta^2 v_\xi = \frac{\partial^2 v_n}{\partial r^2} - \cancel{\frac{2v_s}{a^2}\operatorname{ctg}\theta_1 - \frac{2}{a}\frac{\partial v_s}{\partial s}}$$

$$= \frac{\partial^2 v_n}{\partial r^2} + \left(\frac{2}{a}+\frac{2r}{a^2}\right)\frac{\partial v_n}{\partial r} + \frac{2v_n}{a^2} + \frac{1}{a^2}\frac{\partial^2 v_n}{\partial\theta^2} + \frac{1}{a^2}\operatorname{ctg}\theta\frac{\partial v_n}{\partial\theta}$$

$$\Delta^2 v_\xi = \frac{\partial^2 v_s}{\partial r^2} + \frac{2}{a}\frac{\partial v_s}{\partial r} + 1$$

$$- \frac{1}{a^2 \sin^2\theta_1}\left[v_n \cos\theta_1 \sin\theta_1 + v_s \sin^2\theta_1\right]$$

$$+ \frac{\cos\theta_1}{a^2\sin\theta_1}\left\{\frac{\partial v_n}{\partial\theta}\left[\cancel{-\sin\theta_1\cos\theta + \cos\theta_1\sin\theta\cos\varphi}\right] + \frac{\partial v_s}{\partial\theta}\left[\sin\theta_1\sin\theta + \cos\theta_1\cos\theta\cos\varphi\right]\right.$$
$$\left. + v_n\left[\sin\theta_1\sin\theta + \cos\theta_1\cos\theta\cos\varphi\right] + v_s\left[\cancel{\sin\theta_1\cos\theta - \cos\theta_1\sin\theta\cos\varphi}\right]\right\}$$

$$+ \frac{1}{a^2}\left\{\cancel{\frac{\partial^2 v_n}{\partial\theta^2}} + \frac{\partial^2 v_s}{\partial\theta^2} + 2\frac{\partial v_n}{\partial\theta}\left[\sin^2\theta_1 + \cos^2\theta_1\cos\varphi\right] + \cancel{} + \cancel{v_n \sin\theta_1\cos\theta_1} - v_s\left(\sin^2\theta_1 + \cos^2\theta_1\cos\varphi\right)\right\}$$

$$= \frac{\partial^2 v_s}{\partial r^2} + \frac{2}{a}\frac{\partial v_s}{\partial r} - \cancel{\frac{v_n}{a^2}\operatorname{ctg}\theta_1} - \frac{v_s}{a^2}\operatorname{ctg}^2\theta_1 + \cancel{\frac{v_n}{a^2}\operatorname{ctg}\theta_1} + \frac{\partial v_s}{\partial\theta}\frac{\operatorname{ctg}\theta_1}{a^2}$$
$$+ \frac{1}{a^2}\frac{\partial^2 v_s}{\partial\theta^2} + \frac{2}{a^2}\frac{\partial v_n}{\partial\theta} - \frac{v_s}{a^2}$$

$$\boxed{1 + \operatorname{ctg}^2 = 1 + \frac{\cos^2}{\sin^2} = \frac{1}{\sin^2}}$$

$$= \frac{\partial^2 v_s}{\partial r^2} + \frac{2}{a}\frac{\partial v_s}{\partial r} - \frac{v_s}{a^2\sin^2\theta_1} + \frac{1}{a^2}\frac{\partial^2 v_s}{\partial\theta^2} + \frac{1}{a^2}\frac{\partial v_s}{\partial\theta}\operatorname{ctg}\theta_1 + \frac{2}{a^2}\frac{\partial v_n}{\partial\theta}$$

Np.

$$\Delta r \frac{\partial}{\partial r}\left(4\pi r^2 v_n\right).\sin\theta\,\Delta\theta + 4\pi r \Delta r\frac{\partial}{\partial\theta}\left(\sin\theta . v_s\right)\Delta\theta = 0$$

$$\sin\theta\frac{\partial}{\partial r}\left(v_n . r^2\right) + r\frac{\partial}{\partial\theta}\left(v_s\sin\theta\right) = 0$$

$$2\frac{v_n}{r} + r\frac{\partial v_n}{\partial r} + \frac{\partial v_s}{\partial\theta} + v_s\operatorname{ctg}\theta = 0 \quad = \operatorname{div}$$

$$\frac{\cos}{\sin} : -\frac{\cos^2}{\sin^2} - 1$$

$$\frac{\partial}{\partial\theta} : \quad \frac{2}{a^2}\frac{\partial v_n}{\partial\theta} + \frac{r}{a^2}\frac{\partial^2 v_n}{\partial r\,\partial\theta} + \frac{\partial^2 v_s}{a^2\partial\theta^2} + \frac{\partial v_s}{a^2\partial\theta}\operatorname{ctg}\theta - \frac{v_s}{a^2\sin^2\theta} = 0$$

$$\Delta^2 v_\xi = \frac{\partial^2 v_s}{\partial r^2} + \frac{2}{a}\frac{\partial v_s}{\partial r} - \frac{r}{a^2}\frac{\partial^2 v_n}{\partial r\,\partial\theta}$$

$$\frac{\partial p}{\partial x} = \mu \nabla^2 u + \frac{f}{r^2}\frac{x}{r}$$
$$\frac{\partial p}{\partial y} = \mu \nabla^2 v + \frac{f}{r^2}\frac{y}{r}$$
$$\frac{\partial p}{\partial z} = \mu \nabla^2 w + \frac{f}{r^2}\frac{z}{r}$$

$$\rho\left(\frac{\partial u}{\partial x} + \quad\right) + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} = 0$$

$$\left(u\frac{\partial p}{\partial x} + \cdots\right) - \frac{k}{k-1}R\rho\left(u\frac{\partial \theta}{\partial x} + \quad\right) + \mu\Phi + \kappa\Delta^2\theta = 0$$

$$\frac{1}{a}\left[p^2 + \frac{b}{a}\log(ap^2-b)\right] = \frac{1}{a}\left[p_2^2 - p_1^2 + \frac{b}{a}\log\left(\frac{ap_2^2-b}{ap_1^2-b}\right)\right]\frac{x}{l} + \frac{1}{a}\left[p_1^2 + \frac{b}{a}\log(ap_1^2-b)\right]$$

$$p^2 - p_1^2 + \frac{b}{a}\log\frac{ap^2-b}{ap_1^2-b} = \left[p_2^2 - p_1^2 + \frac{b}{a}\log\left(\frac{ap_2^2-b}{ap_1^2-b}\right)\right]\frac{x}{l}$$

$$2\pi\int \rho u\, r\,dr = \frac{2\pi}{R}\int p\frac{u}{\theta} r\,dr \quad \frac{2\pi}{R}\int\frac{r^2-\delta^2}{4}\,p\,\frac{dp}{dx}\,r\,dr$$

$$= \frac{2\pi}{R}\int r\,dr\left[\theta_0 - \frac{(\delta^2-r^2)^2}{16\mu\kappa}\left(\frac{p_2^2-p_1^2}{l}\right)^2\frac{1}{p^2}\right]\frac{(r^2-\delta^2)}{8R}\;\frac{1}{\theta_0}\left[p_2^2-p_1^2+\frac{b}{a}\log\left(\frac{ap_2^2-b}{ap_1^2-b}\right)\right]$$

$$\log(ap^2-b) = \log ap^2 + \log\left(1-\frac{b}{ap^2}\right) = \log(ap^2) - \frac{b}{ap^2}$$

$$u^2 = \frac{(y\delta-yy)^2c^2}{16\mu^2(a-cx)}$$

$$\frac{(y\delta-y^2)^2c^3}{16\mu^2(a-cx)^2}\frac{1}{Ru}$$

$$\frac{1}{\rho}\frac{\partial u}{\partial x} = \frac{R\theta}{p}\frac{\partial u}{\partial x}$$
$$= \frac{R\theta\,(r^2-\delta^2)\,c^2}{16\mu\sqrt{(\quad)^3}}$$

$$\frac{R\theta\,c^2(r^2-\delta^2)}{16\mu(a-cx)^2} = \frac{(y\delta-y^2)c^2}{4\mu(a-cx)}$$

$$\frac{\partial}{\partial x}\left(\frac{u^2}{2}\right) = \int\frac{(r^2-\delta^2)^2c^3}{128\mu^2(a-cx)^2}\,r\,dr =$$

$$\int\frac{\partial u}{\partial x}\cdot u\,r\,dr = \frac{r^2-\delta^2\cdot c^3}{(a-cx)^2}$$

$$\int p\,u\,r\,dr = \frac{k}{k-1}\frac{\Delta\theta}{\theta}\;\frac{c\int(r^2-\delta^2)r\,dr}{\delta_r}$$